SE207 Modeling and Simulation Unit 3

Translational Mechanical Systems

Dr. Samir Al-Amer

Term 072

Read Sections 2.1 and 2.2

SE207 Modeling and Simulation
Lesson 1: Translational Mechanical Systems

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Read Sections 2.1 and 2.2

Modeling Procedure

- Represent the system by a set of **idealized elements**
- Define a set of suitable **variables**
- Write the appropriate **element laws**
- Write the appropriate **interaction laws**
- **Combine** element and interaction laws to write the **model** of the system
- If the model is nonlinear, select an appropriate equilibrium condition and **linearize the system**

Steps of Modeling

- Translational mechanical systems involve motion in lines and no rotation.
- Translational
- Rotational

Outlines:

- Variables x, v, a, f, w, p
  - x displacement
  - v velocity
  - a acceleration
  - f force
  - w energy,
  - p power,
- Element Laws
  - Mass
  - Friction Element
  - Stiffness Elements

Variables

The equations are expressed in terms of one or more of the following types of variables

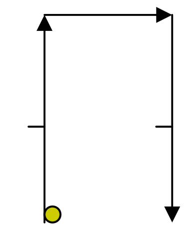
Variables	Unit	symbol
displacement	meter	x
velocity	meter per second	v
acceleration	meter per second per second	a
force	Newton	f
energy	Joule or Newton-meter	w
power	Watt	p

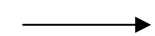
Displacement

- Displacement is defined as the change of position from one point in a frame of reference to another.
- Displacement is a vector quantity (specified by magnitude and direction)
- You need to specify the reference point and the positive direction.

Distance and Displacement

- Distance is a scalar quantity where as displacement is a vector quantity.



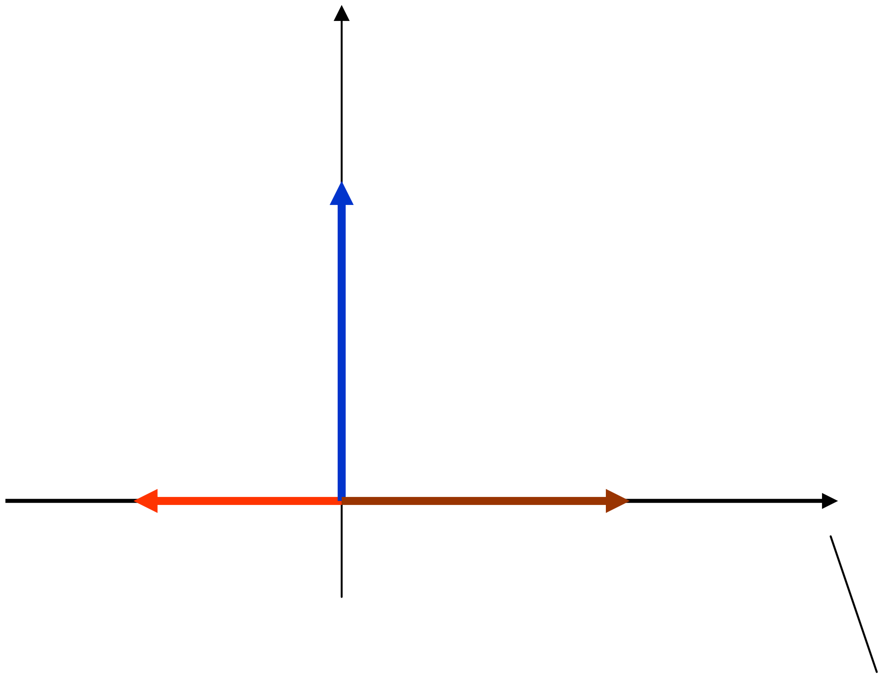
- Distance traveled = 5 units,
- Displacement = 1 unit in the direction 

Direction for displacement, velocity and acceleration

- Usually we select the reference direction to be the same for displacement, velocity and acceleration of a point of interest. As a result, only one direction is shown.
- With this selection we have

$$v = \frac{dx}{dt}, \quad and \quad a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

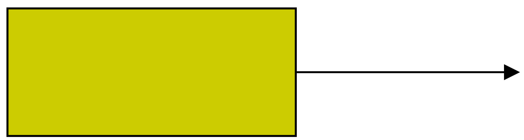
Reference Direction and True Direction



When we draw graphs we show reference directions

Reference Direction and True Direction

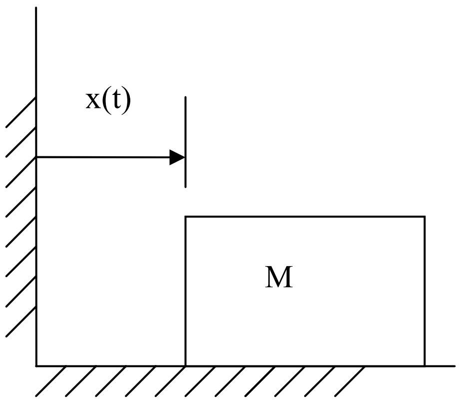
- The reference direction does not carry any information about the actual direction

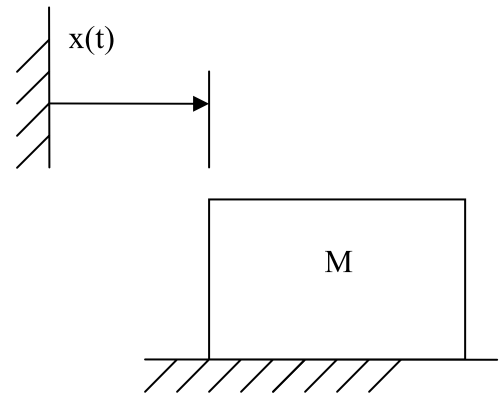


F=cos(0.5t)

- Reference direction
- For $0 < t < \pi$ the actual direction is the same as reference direction.
- For $\pi < t < 3\pi$ the actual direction is opposite to the reference direction.

Reference Position





x (t) represents the distance from the left side of the body to a fixed wall.

The same as in the other figure but the wall is not shown.

Displacement

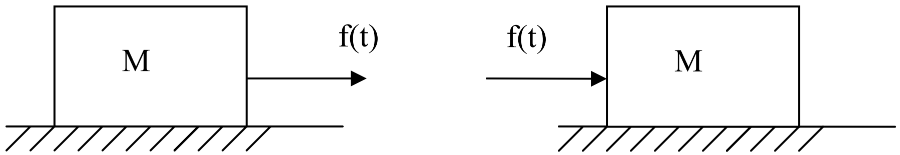
Notation

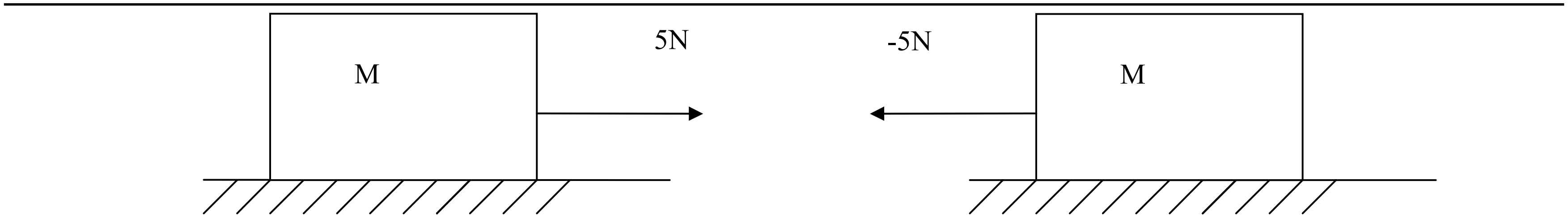
The reference position { $x = 0$ } is usually selected as <u>equilibrium position</u> (position when there is no external force)

Displacement $x = -3$ meters

means 3 meters away from the reference position and in a direction opposite to the reference direction.

- The direction of the force indicates the positive sense.
- $f = -5$ N means that the force is equal to five Newtons and its direction is opposite to the shown reference direction.





- The reference directions, in the above figures are different but the actual forces are the same.

Other variables

Energy & Power

- Energy $$w = f \quad x$$

- Power $$p = f \quad v = \frac{dw}{dt}$$

$$w = w(t_0) + \int_{t_0}^{t} p(z)dz$$

Element Laws

Physical devices are represented by one or more idealized elements. Some approximation is needed.

Elements included in translational mechanical systems:

- Mass
- Friction elements
- Stiffness elements

Mass

- The **mass** is defined as the quantity of mater in an object.
- The standard unit of mass is kilograms.
- When a force f is applied to a mass, its velocity is changed according to

$$f = \frac{d}{dt}(Mv)$$

$$f = M\frac{dv}{dt} \quad \text{if the mass is constant}$$

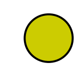
Element Laws
Energy stored in mass

Energy is stored in mass in the form of

1. Kinetic energy (KE) :

Kinetic energy of a mass M is moving at the velocity v is

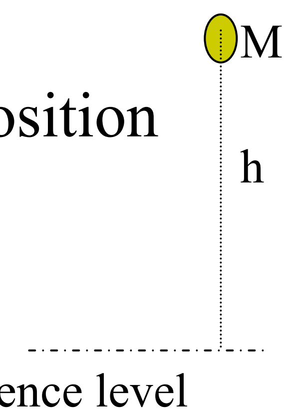
$$KE = \frac{1}{2} M v^2$$



2. Potential energy (PE):

potential energy stored in an object because of its position or shape.

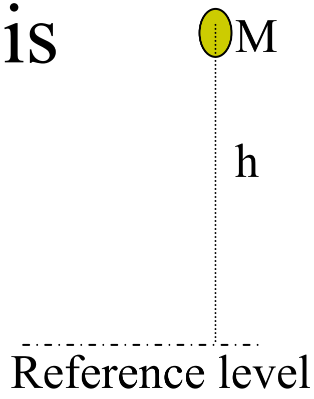
$$PE = M\ g\ h$$



Potential energy

When an object is raised h meters above its reference position, its potential energy is increased by

$$PE = Mgh$$

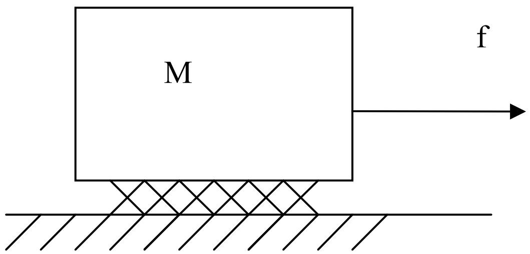


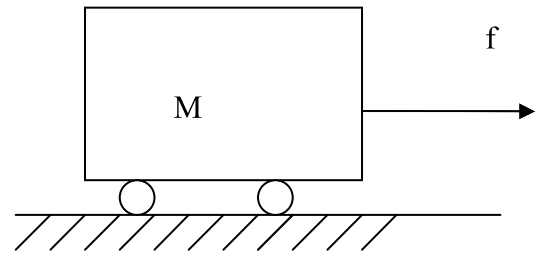
where g is the gravitational constant (9.81 N/s).

