COE 464

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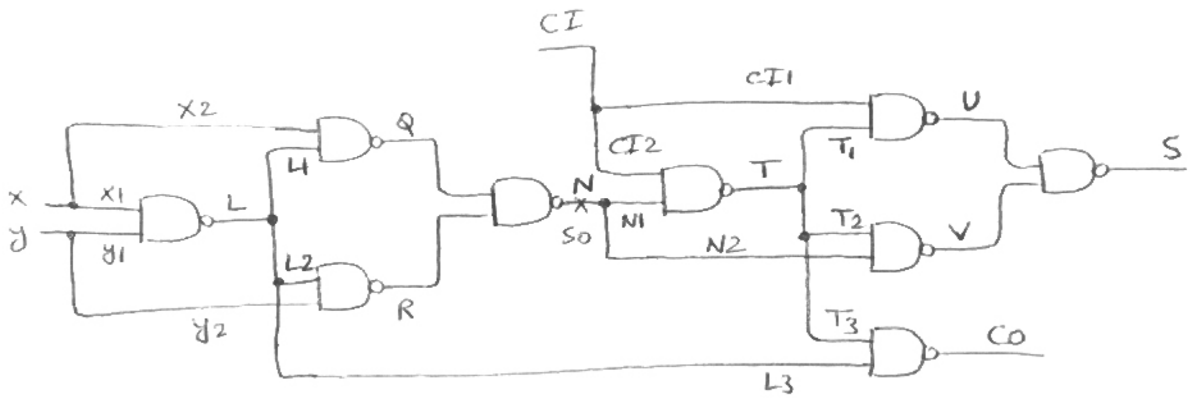
HW #4

Q1



(1) Controllability Costs

line	C0	C1
X	1	1
X1	1	1
X2	1	1
y	1	1
y1	1	1
y2	1	1
CI	1	1
CI1	1	1
CI2	1	1
L	2	1
L1	2	1
L2	2	1
Q	2	1
R	2	1
N	2	2
N1	2	2
N2	2	2
T	3	1
T1	3	1
T2	3	1
T3	3	1
L3	2	1
Co	2	2
U	2	1
V	3	2
S	3	2

Observability Costs

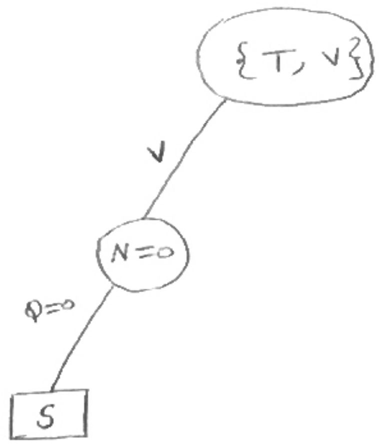
line	O
S	0
Co	0
U	2
V	1
CI1	3
T1	3
T2	3
N2	2
T3	1
L3	1
T	1
CI2	3
N1	2
CI	3
N	2
Q	3
R	3
X2	4
L1	4
L2	4
y2	4
L	1
X1	2
y1	2
X	2
y	2

(ii) D- Algorithm:

Decision Assignments	Implications	D-F	J-F	Comment
	$N=1$, $N1=D$ $N2=D$	$\{T, V\}$	$\{N\}$	Activate the fault
$T2=1$	$T=1$, $V=\overline{D}$, $U=1$, $S=D$, $CI2=0$, $CI=0$, $CI1=0$, $T1=1$, $T3=1$	ϕ	$\{N\}$	V is selected
$Q=0$	$X2=1$, $X=1$, $X1=1$, $L1=1$, $L=1$, $L2=1$, $L3=1$, $C0=0$, $y1=0$, $y=0$, $y2=0$, $R=1$	ϕ	ϕ	Q is selected success

So, a test is generated successfully for the fault as $\{X, y, CI\} = (1, 0, 0)$

The decision tree is as follows:

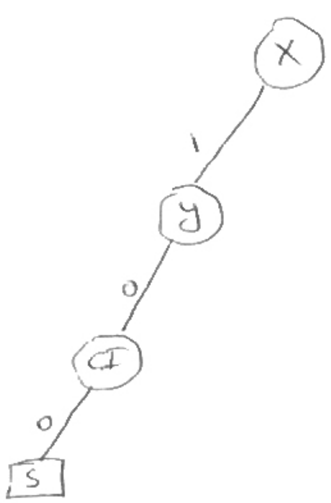


(iii) PODEM algorithm:

objective	PI assignments	Implications	D-F	Comment
$N=1$	$x=1$	$x_1=1$, $x_2=1$	ϕ	Q is selected x_2 is selected
$N=1$	$y=0$	$y_1=0$, $y_2=0$, $L=1$, $L1=1$, $L2=1$, $Q=0$, $R=1$, $N=1$, $L3=1$	{T, V}	Q is selected
$T_2=1$	$CI=0$	$CI1=0$, $CI2=0$, $T=1$, $T1=1$, $T2=1$, $U=1$, $V=\bar{D}$, $S=D$, $T3=1$, $CO=0$		V is selected success

The test $(x, y, CI) = (1, 0, 0)$ is generated successfully for the fault N s-a-0.

