

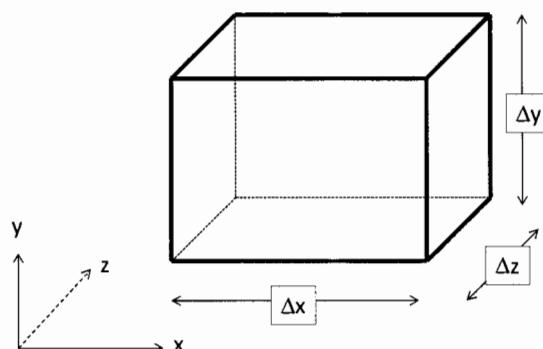
Fluid Statics

CHE204

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Introduce the x y z coordinate system

Introduce a cube of fluid of dimensions $\Delta x \Delta y \Delta z$



Apply Newton's 2nd law of motion:
Sum of forces acting on a static body are equal to zero

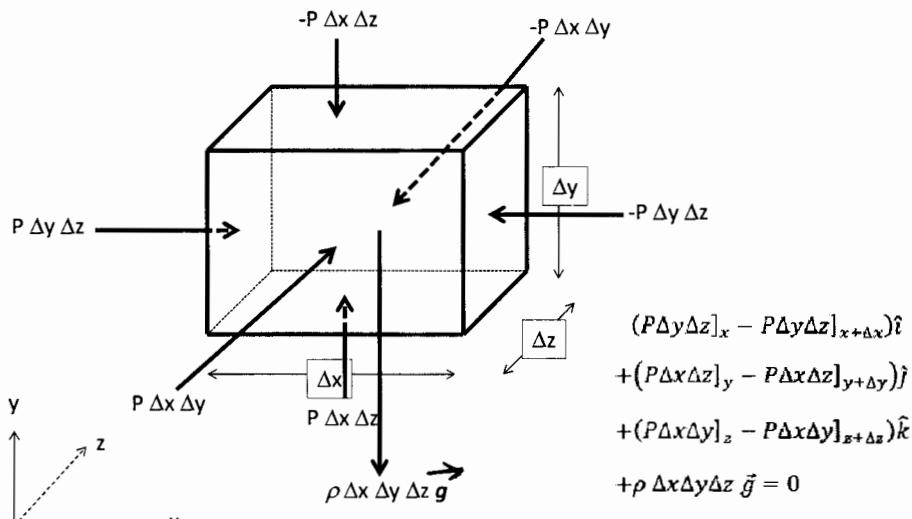
$$\sum F = 0$$

What are the forces acting on the fluid?

1. Pressure forces
2. Body forces (gravity)

First we apply pressure forces

Second we apply gravity force



$$(P\Delta y \Delta z]_x - P\Delta y \Delta z]_{x+\Delta x})\hat{i} + (P\Delta x \Delta z]_y - P\Delta x \Delta z]_{y+\Delta y})\hat{j} + (P\Delta x \Delta y]_z - P\Delta x \Delta y]_{z+\Delta z})\hat{k} + \rho \Delta x \Delta y \Delta z \vec{g} = 0$$

Divide the whole equation by $\Delta x \Delta y \Delta z$
then take the limit when $\Delta x \Delta y \Delta z$ approaches zero

$$\lim_{\Delta x \Delta y \Delta z \rightarrow 0} \left\{ \frac{P]_x - P]_{x+\Delta x}}{\Delta x} \hat{i} + \frac{P]_y - P]_{y+\Delta y}}{\Delta y} \hat{j} + \frac{P]_z - P]_{z+\Delta z}}{\Delta z} \hat{k} + \rho \vec{g} \right\} = 0$$

$$\left(\frac{\partial P}{\partial x} \hat{i} + \frac{\partial P}{\partial y} \hat{j} + \frac{\partial P}{\partial z} \hat{k} \right) + \rho \vec{g} = 0$$

Rearrange $\left(\frac{\partial P}{\partial x} \hat{i} + \frac{\partial P}{\partial y} \hat{j} + \frac{\partial P}{\partial z} \hat{k} \right) = \rho \vec{g}$

Simplify $\left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right) P = \rho \vec{g}$