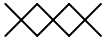
Friction

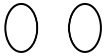
- When a body moves on a surface a friction force is acting in the opposite direction of motion.
- No friction is indicated by small wheels

Friction between surfaces





 Indicates friction between the two surfaces

 Indicates no friction between the two surfaces

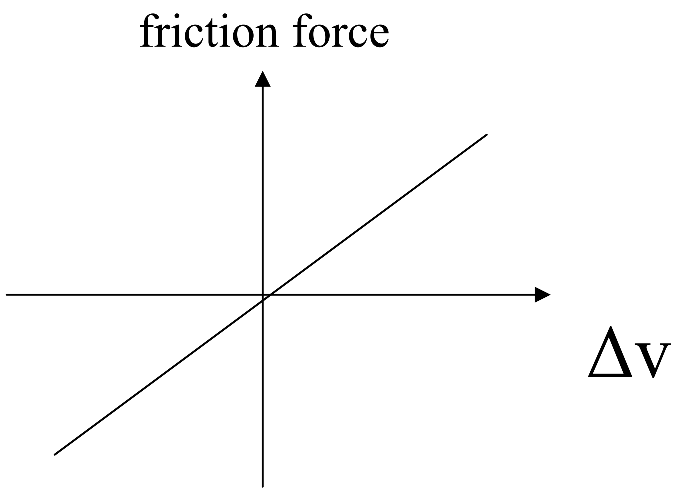
(used when friction can be neglected such as the case when bodies are separated by bearings)

Friction Characteristics

- Linear Friction (viscous friction):
  The friction force is proportional to the difference in velocities of the two objects.

- Nonlinear friction: the friction force is a nonlinear function of difference in velocities
  Examples (drag friction and dry friction)

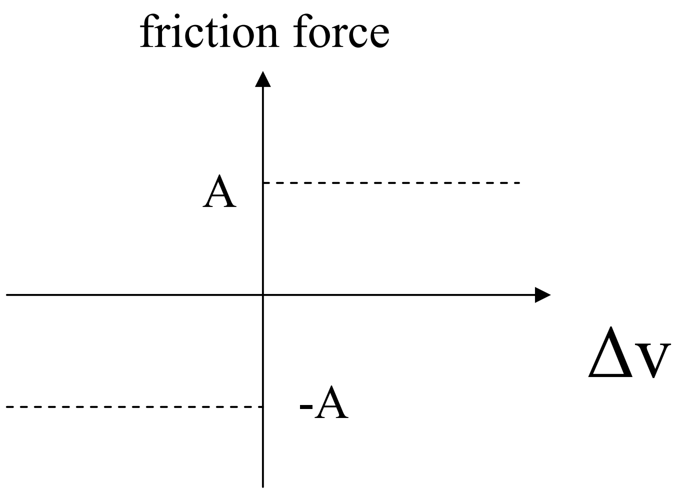
Examples of friction characteristics
Linear Friction (viscous Friction)

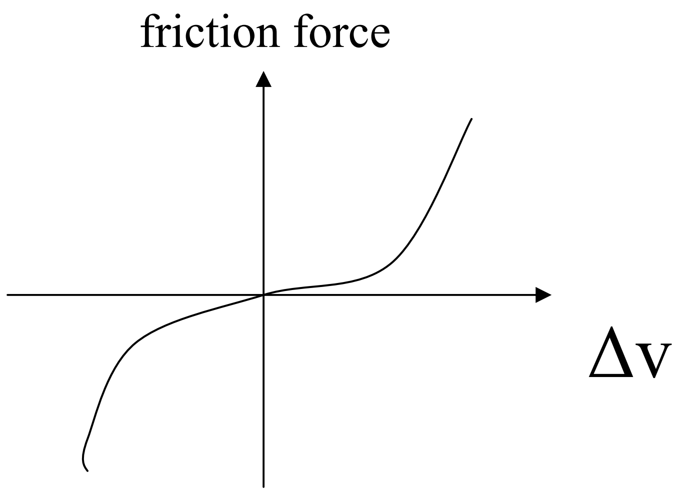


- The viscous friction force is proportional to the relative velocity
- Use to model two bodies with thin layer of oil between them
- used to model dashpot (dampers)

Examples of friction characteristics

nonlinear Friction





Dry Friction

friction force $f = A \, \mathrm{sign}(\Delta v)$

(friction between dry surfaces)

Drag

friction force $f = A \, |\Delta v| \, \Delta v$

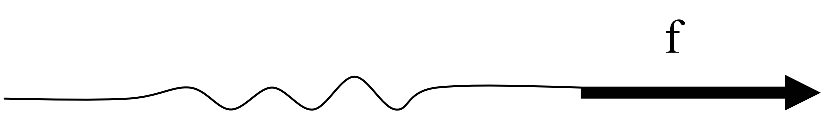
(air resistance)

Stiffness Elements

(springs)

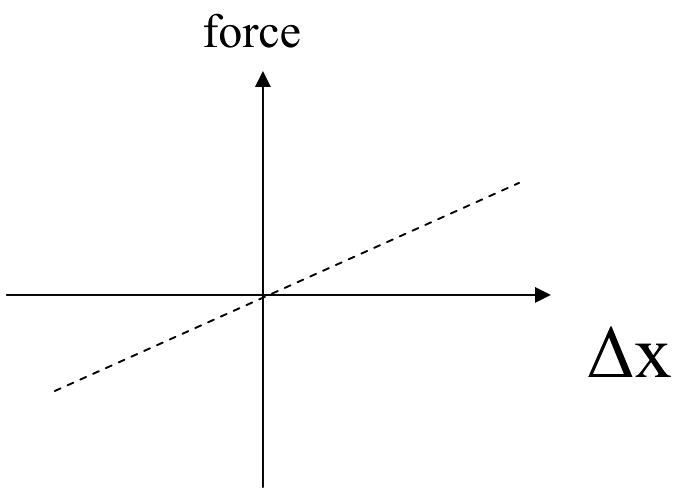
- Mechanical elements that change shape when subjected to force can be modeled by stiffness elements
- Spring is the common stiffness element
- Linear spring $f = k \Delta x$

No force length = d

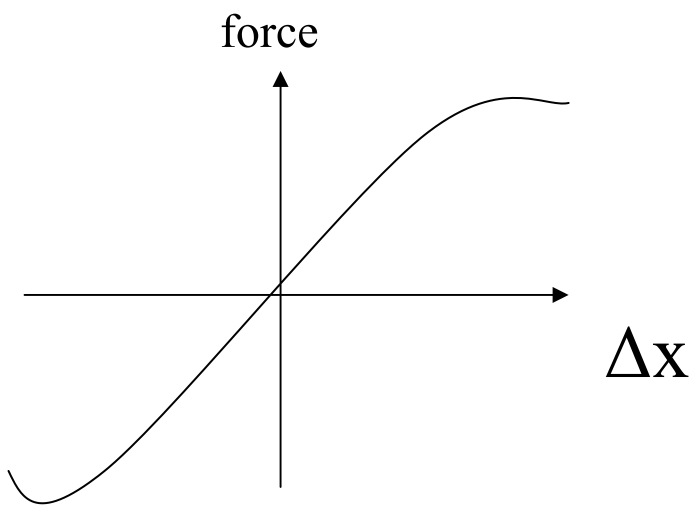


Force =f length = $d + \Delta x$

Examples of Spring characteristics



Linear spring Characteristics



Nonlinear spring Characteristics

Energy stored

- Spring (stiffness elements) stores energy in the form of potential energy

$$PE = \frac{1}{2} k \left(\Delta x \right)^2$$

- Mass stores energy , in the form of kinetic energy or potential energy

$$PE = M g h$$

$$KE = \frac{1}{2} M v^2$$

- Dampers (or Friction elements) dissipate energy.

Reading assignment

- Reading Assignment: Section 2.1-2.2

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Lesson 2: Translational Mechanical Systems2

Dr. Samir Al-Amer
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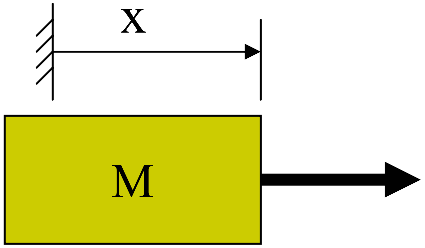
Read Sections 2.3 and 2.4

Variables

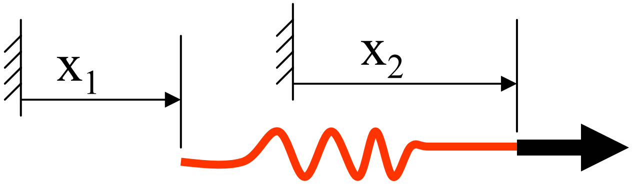
The equations are expressed in terms of one or more of the following types of variables

Variables	Unit	symbol
displacement	meter	x
velocity	meter per second	v
acceleration	meter per second per second	a
force	Newton	f
energy	Joule or Newton-meter	w
power	Watt	p

