Integer Arithmetic

COE 205

Computer Organization and Assembly Language

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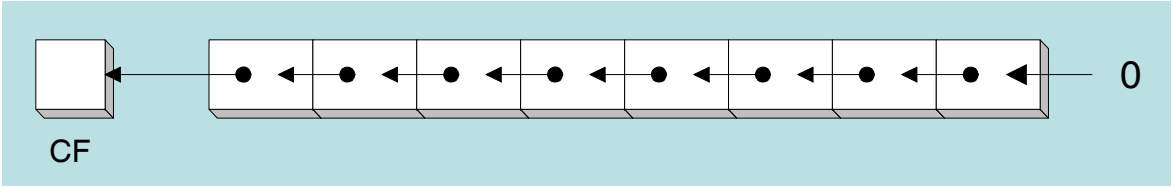
[Adapted from slides of Dr. Kip Irvine: Assembly Language for Intel-Based Computers]

Presentation Outline

- Shift and Rotate Instructions
- Shift and Rotate Applications
- Multiplication and Division Instructions
- Translating Arithmetic Expressions
- Decimal String to Number Conversions

SHL Instruction

- SHL is the Shift Left instruction
  - Performs a logical left shift on the destination operand
  - Fills the lowest bit with zero
  - The last bit shifted out from the left becomes the Carry Flag



- Operand types for SHL:

```
SHL reg,imm8
SHL mem,imm8
SHL reg,CL
SHL mem,CL
```

The shift count is either:

8-bit immediate ***imm8***, or

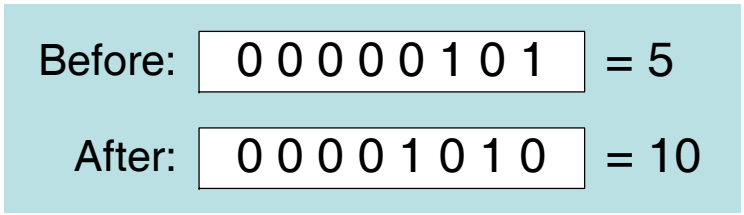
stored in register ***CL***

Only least sig. 5 bits used

Fast Multiplication

Shifting left 1 bit multiplies a number by 2

```
mov dl,5
shl dl,1
```



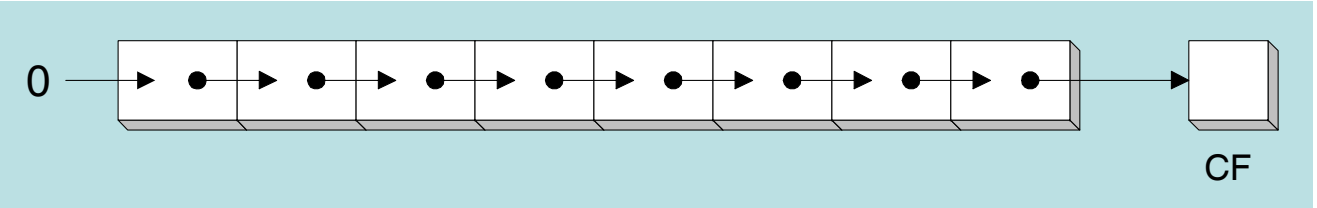
Shifting left *n* bits multiplies the operand by 2^n

For example, $5 * 2^2 = 20$

```
mov dl,5   ; DL = 00000101b
shl dl,2   ; DL = 00010100b = 20, CF = 0
```

SHR Instruction

- SHR is the Shift Right instruction
  - Performs a logical right shift on the destination operand
  - The highest bit position is filled with a zero
  - The last bit shifted out from the right becomes the Carry Flag
  - SHR uses the same instruction format as SHL

