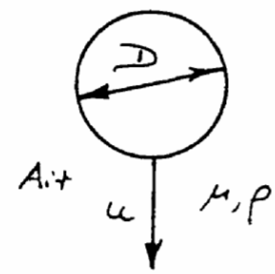
4.7

Terminal Velocity of Hailstones

The following form is useful when the velocity is unknown



$$C_D Re^2 = \frac{4}{3} \frac{g \rho_f D^3}{\mu^2} (\rho_s - \rho_f)$$

(ρ_f negligible)

$$= \frac{4}{3} \frac{32.2 \times 0.0765 \times \left(\frac{1}{12}\right)^3 \times 57.2}{\left(\frac{0.0435}{3600}\right)^2}$$

$$= 7.45 \times 10^8$$

	first estimate		from chart
C_D	0.4	0.45	0.45
$Re = (7.45 \times 10^8 / C_D)^{1/2}$	43,200	40,700	

Terminal Velocity $\quad Re = \dfrac{\rho_f u_t D}{\mu}$

$$\therefore \quad u_t = \frac{\mu Re}{\rho_f D}$$

$$= \frac{0.0435 \times 40,700}{0.0765 \times \left(\frac{1}{12}\right) \times 3,600}$$

$$u_t = 77.1 \ \frac{ft}{s} \qquad \frac{lb_m}{ft\ hr} \ \frac{ft^3}{lb_m} \ \frac{1}{ft} \ \frac{hr}{s}$$

4.10-1

Ascending Hot-Air Balloon

Volume $\quad V = \dfrac{\pi D^3}{6} = \dfrac{\pi \times 40^3}{6} = 33{,}510 \text{ ft}^3$

Mass of gas $= 33{,}510 \times 0.0598 = 2{,}004 \text{ lbm}$

Total mass of balloon (including gas) $= 2{,}504 \text{ lbm}$

Buoyant force

$$F\uparrow = 33{,}510 \times 32.2 \times 0.0774$$

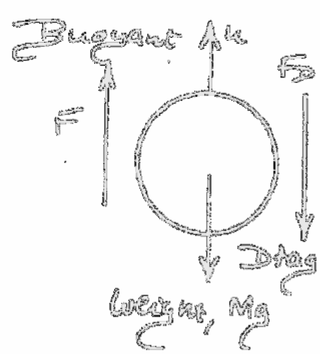
$$= 83{,}516 \ \frac{\text{lbm ft}}{\text{s}^2} \quad (= 2{,}594 \text{ lbf})$$

Drag force $\quad \dfrac{F_D}{\frac{1}{2}\rho u^2 A_p} = 0.6$

or $\quad F_D = \frac{1}{2} \times 0.6 \times 0.0774\, u^2 \times \dfrac{\pi(40^2)}{4} = 29.2\, u^2 \ \dfrac{\text{lbm ft}}{\text{s}^2}$

Momentum Balance $\uparrow$

$$\underset{F}{83{,}516} - \underset{Mg}{2{,}504 \times 32.2} - \underset{F_D}{29.2u^2} = \underset{M}{2{,}504}\ \underset{a}{\frac{du}{dt}}$$



Simplifying,

$$98.87 - u^2 = 85.75 \frac{du}{dt}$$

Terminal velocity occurs when $du/dt = 0$

$$u_t = \sqrt{98.87} = 9.94 \text{ ft/s}$$

Time to attain $0.99\, u_t = 9.84$ ft/s

$$85.75 \int_0^{9.84} \frac{du}{98.87 - u^2} = \int_0^t dt$$

$$t = \frac{85.75}{2 \times 9.94} \ln \frac{9.94 + u}{9.94 - u} \Big|_0^{9.84}$$

$$= \frac{85.75 \times 5.29}{2 \times 9.94}$$

$$t = 22.8 \text{ s}$$

Note that

$$\int \frac{dx}{a - bx^2} = \frac{1}{2\sqrt{ab}} \ln \frac{\sqrt{a} + x\sqrt{b}}{\sqrt{a} - x\sqrt{b}}$$

Q 3.

Ergun equation,

$$\frac{-\Delta p}{\rho u_0^2} \frac{D_p}{L} \frac{\varepsilon^3}{1-\varepsilon} = \underbrace{\frac{150}{Re}}_{\text{Laminar}} + \underbrace{1.75}_{\text{Turbulent}}, \qquad (4.22)$$

And Reynolds number is:

$$Re = \frac{\rho u_0 D_p}{(1-\varepsilon)\mu}.$$

Substitute Re # and rearrange,

$$-\Delta P = \frac{150 u_0 L (1-e)^2 m}{D_p^2 e^3} + 1.75 \frac{r u_0^2 L(1-e)}{D_p e^3}$$

And

$$u_0 = \frac{Q}{A} = \frac{m}{rA} = 3.39\text{E}-04\ \text{m}/\text{s}$$

Plug in data,

$$-\Delta P = 6.982\text{E}+06\ \text{Pa} + 6.757\text{E}-02\ \text{Pa} = 6.982\text{E}+06\ \text{Pa}$$

$\therefore$ Turbulent contributi on is not important.