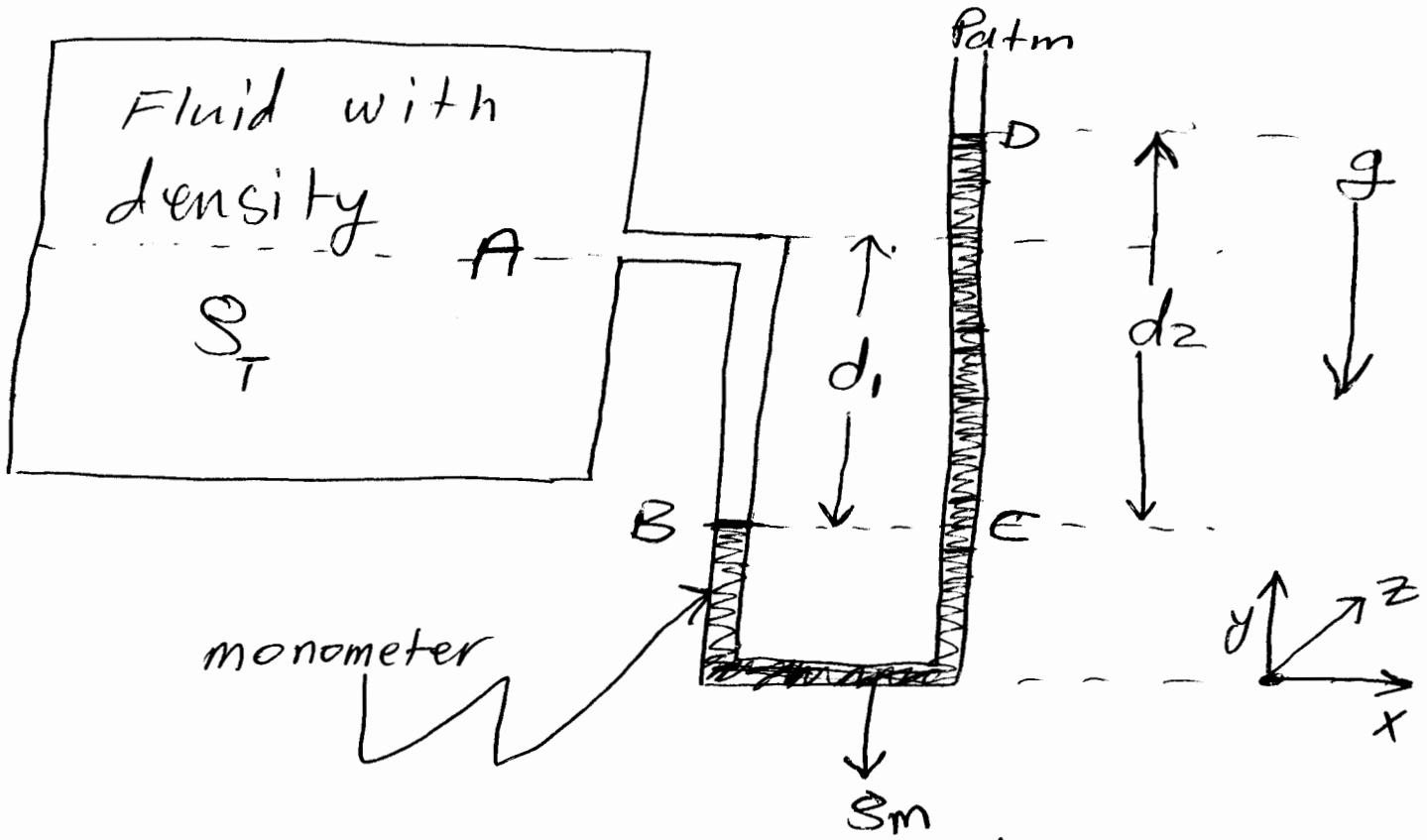
Introduce the gradient operator
(mathematically it means the maximum rate of change)

$$\vec{\nabla} \equiv \left(\frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k} \right)$$

$$\vec{\nabla} P = \rho \vec{g}$$

The maximum rate of change of the hydrostatic pressure occurs in the direction of gravity

Example Find the pressure at Point A



Apply fluid statics equation

$$\nabla \vec{P} = \rho \vec{g}$$

$$\frac{\partial P}{\partial x} \hat{i} + \frac{\partial P}{\partial y} \hat{j} + \frac{\partial P}{\partial z} \hat{k} = \rho \vec{g}$$

In this problem pressure variations in the y-direction only

$$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial z} = 0 \quad \text{and} \quad \vec{g} = -g \hat{j}$$

$$\Rightarrow \frac{\partial P}{\partial y} = -\rho g \quad \dots \quad (1)$$

(5)