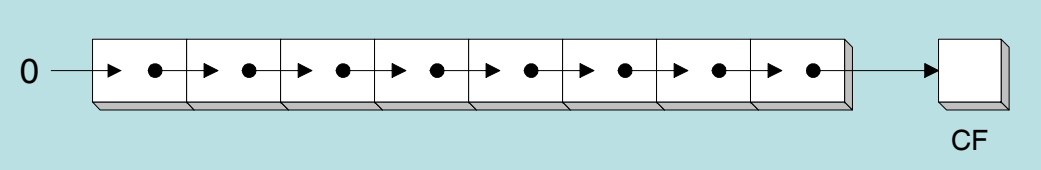


- Shifting right *n* bits divides the operand by 2^n

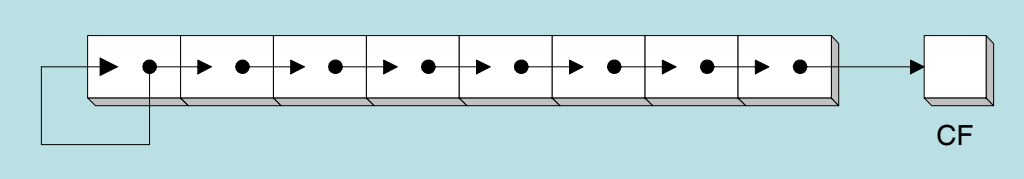
```
mov dl,80   ; DL = 01010000b
shr dl,1    ; DL = 00101000b = 40, CF = 0
shr dl,2    ; DL = 00001010b = 10, CF = 0
```

Logical versus Arithmetic Shifts

- Logical Shift
  - Fills the newly created bit position with zero

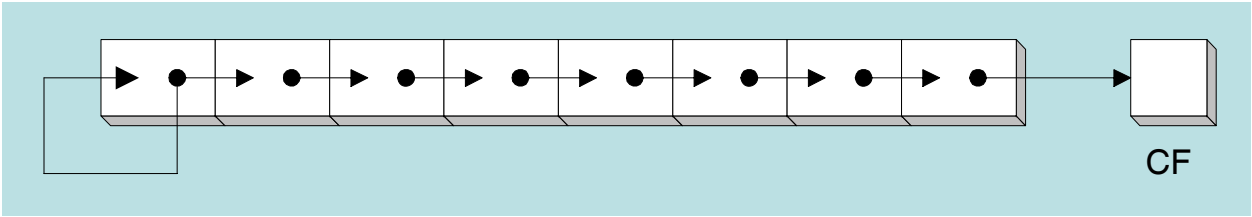


- Arithmetic Shift
  - Fills the newly created bit position with a copy of the sign bit
  - Applies only to Shift Arithmetic Right (SAR)



SAL and SAR Instructions

- SAL: Shift Arithmetic Left is identical to SHL
- SAR: Shift Arithmetic Right
  - Performs a right arithmetic shift on the destination operand



- SAR preserves the number's sign

```
mov dl,-80    ; DL = 10110000b
sar dl,1      ; DL = 11011000b = -40, CF = 0
sar dl,2      ; DL = 11110110b = -10, CF = 0
```

Your Turn . . .

Indicate the value of AL and CF after each shift

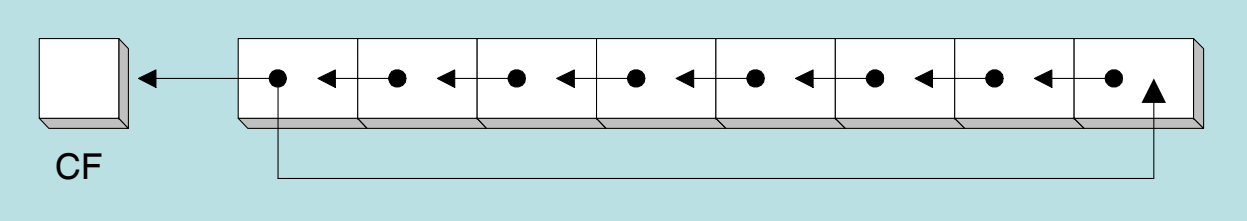
```
mov al,6Bh      ; al = 01101011b
shr al,1        ; al = 00110101b = 35h, CF = 1
shl al,3        ; al = 10101000b = A8h, CF = 1
mov al,8Ch      ; al = 10001100b
sar al,1        ; al = 11000110b = C6h, CF = 0
sar al,3        ; al = 11111000b = F8h, CF = 1
```

Effect of Shift Instructions on Flags

- The CF is the last bit shifted
- The OF is defined for single bit shift only
  - It is 1 if the sign bit changes
- The ZF, SF and PF are affected according to the result
- The AF is unaffected