Q 4.

The minumum fluidizing velocity is given in terms of all other known quantities by:

$$\frac{150(1-\varepsilon_0)\mu u_0}{D_p^2 \varepsilon_0^3} + 1.75\,\frac{\rho_f u_0^2}{D_p \varepsilon_0^3} = g(\rho_s - \rho_f). \qquad (4.43)$$

Rearrange

$$r_s \succ\succ r_s$$

$$\frac{1.75\, r_f}{D_p e_0^3} u_0^2 + \frac{150(1-e_0)m}{D_p^2 e_0^3} u_0 - g r_s = 0$$

$$a u_0^2 + b u_0 + c = 0$$

quadratic equation, solve for the root (u_0), plug in data,

$$5.051\text{E}+05 u_0^2 + 3.008\text{E}+07 u_0 - 9.800\text{E}+03 = 0$$

$$u_0 = 3.258\text{E}-04\ \text{m}/\text{s}$$

reject - ve root

Q 5.

Ergun equation,

$$\frac{-\Delta p}{\rho u_0^2}\,\frac{D_p}{L}\,\frac{\varepsilon^3}{1-\varepsilon} = \underbrace{\frac{150}{Re}}_{\text{Laminar}} + \underbrace{1.75}_{\text{Turbulent}}, \qquad (4.22)$$

And Reynolds number is:

$$Re = \frac{\rho u_0 D_p}{(1-\varepsilon)\mu}.$$

Substitute Re # and rearrange,

$$\frac{-\Delta P D_p e^3}{r u_0^2 L(1-e)} = \frac{150(1-e)m}{r u_0 D_p} + 1.75$$

multiply t hrough by $r u_0 D_p$

$$\frac{-\Delta P e^3}{u_0 L(1-e)} D_p^2 - 1.75 r u_0 D_p - 150(1-e)m = 0$$

$$a D_p^2 + b D_p + c = 0$$

quadratic equation, solve for the root (u_0), plug in data,

$$m \;\&\; r(\text{air from tables}) = 10^{-4}\frac{kg}{m.s} \;\;\&\;\; 1.2\frac{kg}{m^3}$$

$$-\Delta P = 10\,psi \times \frac{atm}{14.7\,psi} \times \frac{1.01325e5\frac{N}{m^2}}{atm} = 6.893\text{E}+04 \text{ Pa}(\frac{N}{m^2})$$

$$u_0 = \frac{Q}{\frac{p}{4}D_e^2} = 0.212\frac{m}{s}$$

$$1.071\text{E}+05 D_p^2 - 4.456\text{E}-01 D_p - 9.750\text{E}-03 = 0$$

$$D_p = 3.038\text{E}-04 \text{ m}$$

reject - ve root

Q 6.

Ergun equation,

$$\frac{-\Delta p}{\rho u_0^2}\,\frac{D_p}{L}\,\frac{\varepsilon^3}{1-\varepsilon} = \underbrace{\frac{150}{Re}}_{\text{Laminar}} + \underbrace{1.75}_{\text{Turbulent}}, \qquad (4.22)$$

And Reynolds number is:

$$Re = \frac{\rho u_0 D_p}{(1-\varepsilon)\mu}.$$

Substitute Re # and rearrange,

$$\frac{e^3}{(1-e)^2} = \frac{150 u_0 L m}{D_p^2(-\Delta P)} + \frac{1.75\, r u_0^2 L(1-e)}{D_p(-\Delta P)}$$

Data

$r = 1.200\ \mathrm{kg}/\mathrm{m}^3 \qquad m = 1.000\mathrm{E}-04\ \mathrm{kg.m}/\mathrm{s} \qquad L = 0.2m \qquad D_p = 1.000\mathrm{E}-03\ \mathrm{m}$

$$\mathrm{Q} = \frac{4\ l/\mathrm{min}}{60\ \mathrm{s}/\mathrm{min} \times 1000\ l/m^3} = 6.667\mathrm{E}-05\ \mathrm{m/s}$$

$$-\Delta P = 10\, psi \times \frac{atm}{14.7\, psi} \times \frac{1.01325e5 \frac{N}{m^2}}{atm} = 6.893\mathrm{E}+04\ \mathrm{Pa}(\frac{N}{m^2})$$

$$u_0 = \frac{Q}{\frac{p}{4} D_e^2} = 0.212 \frac{m}{s}$$

plug in data, solve for $e = 0.1846$ by trial and error.