Element Laws

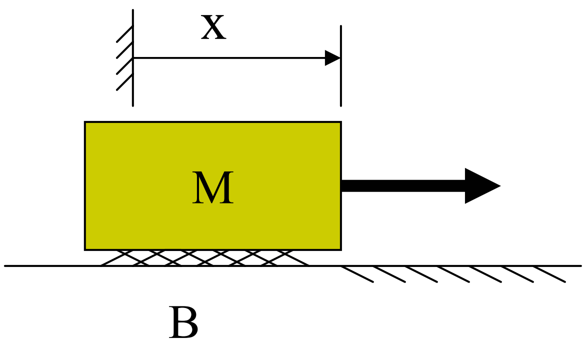


$$f_I = M \ddot{x}$$

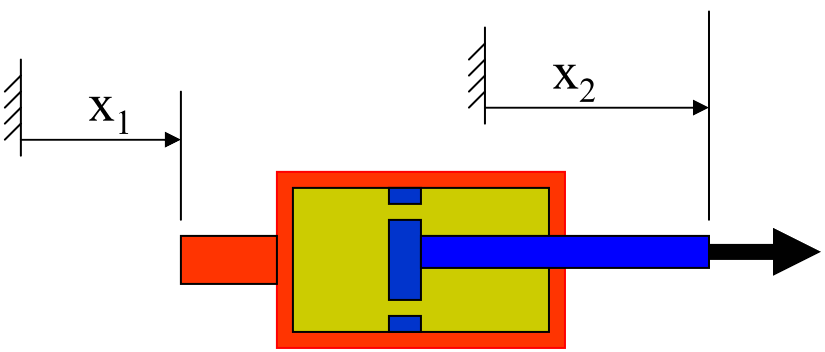


$$f_k = K (x_2 - x_1)$$

Element Laws



$$f_B = B\ \dot{x}$$



$$f_B = B\ (\dot{x}_2 - \dot{x}_1)$$

Interaction Laws

- Interaction laws describe the way different elements are interconnected
- Three important laws will be covered here
  - D'Alembert's Law
  - The Law of Reaction Force
  - The Law of Displacement

D’Alembert’s Law

D’Alembert’s Law is a restatement of the Newton’s Second Law

$$\sum_i (f_{ext}) = M\frac{dv}{dt} \qquad \text{Newton's Second Law}$$

$$\text{The sum of external forces} = M\frac{dv}{dt}$$

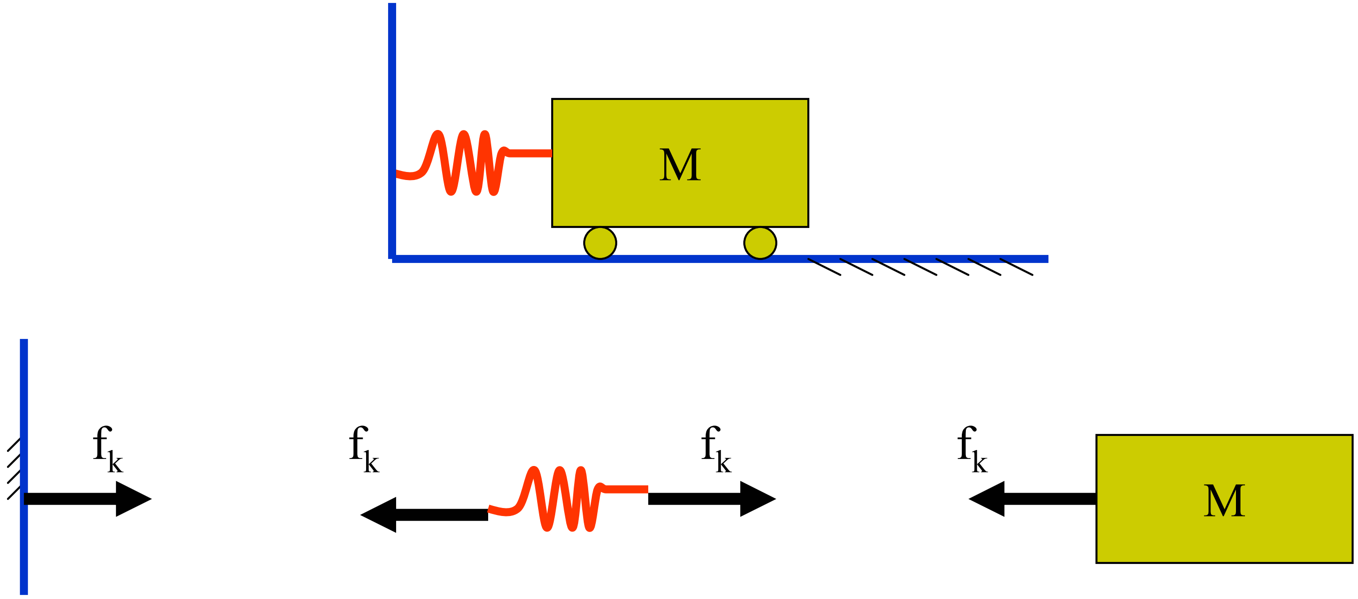
$$Let\ innertial\ \ force = f_I = -M\frac{dv}{dt}$$

innertial force is a fictitious force but will be treated as the external forces

$$\sum_i f_i = 0 \qquad \text{D'Alembert's Law}$$

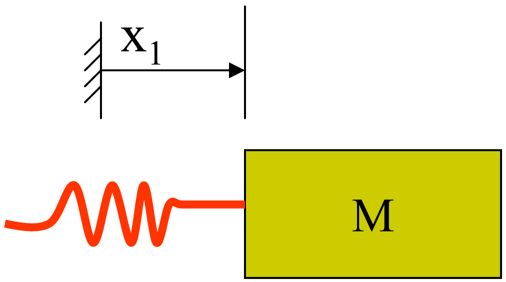
The Law of Reaction Forces

- The Law of Reaction Forces is Newtons' Third Law
- For any force of one element on another, there is a reaction force on the first element that equal in magnitude and opposite in direction.



The Law of Displacement

If the ends of two elements are connected then both end moves at the same velocity.



Since the left end of the mass and the right end of the spring are connected. Both are moving at the same displacement x_1

The Law of Displacement

Alternative statement

The algebraic sum of elongation of elements around any closed path is zero.

This statement is used to emphases similarity between systems of different nature.

Recall Kirchhoff's voltage law:

" The algebraic sum of voltages across elements that make up closed path is zero"

Obtaining The Model

The model is obtained by combining the element laws and interaction laws.

Free Body Diagrams

- Freebody diagram shows all the forces acting on the mass or junction point and their directions.
- Draw freebody diagram for each mass or junction point that moves at unknown velocity.

Reading assignment

- Reading Assignment: Section 2.1-2.3

SE207 Modeling and Simulation Unit 3

Lesson 3: Examples of Translational Mechanical Systems

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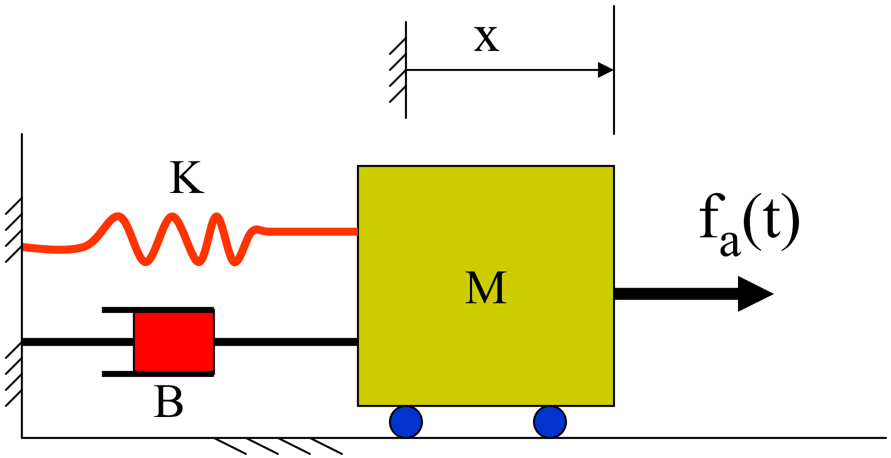
Read Sections 2.3 and 2.4

Obtaining The Model

Procedure

- Apply D'Alembert's Law for every mass and junction point that moves with unknown velocity.
- Draw freebody diagram
- Show all external forces and inertial force
- Use element laws to express all forces in terms of displacement, velocity and acceleration.

Example 1



We need to Apply D'Alembert's Law for the mass

One freebody diagram

<u>Interaction Law:</u>

Elongation in spring = x

Elongation in dashpot = x

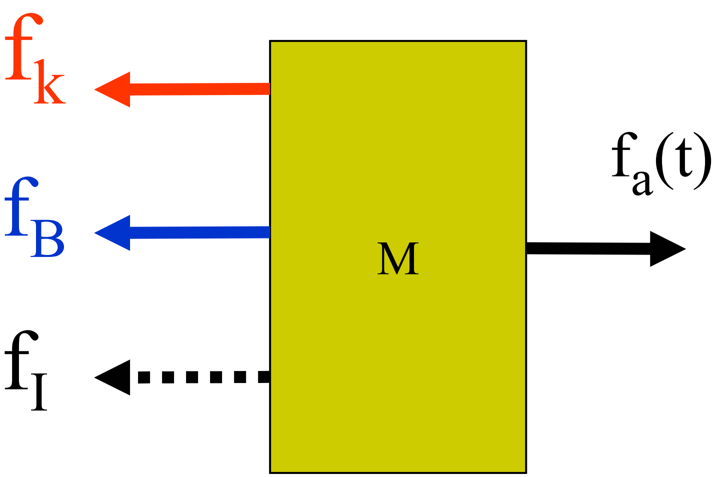
Example 1
Freebody diagram

Only one freebody diagram

Force due to Spring

Force due to dashpot

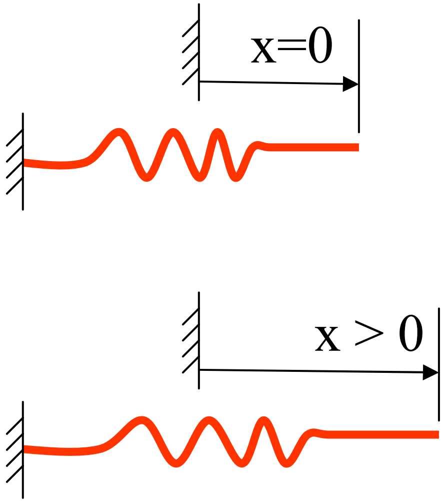
Inertial force



D'Alembert's Law: $f_k + f_B + f_I = f_a(t)$

Element Laws

Spring



Equilibrium position x = 0

When the mass position = x

Elongation of spring = x

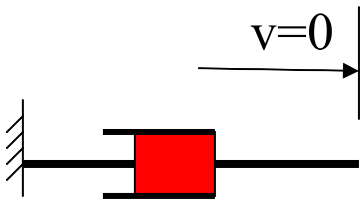
Force generated by the spring

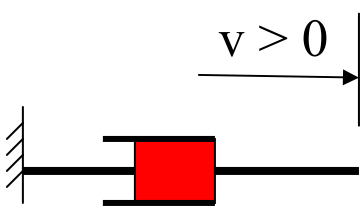
$$f_k = K \; x$$

$$Direction \quad \leftarrow$$

Element Laws

Dashpot





When v = 0 No friction force

When the mass moves at velocity v

Friction force generated by the dashpot

$$f_B = B\ v = B\ \dot{x}$$

$$Direction \quad \leftarrow$$

Element Laws
Mass

When acceleration a = 0

inertial force = 0

When acceleration a > 0

$$f_I = M\,a = M\,\ddot{x}$$

Direction $\leftarrow$

Example 1
Element Laws

D'Alembert's Law: $f_k + f_B + f_I = f_a(t)$

f_k : spring force (tensile force)

