# COE 202, Term 112 <br> Digital Logic Design 

## Quiz\# 2

Date: Monday, Feb. 18

Q1. Simplify the following Boolean functions to the minimum number of literals sum of product s expressions using algebraic manipulation:
(i) $\quad \mathbf{A} \mathbf{D}^{\prime}+\mathbf{A}^{\prime} \mathbf{B} \mathbf{D}^{\prime}+\mathbf{A}^{\prime} \mathbf{C}^{\prime} \mathbf{D}^{\prime}+\mathbf{B}^{\prime} \mathbf{C}^{\prime} \mathbf{D}^{\prime}$

$$
\begin{array}{ll}
=\mathrm{D}^{\prime}\left(\mathrm{A}+\mathrm{A}^{\prime} \mathrm{B}+\mathrm{A}^{\prime} \mathrm{C}+\mathrm{B}^{\prime} \mathrm{C}^{\prime}\right) & \text { by distributive law } \\
=\mathrm{D}^{\prime}\left(\mathrm{A}+\mathrm{A}^{\prime} \mathrm{B}+\mathrm{B}+\mathrm{A}^{\prime} \mathrm{C}+\mathrm{C}+\mathrm{B}^{\prime} \mathrm{C}^{\prime}\right) & \text { by consensus } \\
=\mathrm{D}^{\prime}\left(\mathrm{A}+\mathrm{B}+\mathrm{C}+\mathrm{B}^{\prime} \mathrm{C}^{\prime}\right) & \text { by absorption } \\
=\mathrm{D}^{\prime}\left(\mathrm{A}+\mathrm{B}+\mathrm{C}+\mathrm{B}^{\prime}\right) & \text { by consensus } \\
=\mathrm{D}^{\prime}(1) & \\
=\mathrm{D}^{\prime} &
\end{array}
$$

(ii) $\quad\left\{\left[\left(\begin{array}{ll}\mathbf{A} & \left.)^{\prime} \mathbf{A}\right]^{\prime} \\ {\left[(\mathbf{A ~ B})^{\prime}\right.} & \left.\mathbf{B}]^{\prime}\right\}^{\prime}\end{array}\right.\right.\right.$

$$
\begin{array}{ll}
=(\mathrm{A} \mathrm{~B})^{\prime} \mathrm{A}+(\mathrm{A} \mathrm{~B})^{\prime} \mathrm{B} & \\
=(\mathrm{A} B)^{\prime}(\mathrm{A}+\mathrm{B}) & \\
=\left(\mathrm{A}^{\prime}+\mathrm{B}^{\prime}\right)(\mathrm{A}+\mathrm{B}) & \text { by distributive law } \\
=\mathrm{A}^{\prime} \mathrm{A}+\mathrm{A}^{\prime} \mathrm{B}+\mathrm{A} \mathrm{~B}^{\prime}+\mathrm{B} \mathrm{~B}^{\prime} & \\
=\mathrm{A}^{\prime} \mathrm{B}+\mathrm{A} \mathrm{~B} \text { Demorgan's law }
\end{array}
$$

(iii) $\quad(\mathrm{A}+\mathrm{B}+\mathrm{C})\left(\mathrm{A}+\mathrm{B}+\mathrm{C}^{\prime}\right)\left(\mathrm{A}^{\prime}+\mathrm{C}^{\prime}\right)\left(\mathrm{B}+\mathrm{C}^{\prime}\right)$

We first take the dual to make simplification easier.
Dual $=\mathrm{ABC}+\mathrm{ABC} \mathrm{C}^{\prime}+\mathrm{A}^{\prime} \mathrm{C}^{\prime}+\mathrm{BC}^{\prime}$

$$
\begin{array}{lr}
=\mathrm{A} \mathrm{~B}\left(\mathrm{C}+\mathrm{C}^{\prime}\right)+\mathrm{A}^{\prime} \mathrm{C}^{\prime}+\mathrm{B} \mathrm{C} \mathrm{C}^{\prime} & \text { by distributive law } \\
=\mathrm{A} \mathrm{~B}+\mathrm{A}^{\prime} \mathrm{C}^{\prime}+\mathrm{B} \mathrm{C} \mathrm{C}^{\prime} & \\
=\mathrm{A} \mathrm{~B}+\mathrm{A}^{\prime} \mathrm{C}^{\prime} & \text { by consensus law }
\end{array}
$$

We now take the dual again:

$$
\begin{aligned}
& (\mathrm{A}+\mathrm{B})\left(\mathrm{A}^{\prime}+\mathrm{C}^{\prime}\right) \\
& =\mathrm{A} \mathrm{~A}^{\prime}+\mathrm{A}^{\prime}+\mathrm{A}^{\prime} \mathrm{B} \\
& =\mathrm{A} \mathrm{C}^{\prime}+\mathrm{A}^{\prime} \mathrm{B}+\mathrm{B} \mathrm{C}^{\prime}
\end{aligned}
$$

$$
=\mathrm{A}^{\prime}+\mathrm{A} \mathrm{C}^{\prime}+\mathrm{A}^{\prime} \mathrm{B}+\mathrm{B} \mathrm{C}^{\prime} \quad \text { by distributive law }
$$

$$
=\mathrm{A} \mathrm{C}^{\prime}+\mathrm{A}^{\prime} \mathrm{B} \quad \text { by consensus law }
$$