ROL Instruction

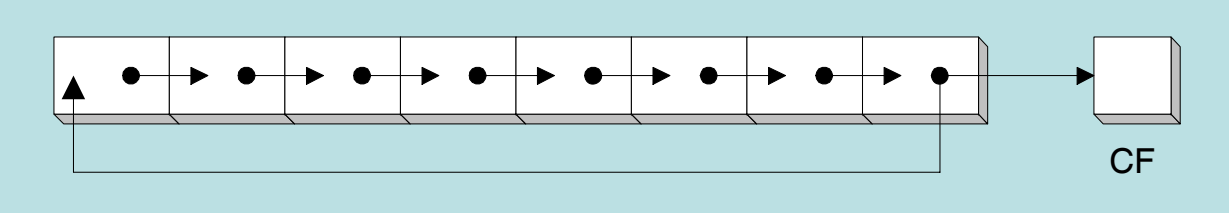
- ROL is the Rotate Left instruction
  - Rotates each bit to the left, according to the count operand
  - Highest bit is copied into the Carry Flag and into the Lowest Bit
- No bits are lost




```
mov al,11110000b
rol al,1            ; AL = 11100001b, CF = 1
mov dl,3Fh          ; DL = 00111111b
rol dl,4            ; DL = 11110011b = F3h, CF = 1
```

ROR Instruction

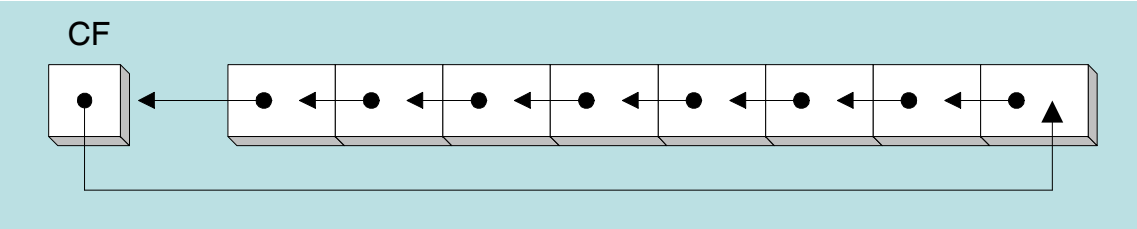
- ROR is the Rotate Right instruction
  - Rotates each bit to the right, according to the count operand
  - Lowest bit is copied into the Carry flag and into the highest bit
- No bits are lost




```
mov al,11110000b
ror al,1                ; AL = 01111000b, CF = 0
mov dl,3Fh              ; DL = 00111111b
ror dl,4                ; DL = F3h, CF = 1
```

RCL Instruction

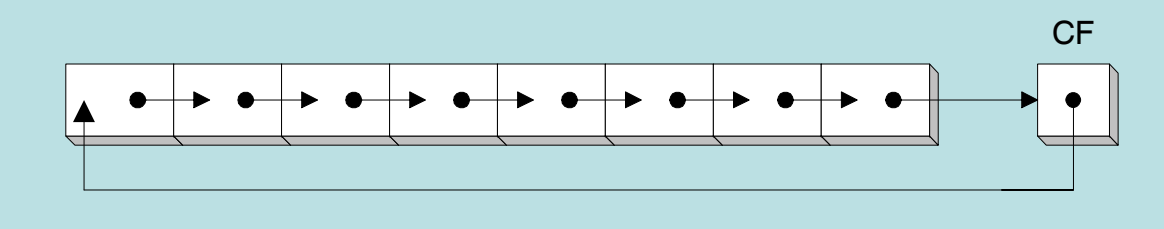
- RCL is the Rotate Carry Left instruction
  - Rotates each bit to the left, according to the count operand
  - Copies the Carry flag to the least significant bit
  - Copies the most significant bit to the Carry flag
- As if the carry flag is part of the destination operand




```
clc                 ; clear carry, CF = 0
mov bl,88h          ; BL = 10001000b
rcl bl,1            ; CF = 1, BL = 00010000b
rcl bl,2            ; CF = 0, BL = 01000010b
```

RCR Instruction

- RCR is the Rotate Carry Right instruction
  - Rotates each bit to the right, according to the count operand
  - Copies the Carry flag to the most significant bit
  - Copies the least significant bit to the Carry flag
- As if the carry flag is part of the destination operand




```
stc                 ; set carry, CF = 1
mov ah,11h          ; AH = 00010001b
rcr ah,1            ; CF = 1, AH = 10001000b
rcr ah,3            ; CF = 0, AH = 00110001b
```

Effect of Rotate Instructions on Flags

- The CF is the last bit shifted
- The OF is defined for single bit rotates only
  - It is 1 if the sign bit changes
- The ZF, SF, PF and AF are unaffected