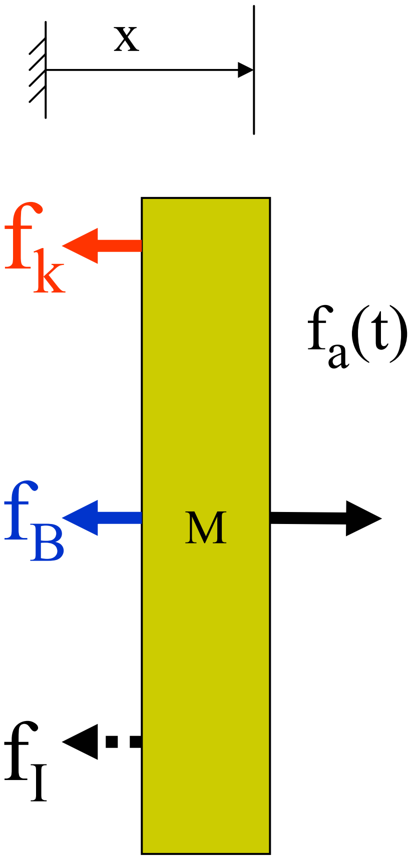
direction $\leftarrow$ magnitude $f_k = k\,x$

f_B : friction force

direction $\leftarrow$ magnitude $f_B = B\,\dot{x}$

f_I : inertial force

direction $\leftarrow$ magnitude $f_I = M\,\ddot{x}$



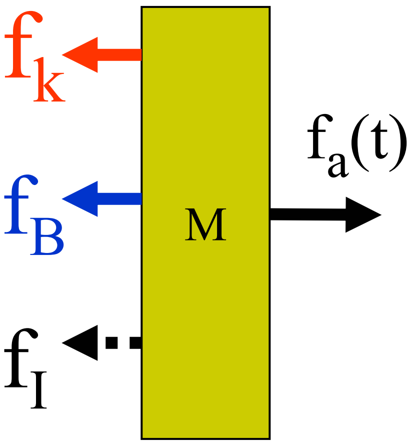
Example 1

Mathematical Model

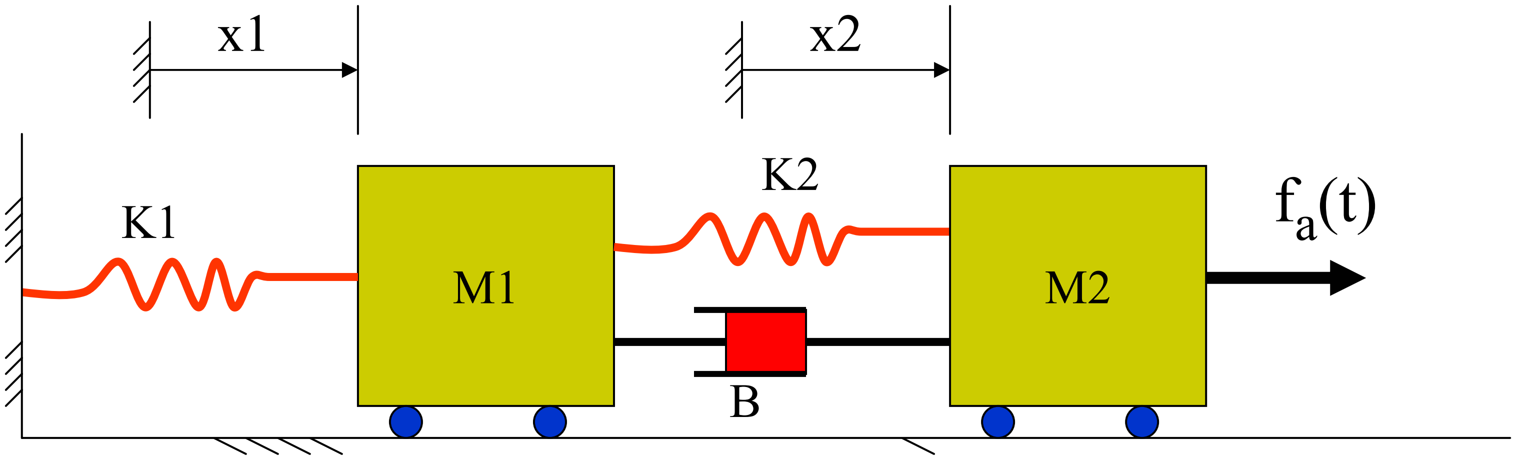
$$Model:$$

$$f_k + f_B + f_I = f_a(t)$$

$$K\,x + B\,\dot{x} + M\,\ddot{x} = f_a(t)$$



Example 2



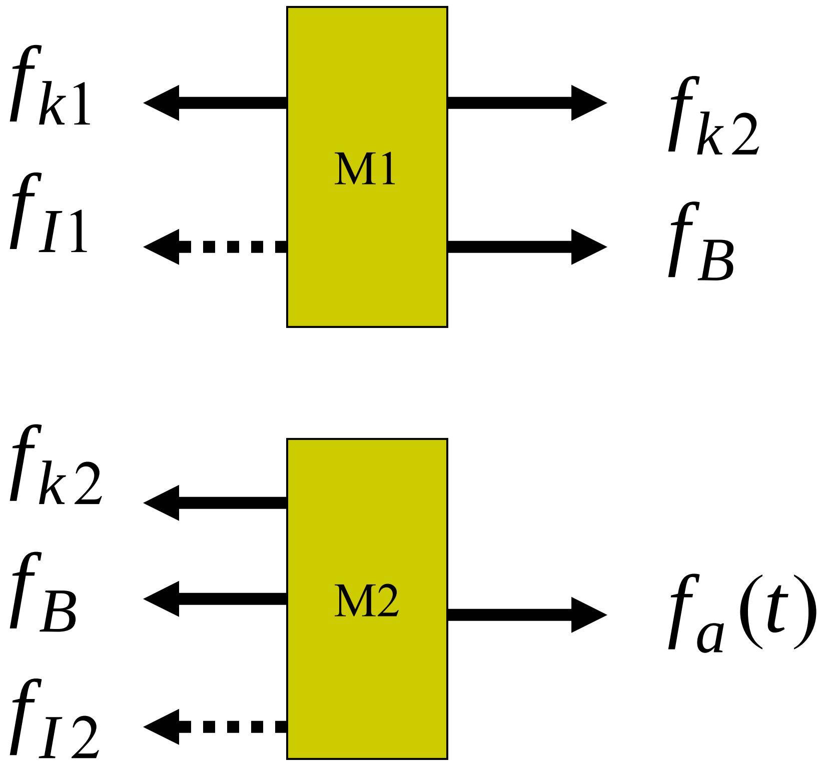
We need to Apply D'Alembert's Law for the two masses

Two freebody diagrams

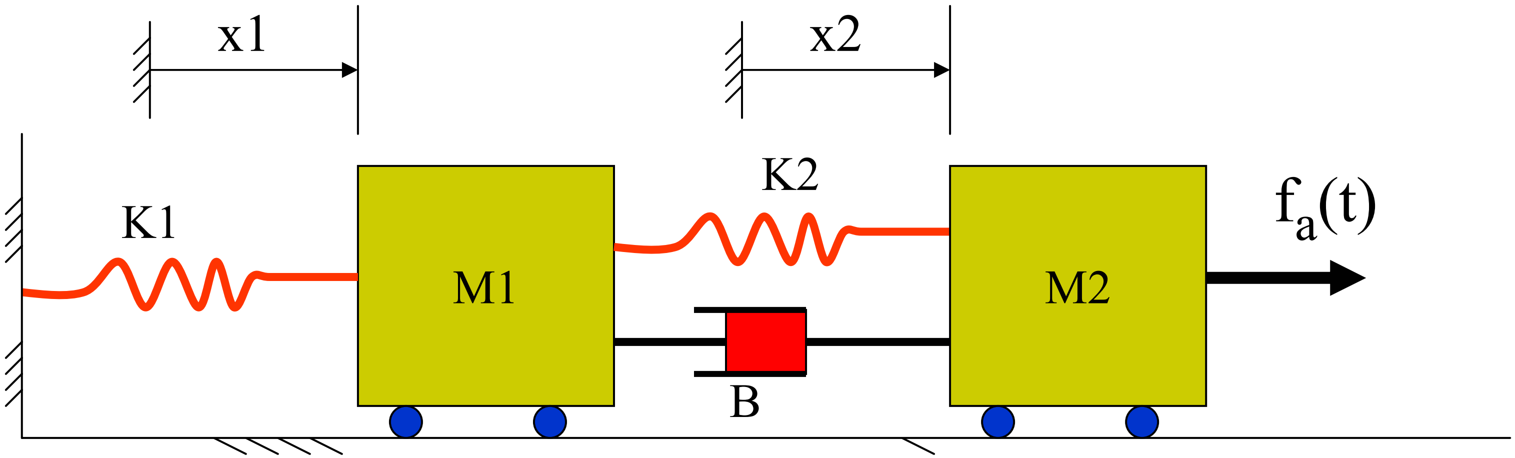
<u>Interaction Laws:</u> **Elongation in spring1 = x1**

