## COE 202, Term 162

## Digital Logic Design

## Assignment\# 2 Solution

Due date: Sunday, April 9
Q.1. It is required to design an iterative combinational circuit that computes the equation $\mathrm{Z}=2 * \mathrm{X}-\mathrm{Y}$, where X and Y are n -bit unsigned numbers.
(i) Determine the number of inputs and outputs needed for your 1-bit cell.

This circuit requires the following 3 pieces of information, which can be encoded using 2 signals:

- No carry or borrow $(\mathrm{C}=0, \mathrm{~B}=0)$
- Carry = $1(\mathrm{C}=1, \mathrm{~B}=0)$
- Borrow $=1(\mathrm{C}=0, \mathrm{~B}=1)$

(ii) Derive the truth table of your 1-bit cell.

| Bin | Cin | X | Y | Bout | Cout | Z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | X | X | X |
| 1 | 1 | 0 | 1 | X | X | X |
| 1 | 1 | 1 | 0 | X | X | X |
| 1 | 1 | 1 | 1 | X | X | X |

(iii) Derive minimized equations for your 1-bit using K-Map method.

|  | $\mathbf{0 0}$ | $\mathbf{0 1}$ | $\mathbf{1 1}$ | $\mathbf{1 0}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{0 0}$ | 0 | 0 | 1 | 1 |
| $\mathbf{1}$ | 3 | 0 | 0 |  |
| $\mathbf{0 1}$ | 1 | 4 | 0 | 5 |
| 0 | 0 | 16 |  |  |
| $\mathbf{1 1}$ | $? 12$ | $? 13$ | $? 15$ | $?$ |
| $\mathbf{1 0}$ | $\mathbf{1}$ | 8 | 0 | 9 |

$\mathrm{Z}=$ Bin' $^{\prime} \operatorname{Cin}{ }^{\prime} \mathrm{Y}+\mathrm{Cin} \mathrm{Y}^{\prime}+$ Bin $\mathrm{Y}^{\prime}=$ Bin' $\operatorname{Cin}{ }^{\prime} \mathrm{Y}+\mathrm{Y}^{\prime}(\mathrm{Cin}+\mathrm{Bin})=\mathrm{Y} \oplus(\mathrm{Cin}+\mathrm{Bin})$

|  | 00 | 01 | 11 | 10 |
| :---: | :---: | :---: | :---: | :---: |
| 00 | 00 | 01 | 0 | 12 |
| 01 | 04 | 05 | 1 | 1 |
| 11 | $? 12$ | ? 13 | ? 15 | $?$ |
| 10 | 08 | 09 | 01 | 0 |

Cout $=\operatorname{Cin} \mathrm{X}+$ Bin' $^{\prime} \mathrm{X} \mathrm{Y}^{\prime}$

|  | $\mathbf{0 0}$ | $\mathbf{0 1}$ | $\mathbf{1 1}$ | $\mathbf{1 0}$ |
| :--- | :---: | :---: | :---: | :---: |
| $\mathbf{0 0}$ | 00 | 1 | 1 | 0 |
| 3 | 0 | 2 |  |  |
| $\mathbf{0 1}$ | 0 | 4 | 0 | 5 |
| 0 | 7 | 06 |  |  |
| $\mathbf{1 1}$ | $? 12$ | $? 13$ | $? 15$ | $? 14$ |
| $\mathbf{1 0}$ | $\mathbf{1}$ | 8 | $\mathbf{1}$ | $0_{1}$ |

Bout $=\operatorname{Bin} \mathrm{X}^{\prime}+\operatorname{Cin} \mathrm{X}^{\prime} \mathrm{Y}$
(iv) Write a Verilog model for modeling your 1-bit cell by using an assign statement for each output.
module Cell2XMY (output Bout, Cout, Z, input Bin, Cin, X, Y);
$\operatorname{assign} \mathrm{Z}=\mathrm{Y}^{\wedge}(\mathrm{Cin} \mid \operatorname{Bin})$;
assign Cout $=\operatorname{Cin} \& X \mid \sim \operatorname{Bin} \& X \& \sim Y$;
assign Bout $=\operatorname{Bin} \& \sim \mathrm{X} \mid \sim \operatorname{Cin} \& \sim \mathrm{X} \& \mathrm{Y}$;
endmodule
(v) Write a Verilog model for modeling a 4-bit circuit based on the 1-bit model you have.
module D2XMY (output Bout, Cout, output [3:0] Z, input [3:0] X, Y);
wire [2:0] C, B;
Cell2XMY M1 (B[0], C[0], Z[0], 1'b0, 1'b0, X[0], Y[0]);
Cell2XMY M2 (B[1], C[1], Z[1], B[0], C[0], X[1], Y[1]);
Cell2XMY M3 (B[2], C[2], Z[2], B[1], C[1], X[2], Y[2]);
Cell2XMY M4 (Bout, Cout, Z[3], B[2], C[2], X[3], Y[3]);
endmodule
(vi) Write a Verilog test bench to test the correctness of your design for the following input values: $\{\mathrm{X}=1, \mathrm{Y}=1\}, \quad\{\mathrm{X}=3, \mathrm{Y}=2\},\{\mathrm{X}=5, \mathrm{Y}=1\},\{\mathrm{X}=4, \mathrm{Y}=5\}$, and $\{\mathrm{X}=15$, $\mathrm{Y}=15$ \}.
module D2XMY_Test();
reg [3:0] X, Y;
wire Bout, Cout;
wire [3:0] Z;
D2XMY M1 (Bout, Cout, Z, X, Y);
initial begin
$\mathrm{X}=4 \mathrm{~b} 0001 ; \mathrm{Y}=4$ 'b0001;
\#100 X=4'b0011; Y=4'b0010;
\#100 X=4'b0101; Y=4'b0001;
\#100 X=4'b0100; Y=4'b0101;
\#100 X=4'b1111; Y=4'b1111;
end
endmodule

The simulations results are shown below and it is clear that the circuit is implementing the function $\mathrm{Z}=2 * \mathrm{X}-\mathrm{Y}$ correctly.

| $\pm$ /- /D2XMY_Test/X | 1111 | 0001 | 0011 | 0101 | 0100 | 1111 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\pm$ /D2XMY_Test/Y | 1111 | 0001 | 0010 | 0001 | 0101 | 1111 |
| $\pm \downarrow / \mathrm{D} 2 \mathrm{XMY}$ _Test/Z | 1111 | 0001 | 0100 | 1001 | 0011 | 1111 |
| 4 /D2XMY_Test/Cout | St0 |  |  |  |  |  |
| 4/D2XMY_Test/Bout | St0 |  |  |  |  |  |
|  |  |  |  |  |  |  |