The Decision tree is:



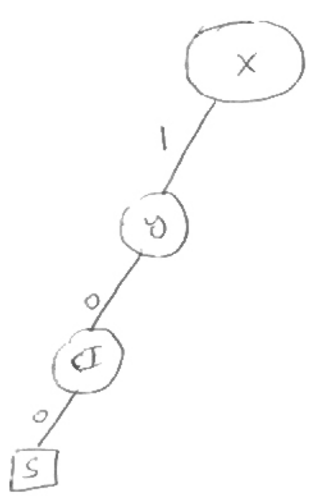
(iv) FAN Algorithm:

current objectives	Processed Entry	Stem objective	Head objective	Assignment	Implications	D-f	δ-f
					N=1, N1=D, N2=D	{T, V}	{N}
						{T, V}	{N}
{N, T_2}	T_2	T				{T, V}	{N}
{N}	N	T				{T, V}	{N}
{Q}	Q	T				{T, V}	{N}
{X2, L1}	L1	T, L				{T, V}	{N}
{X2}	X2	T, L, X				{T, V}	{N}
{T}	T	L, X				{T, V}	{N}
{CI2}	CI2	L, X, CI				{T, V}	{N}
{L}	L	X, CI				{T, V}	{N}
{X_1}	X_1	X / CI		X=1	X_1=1, X2=1	{T, V}	{N}
		CI				{T, V}	{N}
{N, T_2}	T_2	T				{T, V}	{N}
{N}	N	T				{T, V}	{N}
{Q}	Q	T				{T, V}	{N}
{L1}	L1	T, L				{T, V}	{N}
{T}	T	L				{T, V}	{N}
{CI2}	CI2	L, CI				{T, V}	{N}
{L}	y1	CI, y				{T, V}	{N}
{y}		CI	y			{T, V}	{N}
{CI}			y, CI				

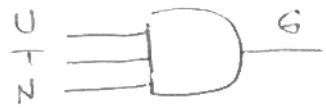
current objectives	Processed Entry	Stem Objective	Head objective	Assignment	Implications	D-F	J-F
			CI	$y = 0$	$y_1 = 0$, $y_2 = 0$ $L = 1$, $L_1 = 1$, $L_3 = 1$ $L_2 = 1$, $R = 1$, $Q = 0$, $N = 1$	$\{T, V\}$	ϕ
$\{T_2\}$	T_2	T				$\{T, V\}$	ϕ
$\{T\}$	T					$\{T, V\}$	ϕ
$\{CI_2\}$	CI_2	CI				$\{T, V\}$	ϕ
$\{CI\}$	CI		CI			$\{T, V\}$	ϕ
				$CI = 0$	$CI_1 = 0$, $CI_2 = 0$ $T = 1$, $T_1 = 1$, $T_2 = 1$, $U = 1$, $V = \overline{D}$, $S = D$, $L_0 = 0$ success	ϕ	ϕ

Thus, a test is generated successfully for the fault as $(x, y, CI) = (1, 0, 0)$.

The decision tree is:



(v) To detect the fault N s-a-0 across the path {N, N2, V, S}, this requires that $N = 1$, $T = 1$ and $U = 1$. Thus, the auxiliary gate G is formed as:



The probability of detecting the fault N s-a-0

$d_{N,s0} \geq P_G$.

$$P_G = P_U \cdot P_T \cdot P_N$$

$$P_X = 0.5, \quad P_y = 0.5, \quad P_L = \frac{3}{4}$$

$$P_Q = \frac{5}{8}, \quad P_R = \frac{5}{8}, \quad P_N = \frac{39}{64}$$

$$P_T = \frac{89}{128}, \quad P_U = \frac{167}{256}$$

$$\Rightarrow P_G = \frac{167}{256} \times \frac{89}{128} \times \frac{39}{64} = \frac{579657}{2^{21}} = 0.276$$

$$\Rightarrow d_{N,s0} \geq 0.276$$

(vi) To compute the exact detection prob. of the fault N s-a-0, we need to find all the tests that detect this fault. This can be found by the following equation:

$$(S \oplus S^f) + (CO \oplus CO^f) = 1$$

$$S = X \oplus y \oplus CI \qquad S^f = CI$$

$$CO = xy + CI(x \oplus y) \qquad CO^f = xy$$

$$\Rightarrow \quad [(x \oplus y \oplus CI) \oplus CI] + [(xy + CI(x \oplus y)) \oplus xy] = 1$$

$$\Rightarrow \quad [x \oplus y] + CI\, x\bar{y} + CI\, \bar{x} y = 1$$

Thus, the test set that detects this fault is

$(x, y, CI) = \{010, 011, 100, 101\}$

Thus, the exact detection probability for this fault is $\frac{4}{8} = 0.5$

Note that the lower bound we estimated in part (v) was based on the sum output only. The exact detection probability of the fault on the sum output is also 0.5.