SHLD Instruction

- SHLD is the Shift Left Double instruction
- Syntax: **`SHLD`** ***`destination, source, count`***
  - Shifts a *destination* operand a given *count* of bits to the left
- The rightmost bits of *destination* are filled by the leftmost bits of the *source* operand
- The *source* operand is not modified
- Operand types:

```
SHLD reg/mem16, reg16, imm8/CL
SHLD reg/mem32, reg32, imm8/CL
```

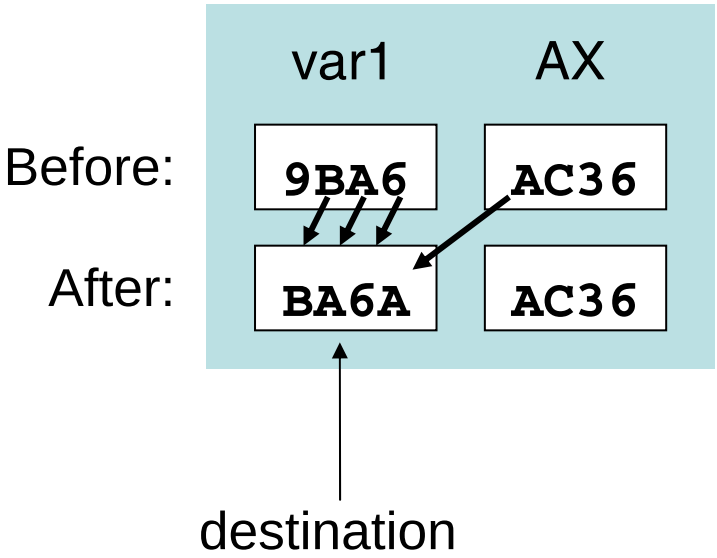
SHLD Example

Shift variable **`var1`** 4 bits to the left

Replace the lowest 4 bits of **`var1`** with the high 4 bits of AX

```
.data
var1 WORD 9BA6h
.code
mov  ax, 0AC36h
shld var1, ax, 4
```

destination source count



Only the *destination* is modified, not the *source*

SHRD Instruction

- SHRD is the Shift Right Double instruction
- Syntax: **SHRD *destination, source, count***
  - Shifts a *destination* operand a given *count* of bits to the left
- The leftmost bits of *destination* are filled by the rightmost bits of the *source* operand
- The *source* operand is not modified
- Operand types:

```
SHRD reg/mem16, reg16, imm8/CL
SHRD reg/mem32, reg32, imm8/CL
```

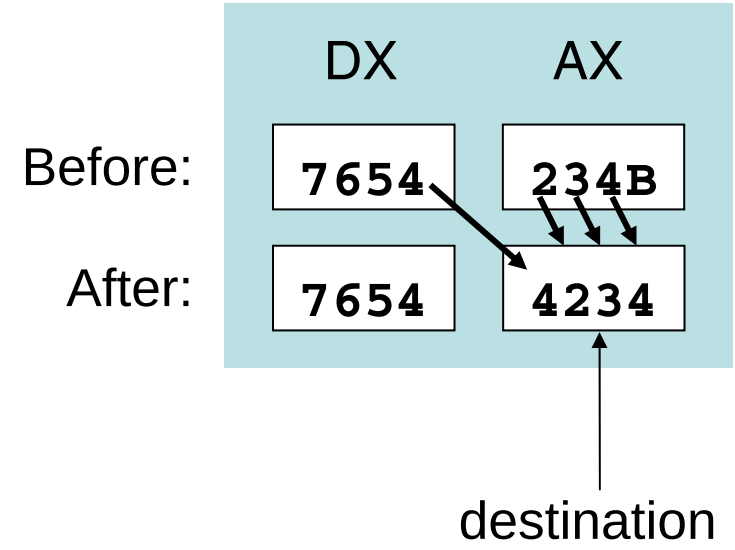
SHRD Example

Shift AX 4 bits to the right

Replace the highest 4 bits of AX with the low 4 bits of DX

```
mov   ax,234Bh
mov   dx,7654h
shrd  ax, dx, 4
```

destination source count



Only the *destination* is modified, not the *source*

Your Turn . . .

