# COE 202, Term 162 <br> Fundamentals of Computer Engineering 

## Quiz\# 4 Solution

Date: Sunday, April 16

Q1. In designing a combinational circuit that computes the function $f(X)=X^{2}-X$ for a 3-bit 2's complement signed number $X$, where the output $f(X)$ is an un-signed integer:
(i) How many bits do we need for the output?
$X$ that produces that largest $f(x)$ is -4 . In this case $f(x)=16+4=20$. So, the number of bit needed for the output is 5 bits.
(ii) Obtain the truth table for this circuit.

| $\mathrm{X}_{2} \mathrm{X}_{1} \mathrm{X}_{0}$ |  |  | Decimal value of $X$ | Decimal value of $f(X)$ | $\mathrm{F}_{4} \mathrm{~F}_{3} \mathrm{~F}_{2} \mathrm{~F}_{1} \mathrm{~F} 0$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0000 | 0 |
| 0 | 0 | 1 | +1 | 0 | 0000 | 0 |
| 0 | 1 | 0 | +2 | 2 | 0001 | 0 |
| 0 | 1 | 1 | +3 | 6 | 00110 | 0 |
| 1 | 0 | 0 | -4 | 20 | 10100 | 0 |
| 1 | 0 | 1 | -3 | 12 | 0110 | 0 |
| 1 | 1 | 0 | -2 | 6 | 00110 | 0 |
| 1 | 1 | 1 | -1 | 2 | 0001 | 0 |

(iii) Obtain simplified Boolean expressions of the circuit outputs in SOP form.
[4 points]
$F_{4} F_{3} F_{1} F_{0} C_{0}$ be obtained directly from the truth table (no minimization can be done)
$F_{4}=X_{2} X_{1}^{\prime} X_{0}^{\prime}$
$F_{3}=X_{2} X_{1}^{\prime} X_{0}$
$F_{1}=X_{1}$
$F_{0}=0$

K-map for $\mathrm{F}_{2}$ :

$$
F_{2}=X_{2}^{\prime} X_{1} X_{0}+X_{2} X_{1}^{\prime}+X_{2} X_{0^{\prime}}
$$

| $\mathrm{XI}^{\prime}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 00 | 01 | 11 | 10 |
| $\mathrm{X}^{\prime}{ }^{\prime} 0$ | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 |

Q2.
(i) What is the minimum number of bits needed to represent integers in the range from -100 to +100 using sign-magnitude representation?

## 8-bits

(ii) Show the binary representations of $\mathbf{+ 4 9}$ and $\mathbf{- 4 9}$ using $\mathbf{1 0 - b i t s}$ signed-magnitude, 1 's complement and 2's complement representations (record your answers in the table below). [4 points]

| Decimal | Binary Signed-magnitude representation | Binary Signed-1's complement representation | Binary Signed-2's complement representation |
| :---: | :---: | :---: | :---: |
| -49 | 1_000_110_001 | 1_111_001_110 | 1_111_001_111 |
| + 49 | 0_000_110_001 | 0_000_110_001 | 0_000_110_001 |

(iii) Perform the following operations on $\mathbf{6}$-bits signed numbers using 2'complement representation. Check for overflow and mark clearly any overflow occurrences. [4 points]