Elongation in spring 2 = x2-x1 **Relative velocity in dashpot = v2-v1**

Example 2



Derive the model

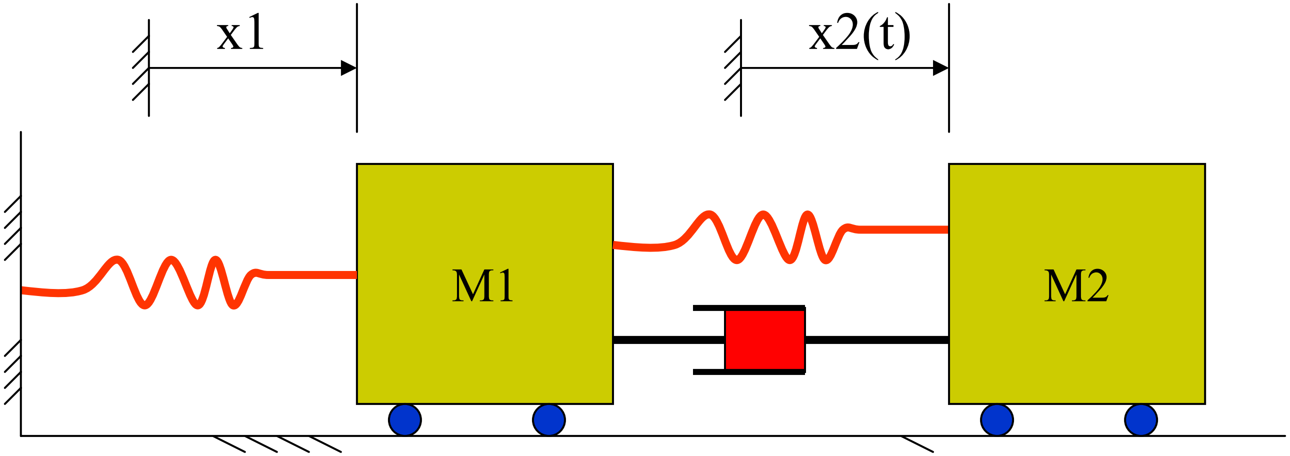


$f_{k1} =$, $f_B =$, $f_{I1} =$

$f_{k2} =$, $f_{I2} =$

$Model:$

Example 3



X2(t) is known

One freebody diagram

We need to Apply D'Alembert's Law for the M1 only

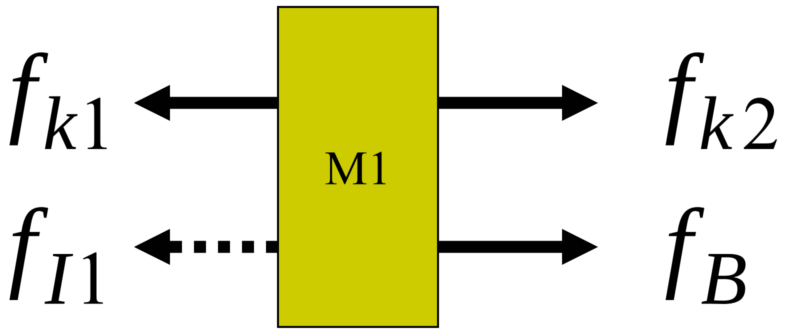
<u>Interaction Law:</u>

Elongation in spring 1 = x1

Elongation in spring 2 = x2-x1

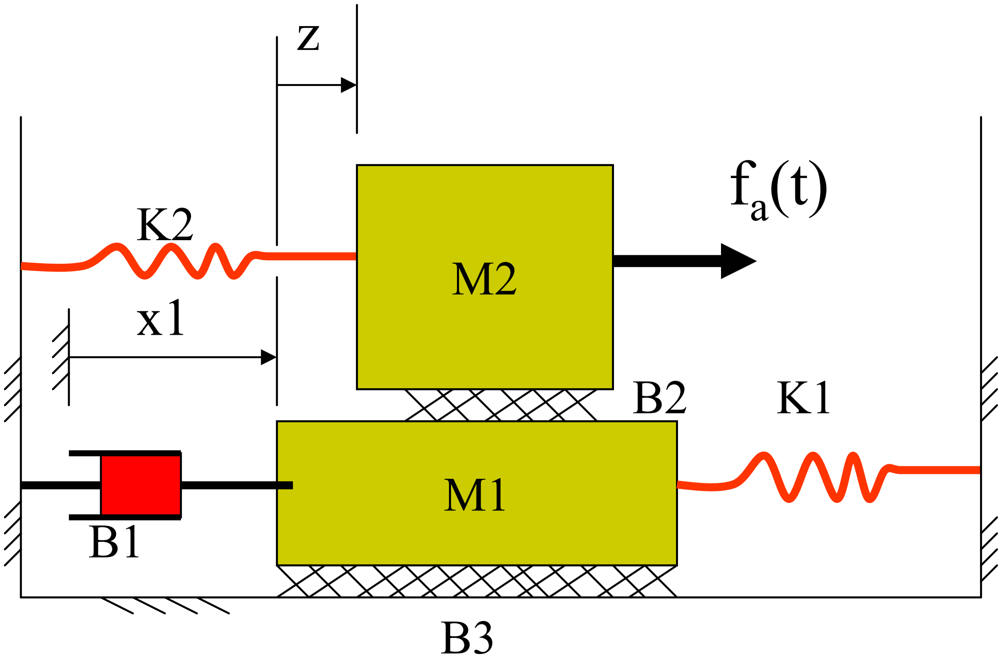
Elongation in dashpot = x2-x1

Example 3



$Model:$

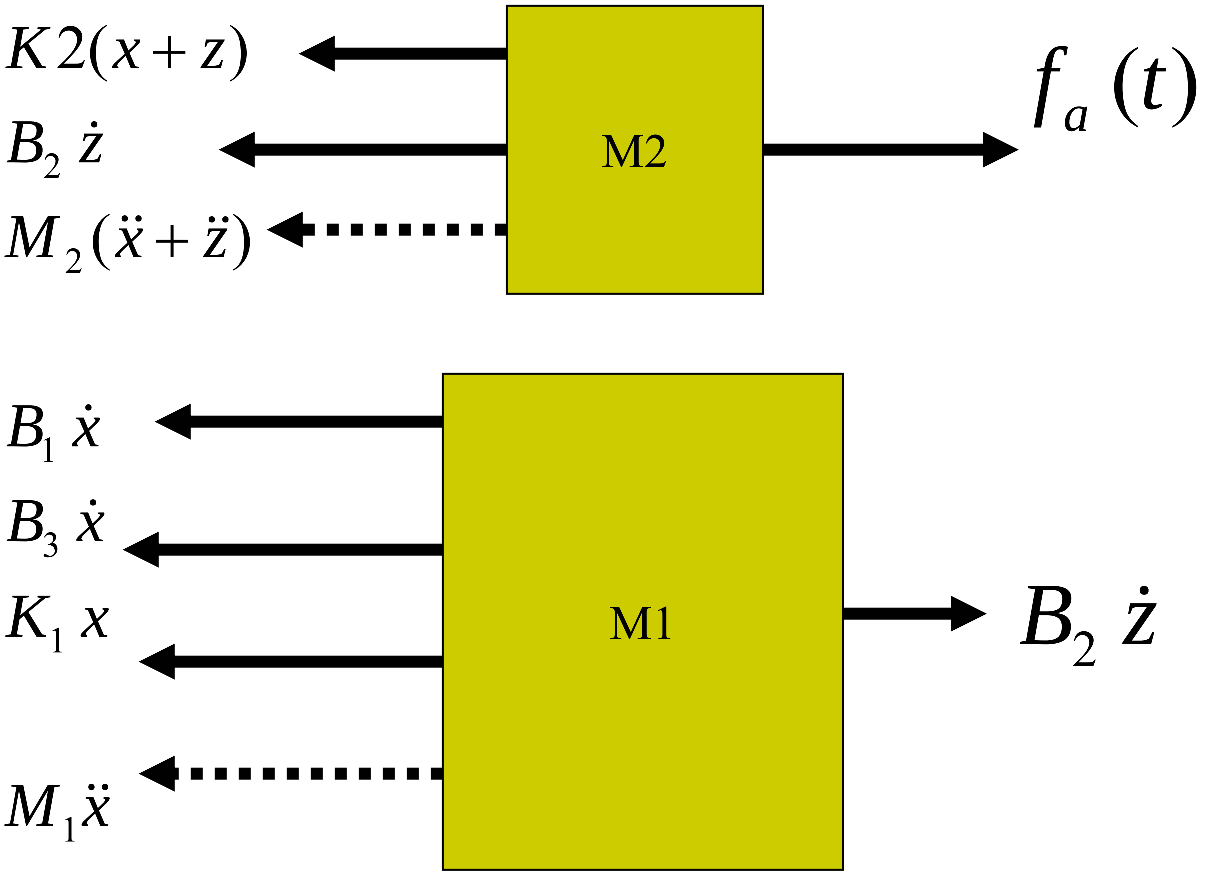
Example 4



Two masses moving at unknown velocities.

Two freebody diagrams

Example 4



$K2(x+z)$
$B_2\,\dot{z}$
$M_2(\ddot{x}+\ddot{z})$
M2
$f_a(t)$
$B_1\,\dot{x}$
$B_3\,\dot{x}$
$K_1\,x$
$M_1\ddot{x}$
M1
$B_2\,\dot{z}$

Model

$$M_1\ddot{x} + (B_1 + B_3)\dot{x} + K_1 x - B_2 \dot{z} = 0$$

$$M_2\ddot{x} + K_2 x + M_2\ddot{z} + B_2 \dot{z} + K_2 z = f_a(t)$$

Alternative Model

Define

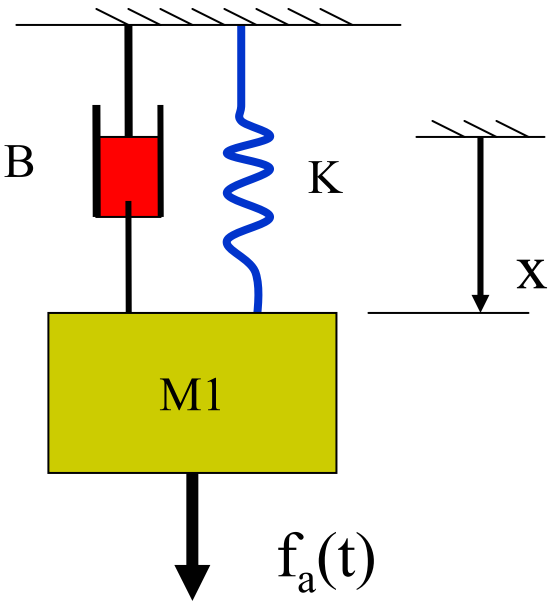
$$x_1 = x$$

$$x_2 = x + z$$

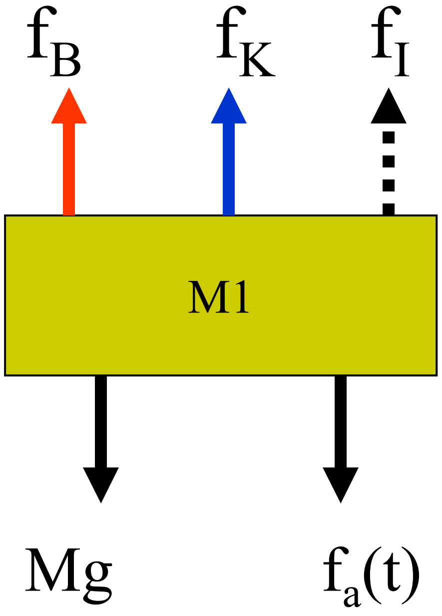
$$M_1 \ddot{x}_1 + (B_1 + B_2 + B_3)\dot{x}_1 + K_1 x_1 - B_2 \dot{x}_2 = 0$$

$$M_2 \ddot{x}_2 + K_2 x_2 + B_2 \dot{x}_2 - B_2 \dot{x}_1 = f_a(t)$$

Example 5



How many free body diagrams?