Indicate the values (in hex) of each destination operand

```
mov  ax,7C36h
mov  dx,9FA6h
shld dx,ax,4      ; DX = FA67h
shrd ax,dx,8      ; AX = 677Ch
```

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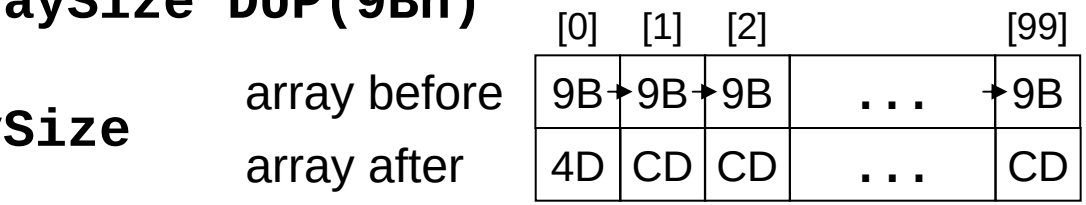
Shifting Bits within an Array

- Sometimes, we need to shift all bits within an array
  - Example: moving a bitmapped image from one screen to another
- Task: shift an array of bytes 1 bit right, starting a first byte

```
.data
    ArraySize  EQU 100
    array BYTE ArraySize DUP(9Bh)
.code
    mov ecx, ArraySize
    mov esi, 0
    clc                     ; clear carry flag
L1:
    rcr array[esi], 1       ; propagate the carry flag
    inc esi                 ; does not modify carry
    loop L1                 ; does not modify carry
```



Binary Multiplication

- You know that SHL performs multiplication efficiently
  - When the multiplier is a power of 2
- You can factor any binary number into powers of 2
  - Example: multiply EAX by 36
    - Factor 36 into (4 + 32) and use distributive property of multiplication
  - EAX * 36 = EAX * (4 + 32) = EAX * 4 + EAX * 32

```
mov ebx, eax          ; EBX = number
shl eax, 2            ; EAX = number * 4
shl ebx, 5            ; EBX = number * 32
add eax, ebx          ; EAX = number * 36
```

Your Turn . . .

Multiply EAX by 26, using shifting and addition instructions

Hint: 26 = 2 + 8 + 16

```
mov   ebx, eax          ; EBX = number
shl   eax, 1            ; EAX = number * 2
shl   ebx, 3            ; EBX = number * 8
add   eax, ebx          ; EAX = number * 10
shl   ebx, 1            ; EBX = number * 16
add   eax, ebx          ; EAX = number * 26
```

Multiply EAX by 31, Hint: 31 = 32 – 1

```
mov   ebx, eax          ; EBX = number
shl   eax, 5            ; EAX = number * 32
sub   eax, ebx          ; EAX = number * 31
```

Convert Number to Binary String

Task: Convert Number in EAX to an ASCII Binary String

Receives: EAX = Number
ESI = Address of binary string

Returns: String is filled with binary characters '0' and '1'

```
ConvToBinStr PROC USES ecx esi
    mov  ecx,32
L1: rol  eax,1
    mov  BYTE PTR [esi],'0'
    jnc  L2
    mov  BYTE PTR [esi],'1'
L2: inc  esi
    loop L1
    mov  BYTE PTR [esi], 0
    ret
ConvToBinStr ENDP
```

Rotate left most significant bit of EAX into the Carry flag; If CF = 0, append a '0' character to a string; otherwise, append a '1'; Repeat in a loop 32 times for all bits of EAX.

Convert Number to Hex String

Task: Convert EAX to a Hexadecimal String pointed by ESI

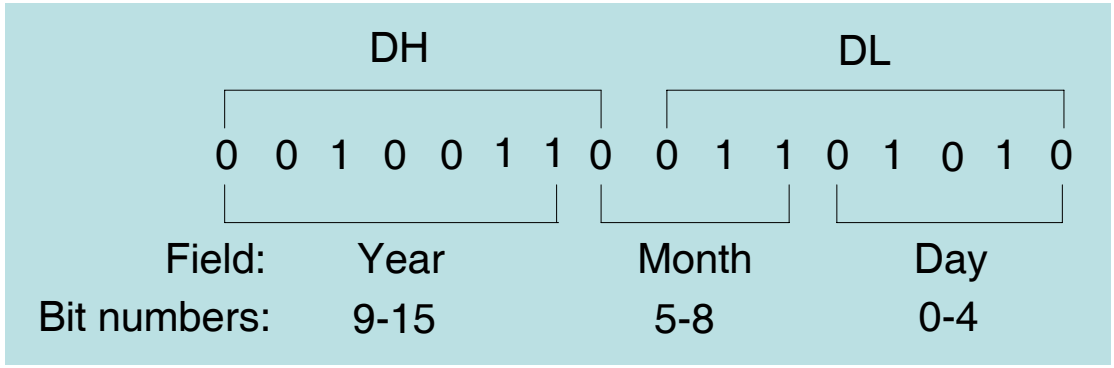
Receives: EAX = Number, ESI= Address of hex string