The length of the random sequence required to detect this fault can be found by

$$N = \frac{\ln(1-c)}{\ln(1-d_f)}$$

Let us assume that the random sequence will detect the fault with a probability

$c = 0.95$

$$\Rightarrow N = \frac{\ln(1-0.95)}{\ln(1-0.5)} = 4.32$$

Thus, the fault will be detected with a prob. of 95% if more than 4 vectors are applied.

Q2

Primitive Cubes:

A	B	C	Z
1	0	0	1
0	1	0	1
0	0	1	1
1	1	1	1
1	1	0	0
1	0	1	0
0	1	1	0
0	0	0	0

Propagation D-cubes:

A	B	C	Z
D	0	0	D
D	0	1	$\bar{D}$
D	1	0	$\bar{D}$
D	1	1	D
0	D	0	D
1	D	0	$\bar{D}$
0	D	1	$\bar{D}$
1	D	1	D
0	0	D	D
1	0	D	$\bar{D}$
0	1	D	$\bar{D}$
1	1	D	D
D	D	D	D
D	$\bar{D}$	D	$\bar{D}$
$\bar{D}$	D	D	$\bar{D}$
D	D	$\bar{D}$	$\bar{D}$

A	B	C	Z
$\bar{D}$	0	0	$\bar{D}$
$\bar{D}$	0	1	D
$\bar{D}$	1	0	D
$\bar{D}$	1	1	$\bar{D}$
0	$\bar{D}$	0	$\bar{D}$
1	$\bar{D}$	0	D
0	$\bar{D}$	1	D
1	$\bar{D}$	1	$\bar{D}$
0	0	$\bar{D}$	$\bar{D}$
1	0	$\bar{D}$	D
0	1	$\bar{D}$	D
1	1	$\bar{D}$	$\bar{D}$
$\bar{D}$	$\bar{D}$	$\bar{D}$	$\bar{D}$
$\bar{D}$	D	$\bar{D}$	D
D	$\bar{D}$	$\bar{D}$	D
$\bar{D}$	$\bar{D}$	D	D

Q 4

(i) We need to generate a test sequence
for the fault G3 s-a-1.
Let us start at time frame $T=0$ to
excite the fault.

$T=0$:

To excite the fault, we need to set $G3=0$.
There are two possible solutions either to
set $G1=0$ or $G2=0$. Note that if
we select $G2=0$, this will result in
$I_2=0$ & $I_3=0$ which will not propagate the
fault in the next time frame. Thus, we
select $G1=0$. To set $G1=0$, we have
two solutions: $I_1=0$ & $I_2=0$ or $I_1=1$ & $I_2=1$.
We select the solution $I_1=1$ & $I_2=1$ since
the other choice will not propagate the
fault in the next time frame.
By setting $I_1=1$ and $I_2=1$, we get $G_1=0$
and $G3=\overline{D}$. Thus, we need to propagate
the fault to the next time frame.

$T=1$:

In this time frame, $F2=\overline{D}$, $F1=1$, $F3=X$,
and $F4=X$. To propagate the fault to
the primary output Z, this requires $F1=1$,
and $G8=0$. $F1=1$ is already satisfied.

To have $G8=0$, this requires that $F3=1$ and $F4=1$. These have to be justified in time frame $T=0$. This requires that $I4=1$ and $F3=0$ and $F4=1$. This has to be justified in $T=-1$

<u>$T=-1$:</u>

To justify $F3=0$ and $F4=1$, we need $I4=1$ and $F3=1$ and $F4=0$. This has to be justified in $T=-2$.

<u>$T=-2$:</u>

To justify $F3=1$ and $F4=0$, this requires that $I4=1$ and $F3=0$ and $F4=0$. This has to be justified in $T=-3$.

<u>$T=-3$:</u>

To justify $F3=0$ and $F4=0$, this requires $I4=0$ or $F3=1$ & $F4=1$. We choose $I4=0$.

Thus, a justification sequence is found.

The test sequence to detect the fault is

$$\begin{array}{cccccc} & T=-3 & T=-2 & T=-1 & T=0 & T=1 \\ I_1 I_2 I_3 I_4 = \{ & xxx0, & xxx1, & xxx1, & 11x1, & xxxx \} \end{array}$$