Example 5

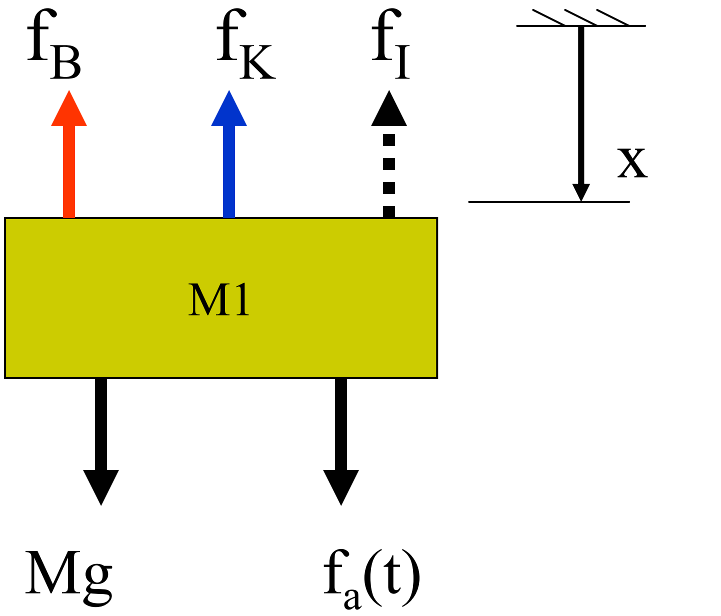
$$f_B =$$

$$f_k =$$

$$f_I =$$

$$Model:$$

$$f_B + f_k + f_I = Mg + f_a(t)$$



Model

$$M\ddot{x} + B\dot{x} + Kx = Mg + f_a(t)$$

Model

$$M\ddot{x} + B\dot{x} + K\,x = f_a(t) + Mg$$

Do we need to include the weight in the model?

$$Suppose\ \ x = x_0 \quad (when\ \ f_a(t) = 0),$$

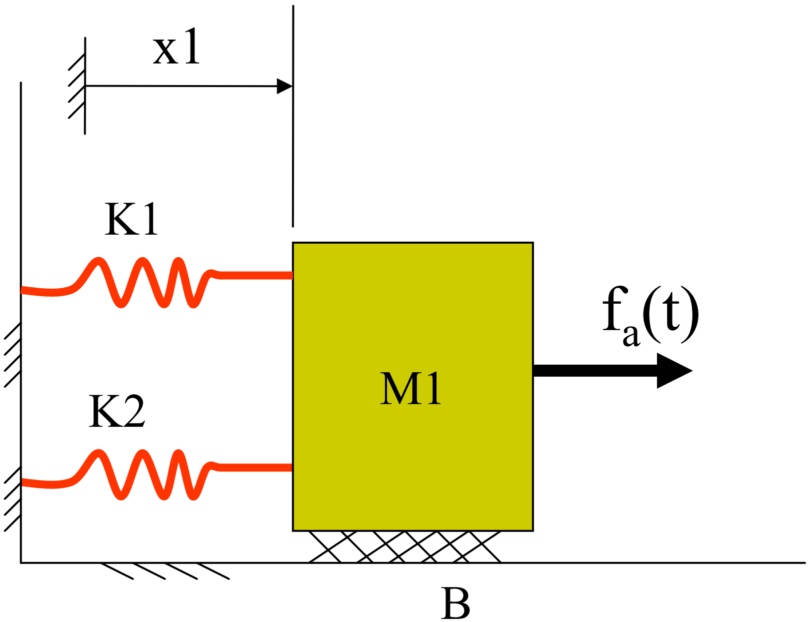
M is not moving

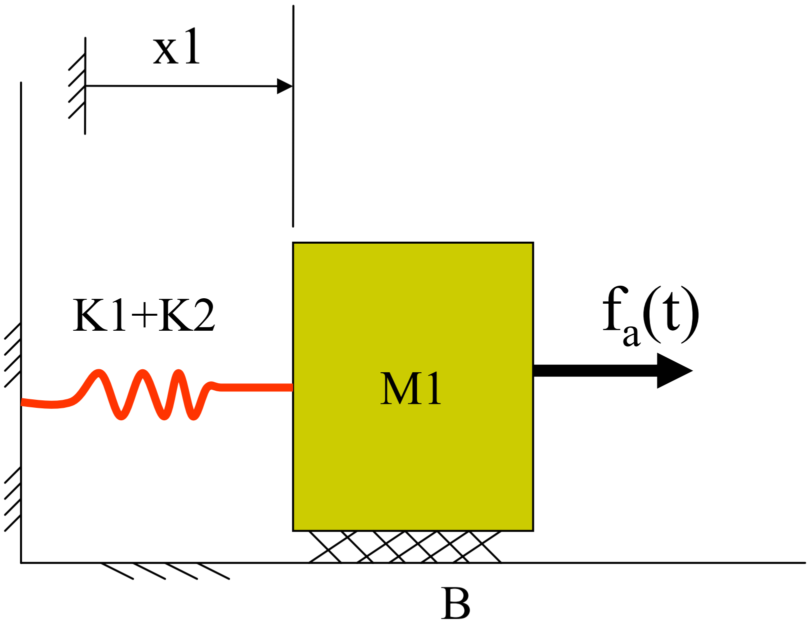
$$x = z + x_0$$

Model

$$M\ddot{z} + B\dot{z} + K\,z = f_a(t)$$

Example 6





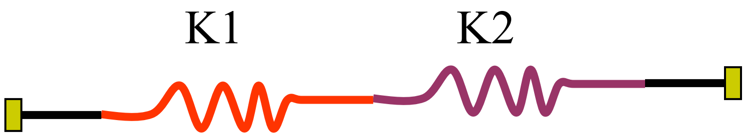
$$M\ddot{x} + B\dot{x} + (K_1 + K_2)x = f_a(t)$$