Returns: String pointed by ESI is filled with hex characters '0' to 'F'

```
ConvToHexStr PROC  USES ebx ecx esi
    mov  ecx, 8                ; 8 iterations, why?
L1: rol  eax, 4                ; rotate upper 4 bits
    mov  ebx, eax
    and  ebx, 0Fh              ; keep only lower 4 bits
    mov  bl,  HexChar[ebx]     ; convert to a hex char
    mov  [esi], bl             ; store hex char in string
    inc  esi
    loop L1                    ; loop 8 times
    mov  BYTE PTR [esi], 0     ; append a null byte
    ret
HexChar BYTE "0123456789ABCDEF"
ConvToHexStr ENDP
```

Isolating a Bit String

- MS-DOS date packs the year, month, & day into 16 bits
  - Year is relative to 1980



In this example:

Day = 10
Month = 3
Year = 1980 + 19

Date = March 10, 1999

Isolate the Month field:

```
mov ax,dx           ; Assume DX = 16-bit MS-DOS date
shr ax,5            ; shift right 5 bits
and al,00001111b    ; clear bits 4-7
mov month,al        ; save in month variable
```

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MUL Instruction

- The MUL instruction is used for unsigned multiplication
- Multiplies 8-, 16-, or 32-bit operand by AL, AX, or EAX
- The instruction formats are:

```
MUL r/m8      ; AX      = AL  * r/m8
MUL r/m16     ; DX:AX   = AX  * r/m16
MUL r/m32     ; EDX:EAX = EAX * r/m32
```

Multiplicand	Multiplier	Product
AL	*r/m8*	AX
AX	*r/m16*	DX:AX
EAX	*r/m32*	EDX:EAX

MUL Examples

Example 1: Multiply 16-bit var1 (2000h) * var2 (100h)

```
.data
var1 WORD 2000h
var2 WORD 100h
.code
mov ax,var1
mul var2     ; DX:AX = 00200000h, CF = OF = 1
```

The Carry and Overflow flags are set if upper half of the product is non-zero

Example 2: Multiply EAX (12345h) * EBX (1000h)

```
mov eax,12345h
mov ebx,1000h
mul ebx      ; EDX:EAX = 0000000012345000h, CF=OF=0
```

Your Turn . . .

What will be the hexadecimal values of DX, AX, and the Carry flag after the following instructions execute?

```
mov ax, 1234h
mov bx, 100h
mul bx
```

Solution

DX = 0012h, AX = 3400h, CF = 1

What will be the hexadecimal values of EDX, EAX, and the Carry flag after the following instructions execute?

```
mov eax,00128765h
mov ecx,10000h
mul ecx
```

Solution

EDX = 00000012h,

EAX = 87650000h, CF = OF = 1

IMUL Instruction

- The IMUL instruction is used for signed multiplication
  - Preserves the sign of the product by sign-extending it
- One-Operand formats, as in MUL

```
IMUL r/m8       ; AX       = AL  * r/m8
IMUL r/m16      ; DX:AX    = AX  * r/m16
IMUL r/m32      ; EDX:EAX  = EAX * r/m32
```

- Two-Operand formats:

```
IMUL r16, r16/m16/imm8/imm16
IMUL r32, r32/m32/imm8/imm32
```

- Three-Operand formats:

```
IMUL r16, r16/m16, imm8/imm16
IMUL r32, r32/m32, imm8/imm32
```

The Carry and Overflow flags are set if the upper half of the product is not a sign extension of the lower half

IMUL Examples

- Multiply AL = 48 by BL = 4

```
mov  al,48
mov  bl,4
imul bl          ; AX = 00C0h, CF = OF = 1
```

OF = 1 because AH is not a sign extension of AL

- Your Turn: What will be DX, AX and OF ?

```
mov  ax,8760h
mov  bx,100h
imul bx
```