Apply equation ① for fluid inside tank and for manometer fluid.

$$\frac{dp}{dy} = - \gamma_T g \quad (1-a)$$

$$\frac{dp}{dy} = - \gamma_m g \quad (1-b)$$

Solve (1-a) from point A to B:

$$\int_A^B dp = - \int_A^B \gamma_T g dy$$

$$P_B - P_A = - \gamma_T g (y_B - y_A)$$

\downarrow

$-d_1$

$$\Rightarrow P_A = P_B - \gamma_T g d_1 \quad \dots \quad (2)$$

Solve (1-b) from C to D

$$\int_C^D dp = \int_C^D - \gamma_m g dy$$

(6)

$$P_D - P_C = - \gamma_m g \underbrace{(y_D - y_C)}_{d_2}$$

$$P_C = \underbrace{P_D}_{\downarrow} + \gamma_m g d_2.$$

$$P_{atm}$$

$$\Rightarrow P_C = P_{atm} + \gamma_m g d_2 \quad \dots \textcircled{3}$$

From Figure $P_B = P_C$

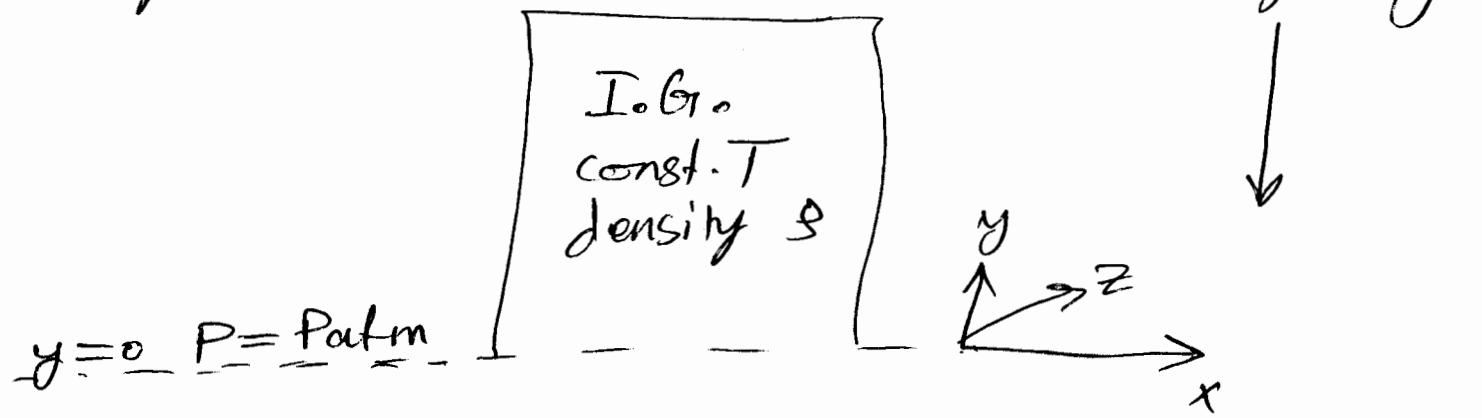
$$P_A + \gamma_T g d_1 = P_{atm} + \gamma_m g d_2.$$

solve for P_A

$$P_A = P_{atm} + [\gamma_m d_2 - \gamma_T d_1] g$$

Example Fluid statics for an ideal gas ⑦

Derive an equation for the pressure variation ^{with elevation} of an ideal gas whose temperature is constant.



$$\vec{F}_P = \delta \vec{g}$$

simplify pressure change in y direction only and $\vec{g} = -\vec{j} \therefore$

$$\frac{dP}{dy} = -\delta g$$

Before we integrate the above equation we have to be careful! $\delta_{FG} = f(P)$

$$\delta = \frac{PM_w}{RT}$$

$$\frac{dP}{dy} = - \frac{PM_w}{RT} g .$$

(8)

separate the variables

$$\frac{dP}{P} = - \frac{M_w g}{RT} dy .$$

integrate from $y=0$ to any arbitrary location y .

$$\int_{P_{atm}}^P \frac{dP}{P} = \int_0^y - \frac{M_w g}{RT} dy .$$

$$\ln\left(\frac{P}{P_{atm}}\right) = - \frac{M_w g}{RT} y .$$

$$P = P_{atm} e^{-\frac{M_w g}{RT} y} .$$