$$M\ddot{x} + B\dot{x} + K_{eq}x = f_a(t)$$

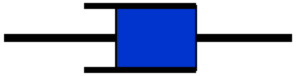
$$K_{eq} = K_1 + K_2$$

Parallel combination

Serial Combination



$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}$$



$$\frac{1}{B_{eq}} = \frac{1}{B_1} + \frac{1}{B_2}$$

SE207 Modeling and Simulation

Lecture 6: Translational Mechanical Systems Examples

Dr. Samir Al-Amer
Term 052

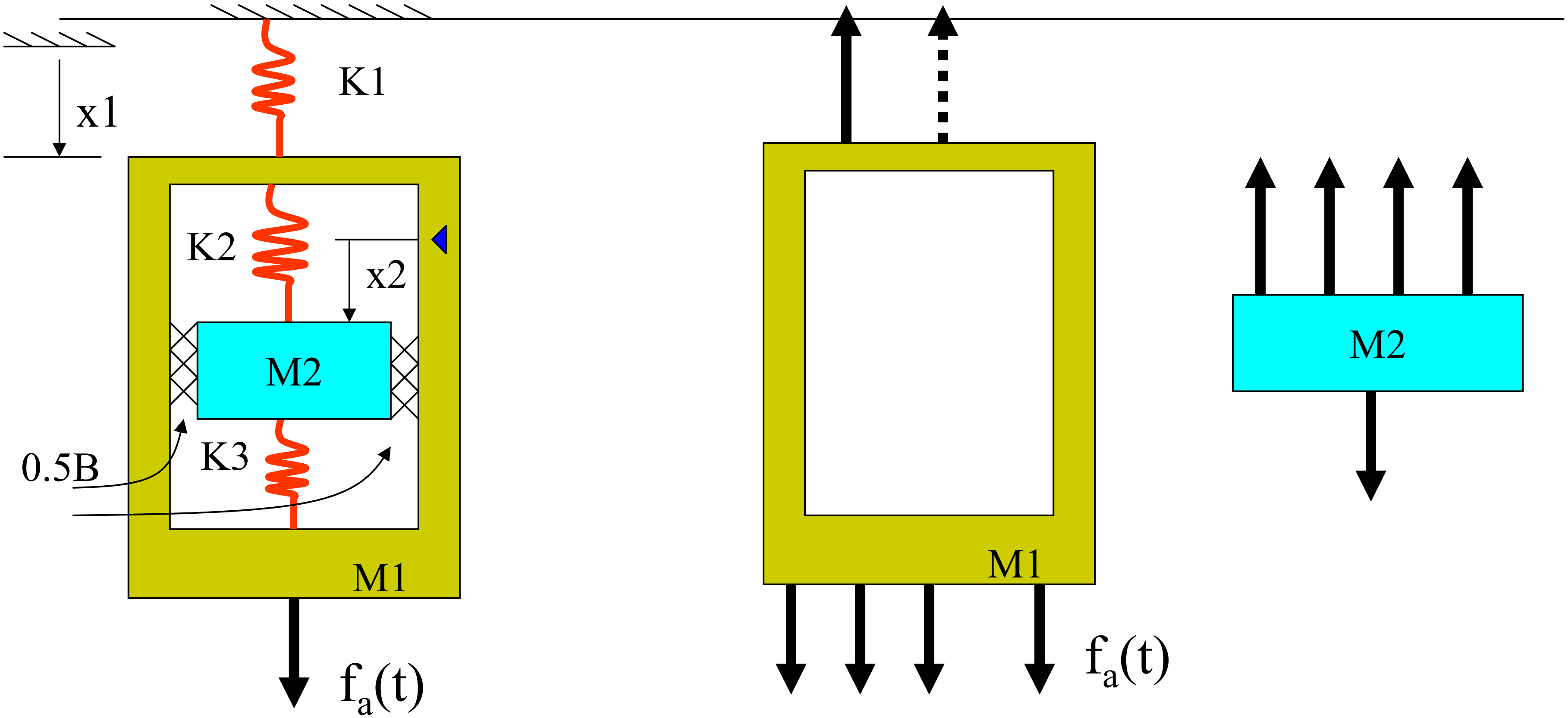
Read Sections 2.3 and 2.4

Obtaining The Model

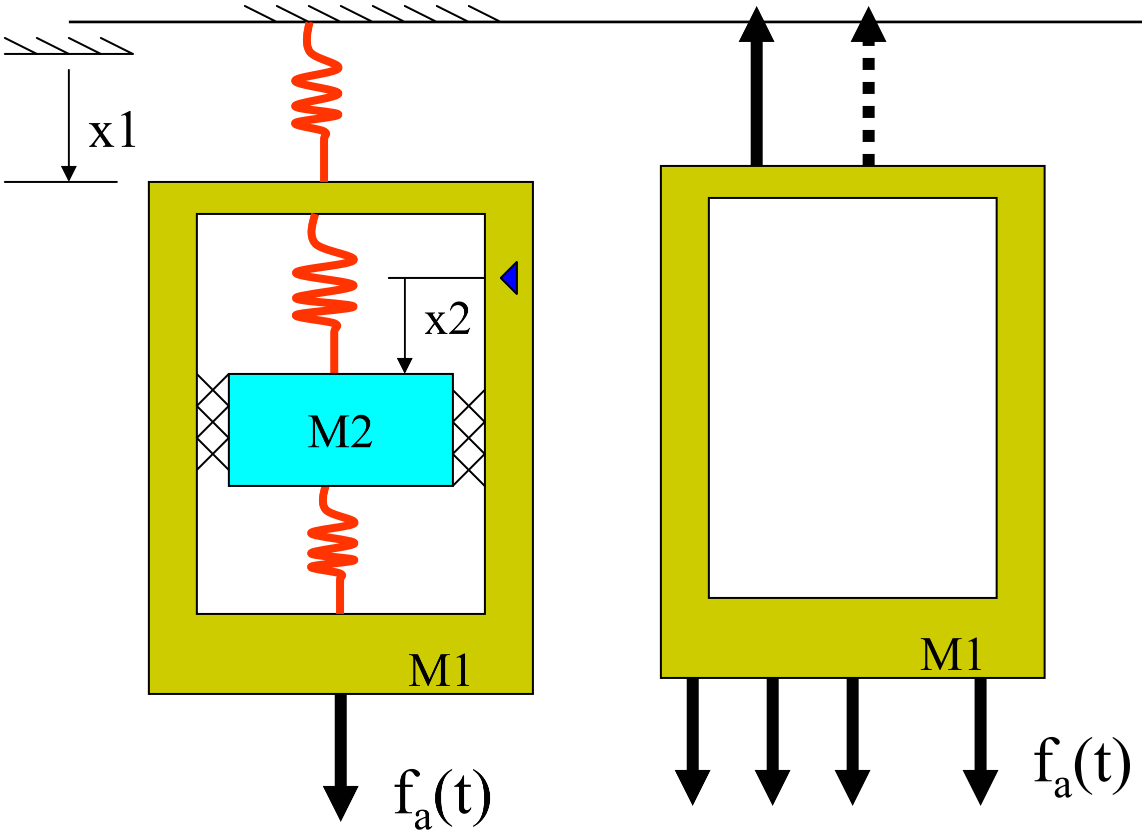
Procedure

- Apply D'Alembert's Law for every mass and junction point that moves with unknown velocity.
- Draw freebody diagram
- Show all external forces and inertial force
- Use element laws to express all forces in terms of displacement, velocity and acceleration.

Example 1

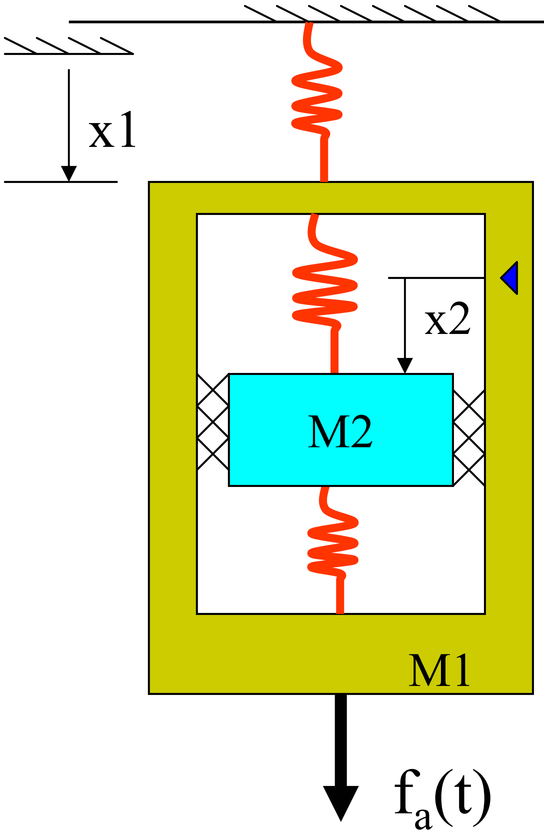


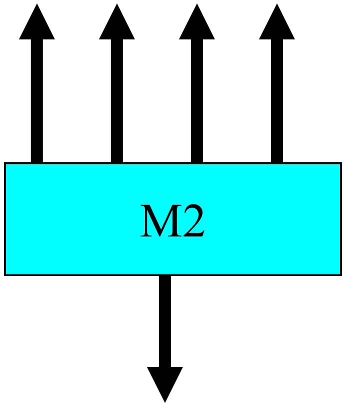
Example 1



$$M_1\ddot{x}_1$$
$$+K_1x_1$$
$$-K_2x_2$$
$$-K_3x_2$$
$$-B\dot{x}_2$$
$$-M_1g$$
$$-\ f_a(t)$$
$$=0$$

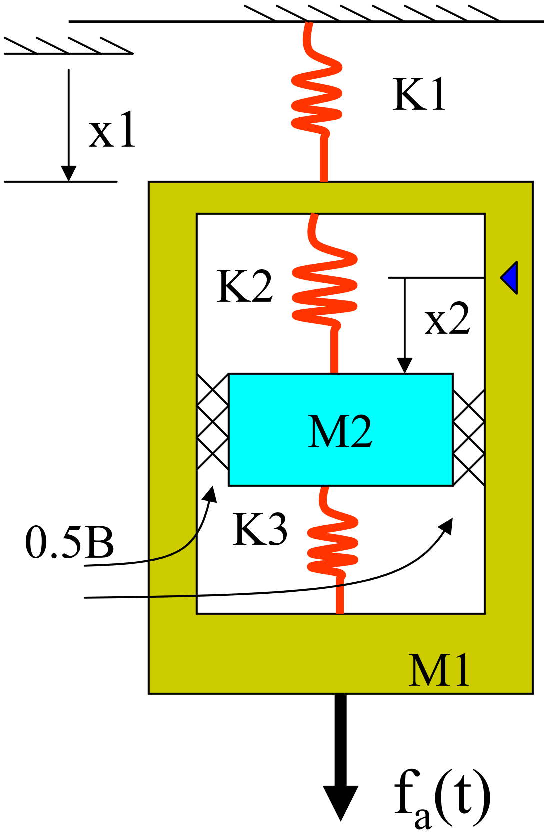
Example 1





$$
\begin{aligned}
&M_2(\ddot{x}_1 + \ddot{x}_2) \\
&+ B\dot{x}_2 \\
&+ K_2 x_2 \\
&+ K_3 x_2 \\
&- M_2 g \\
&= 0
\end{aligned}
$$

Example 1
Model

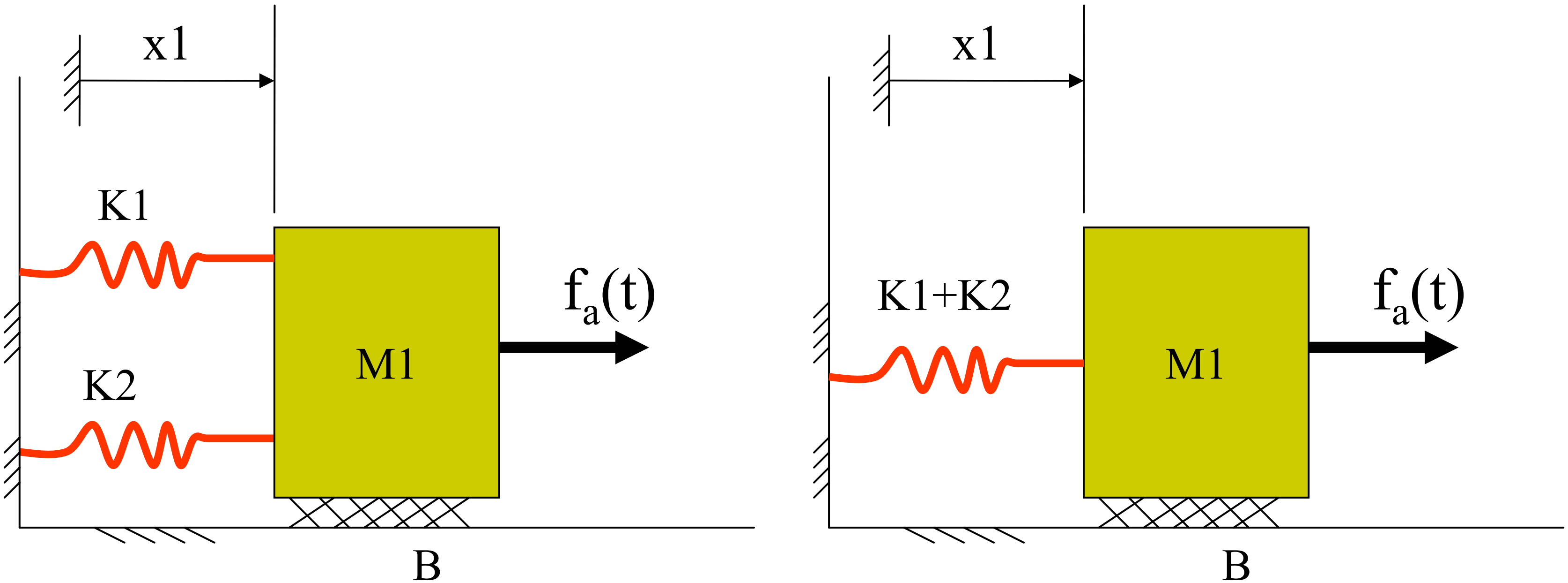


Model :