DX = FF87h, AX = 6000h, OF = CF = 1

Two and Three Operand Formats

```
.data
wval SWORD -4
dval SDWORD 4
.code
mov  ax, -16
mov  bx, 2
imul bx, ax                 ; BX  = BX * AX    = -32
imul bx, 2                  ; BX  = BX * 2     = -64
imul bx, wval               ; BX  = BX * wval  = 256
imul bx, 5000               ; OF  = CF = 1
mov  edx,-16
imul edx,dval               ; EDX = EDX * dval = -64
imul bx, wval,-16           ; BX  = wval * -16 =  64
imul ebx,dval,-16           ; EBX = dval * -16 = -64
imul eax,ebx,2000000000     ; OF  = CF = 1
```

DIV Instruction

- The DIV instruction is used for unsigned division
- A single operand (divisor) is supplied
  - Divisor is an 8-bit, 16-bit, or 32-bit register or memory
  - Dividend is implicit and is either AX, DX:AX, or EDX:EAX
- The instruction formats are:

```
DIV r/m8
DIV r/m16
DIV r/m32
```

Dividend	Divisor	Quotient	Remainder
AX	*r/m8*	AL	AH
DX:AX	*r/m16*	AX	DX
EDX:EAX	*r/m32*	EAX	EDX

DIV Examples

Divide AX = 8003h by CX = 100h

```
mov dx,0          ; clear dividend, high
mov ax,8003h      ; dividend, low
mov cx,100h       ; divisor
div cx            ; AX = 0080h, DX = 3 (Remainder)
```

Your turn: what will be the hexadecimal values of DX and AX after the following instructions execute?

```
mov dx,0087h
mov ax,6023h
mov bx,100h
div bx
```

Solution: DX = 0023h, AX = 8760h

Divide Overflow

- Divide Overflow occurs when …
  - Quotient cannot fit into the destination operand, or when
  - Dividing by Zero
- Divide Overflow causes a CPU interrupt
  - The current program halts and an error dialog box is produced
- Example of a Divide Overflow

```
mov dx,0087h
mov ax,6002h
mov bx,10h
div bx
```

```
Divide overflow:
Quotient = 87600h
Cannot fit in AX
```

Signed Integer Division

- Signed integers must be sign-extended before division
  - Fill high byte, word, or double-word with a copy of the sign bit
- CBW, CWD, and CDQ instructions
  - Provide important sign-extension operations before division
  - CBW: Convert Byte to Word, sign-extends AL into AH
  - CWD: Convert Word to Double, sign-extends AX into DX
  - CDQ: Convert Double to Quad, sign-extends EAX into EDX
- Example:

```
mov ax, 0FE9Bh          ; AX = -357
cwd                     ; DX:AX = FFFFFF9Bh
```

IDIV Instruction

- IDIV performs signed integer division
- Same syntax and operands as DIV instruction

```
IDIV r/m8
IDIV r/m16
IDIV r/m32
```

Dividend	Divisor	Quotient	Remainder
AX	*r/m8*	AL	AH
DX:AX	*r/m16*	AX	DX
EDX:EAX	*r/m32*	EAX	EDX

- Example: divide eax (-503) by ebx (10)

```
mov  eax, -503
cdq
mov  ebx, 10
idiv ebx        ; EAX = -50, EDX = -3
```

All status flags are undefined after executing DIV and IDIV

IDIV Examples

Example: Divide DX:AX (-48) by BX (-5)

```
mov  ax,-48
cwd              ; sign-extend AX into DX
mov  bx,-5
idiv bx          ; AX = 9,  DX = -3
```

Example: Divide EDX:EAX (48) by BX (-5)

```
mov  eax,48
cdq              ; sign-extend EAX into EDX
mov  bx,-5
idiv bx          ; AX = -9,  DX = 3
```

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Translating Arithmetic Expressions

- Some good reasons to translate arithmetic expressions
  - Learn how compilers do it
  - Test your understanding of MUL, IMUL, DIV, and IDIV
  - Check for Carry and Overflow flags
- Two Types of Arithmetic Expressions
  - Unsigned arithmetic expressions
    - Unsigned variables and values are used only
    - Use MUL and DIV for unsigned multiplication and division
  - Signed arithmetic expressions
    - Signed variables and values
    - Use IMUL and IDIV for signed multiplication and division

Unsigned Arithmetic Expressions

- Example: **`var4 = (var1 + var2) * var3`**
- All variables are 32-bit unsigned integers
- Translation:

```
    mov  eax, var1
    add  eax, var2      ; EAX = var1 + var2
    jc   tooBig         ; check for carry
    mul  var3           ; EAX = EAX * var3
    jc   tooBig         ; check for carry
    mov  var4, eax      ; save result
    jmp  next
tooBig:
    . . .               ; display error message
next:
```

