

How to determine the steady state value of $x(t)$ from its Laplace Transform expression?

Given $X(s)$ what is $\lim_{t \rightarrow \infty} x(t)$

Approach 1: Use Final Value Theorem

Approach 2: Find the expression of $x(t)$ using Inverse Laplace Transform then take the limit.

Final Value Theorem

$$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$$

provided all poles of $\{sX(s)\}$ have negative real part otherwise the limit does not exist.

Examples

Example 1: find $x(\infty)$ when $X(s) = \frac{s+2}{s(s+1)(s+10)}$

Solution :

all poles of $\{sX(s)\}$ have negative real part $\{-1, -10\}$

$$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s) = \lim_{s \rightarrow 0} \frac{s+2}{(s+1)(s+10)} = 0.2$$

Example 2: find $x(\infty)$ when $X(s) = \frac{s-2}{(s-1)(s+3)}$

Solution :

one of the poles of $\{sX(s)\}$ have positive real part $\{1\}$

Final value Theorem can not be applied.