Signed Arithmetic Expressions

Example: **var4 = (-var1 * var2) + var3**

```
mov  eax, var1
neg  eax
imul var2             ; signed multiplication
jo   tooBig           ; check for overflow
add  eax, var3
jo   tooBig           ; check for overflow
mov  var4, eax        ; save result
```

Example: **var4 = (var1 * 5) / (var2 – 3)**

```
mov  eax, var1
mov  ebx, 5
imul ebx              ; EDX:EAX = product
mov  ebx, var2        ; right side
sub  ebx, 3
idiv ebx              ; EAX = quotient
mov  var4, eax
```

Your Turn . . .

Translate: **`var5 = (var1 * -var2)/(var3 – var4)`**

Assume signed 32-bit integers

```
mov  eax, var1
mov  edx, var2
neg  edx
imul edx                    ; EDX:EAX = product
mov  ecx, var3
sub  ecx, var4
idiv ecx                    ; EAX = quotient
mov  var5, eax
```

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Convert Decimal String to Number

Task: Convert decimal string pointed by ESI to a number

Receives: ESI = address of decimal string

Returns: EAX = number in binary format

Algorithm:

Start by initializing EAX to 0

For each decimal character in string (example: "1083")

Move one decimal character of string into EDX

Convert EDX to digit (0 to 9): **`EDX = EDX – '0'`**

Compute: **`EAX = EAX * 10 + EDX`**

Repeat until end of string (NULL char)

Convert Decimal String - cont'd

```
; Assumes: String should contain only decimal chars
;          String should not be empty
;          Procedure does not detect invalid input
;          Procedure does not skip leading spaces

ConvDecStr PROC USES edx esi
    mov   eax, 0                ; Initialize EAX
L1: imul  eax, 10               ; EAX = EAX * 10
    movzx edx, BYTE PTR [esi]   ; EDX = '0' to '9'
    sub   edx, '0'              ; EDX =  0  to  9
    add   eax, edx              ; EAX = EAX*10 + EDX
    inc   esi                   ; point at next char
    cmp   BYTE PTR [esi],0      ; NULL byte?
    jne   L1
    ret                         ; return
ConvDecStr ENDP
```

Convert Number to Decimal String

Task: Convert Number in EAX to a Decimal String

Receives: EAX = Number, ESI = String Address

Returns: String is filled with decimal characters '0' to '9'

Algorithm: Divide EAX by 10 (Example: EAX = 1083)

```
mov EBX, 10   ; divisor   = EBX = 10
mov EDX, 0    ; dividend  = EDX:EAX
div EBX       ; EDX (rem) = 3, EAX = 108
add dl, '0'   ; DL = '3'
```

Repeat division until EAX becomes 0

Remainder chars are computed backwards: '3', '8', '0', '1'

Store characters in reverse order in string pointed by ESI

Convert to Decimal String – cont'd

```
ConvToDecStr PROC
    pushad                      ; save all since most are used
    mov  ecx, 0                 ; Used to count decimal digits
    mov  ebx, 10                ; divisor = 10
L1: mov  edx, 0                 ; dividend = EDX:EAX
    div  ebx                    ; EDX = remainder = 0 to 9
    add  dl, '0'                ; convert DL to '0' to '9'
    push dx                     ; save decimal character
    inc  ecx                    ; and count it
    cmp  eax, 0
    jnz  L1                     ; loop back if EAX != 0
L2: pop  dx                     ; pop in reverse order
    mov  [esi], dl              ; store decimal char in string
    inc  esi
    loop L2
    mov  BYTE PTR [esi], 0      ; Terminate with a NULL char
    popad                       ; restore all registers
    ret                         ; return
ConvToDecStr ENDP
```

Summary

- Shift and rotate instructions
  - Provide finer control over bits than high-level languages
  - Can shift and rotate more than one bit left or right
  - SHL, SHR, SAR, SHLD, SHRD, ROL, ROR, RCL, RCR
  - Shifting left by *n* bits is a multiplication by 2^n
  - Shifting right does integer division (use SAR to preserve sign)
- MUL, IMUL, DIV, and IDIV instructions
  - Provide signed and unsigned multiplication and division
  - One operand format: one of the operands is always implicit
  - Two and three operand formats for IMUL instruction only
  - CBW, CDQ, CWD: extend AL, AX, and EAX for signed division