$$M_1\ddot{x}_1 + K_1 x_1 - B\dot{x}_2 - (K_3 + K_2)x_2 = M_1 g + f_a(t)$$

$$M_2\ddot{x}_1 + M_2\ddot{x}_2 + B\dot{x}_2 + (K_3 + K_2)x_2 = M_2 g$$

Example 2



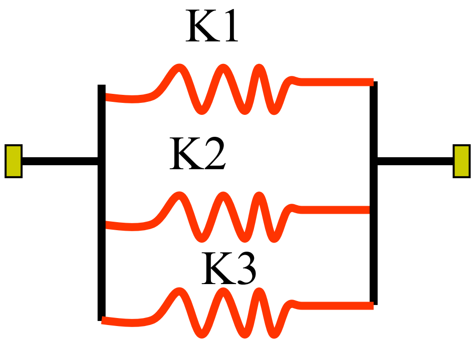
$$M\ddot{x} + B\dot{x} + (K_1 + K_2)x = f_a(t)$$

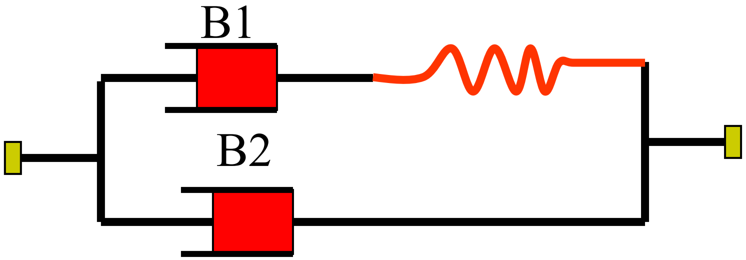
$$M\ddot{x} + B\dot{x} + K_{eq}x = f_a(t)$$

$$K_{eq} = K_1 + K_2$$

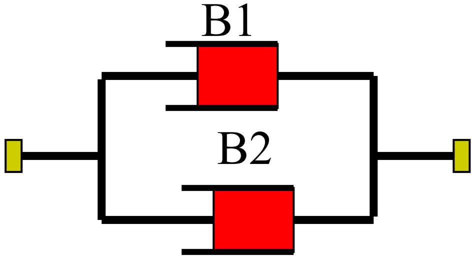
Parallel combination

- Two springs are in parallel if
  - first end of both springs are attached to one body
  - Second end of both springs are attached to another body.
- Similar definition can be used for dashpots





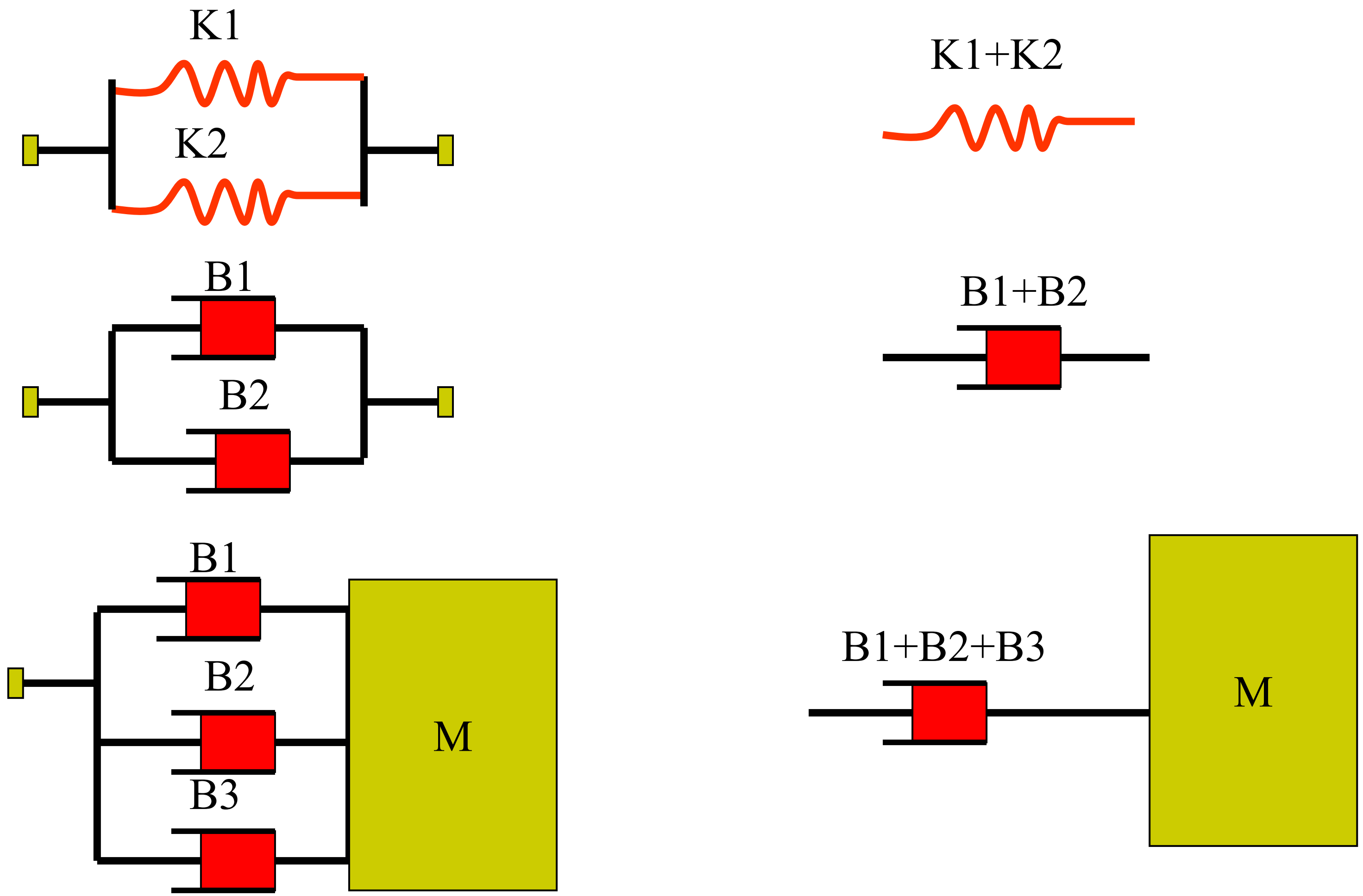
Not in parallel



In parallel

Parallel combination

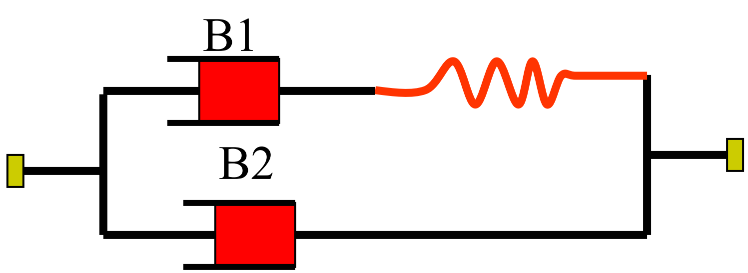
Reduction



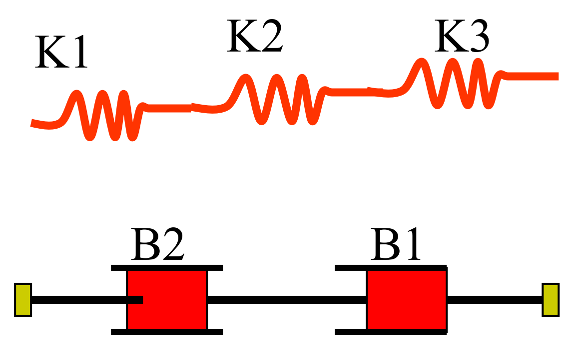
Serial combination

- Two springs are in series if they are joined at one end only and no other element is connected to their common junction

- Similar definition can be used for dashpots

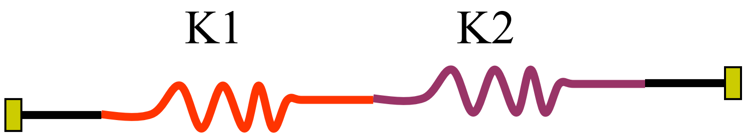


Not in series

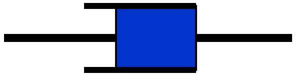


serial Combination

Serial Combination



$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}$$

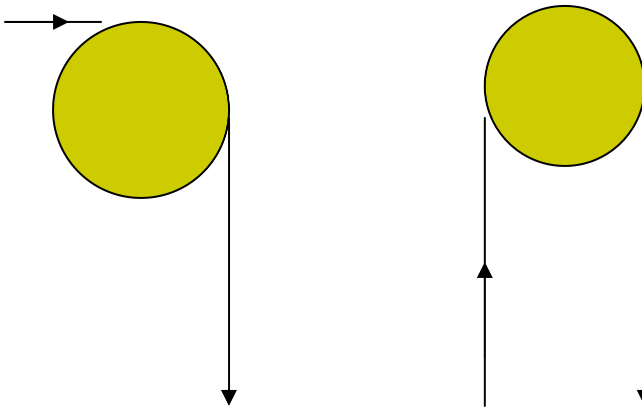


$$\frac{1}{B_{eq}} = \frac{1}{B_1} + \frac{1}{B_2}$$

Ideal Pulley

Pulley:

Consists of a cylinder that rotates about its center. Used to change direction of force.



Ideal Pulley

- No mass
- No friction
- No slipping between pulley and cable
- Cable resting on the pulley surface.

Ideal Cable

Ideal cables can not be stretched.

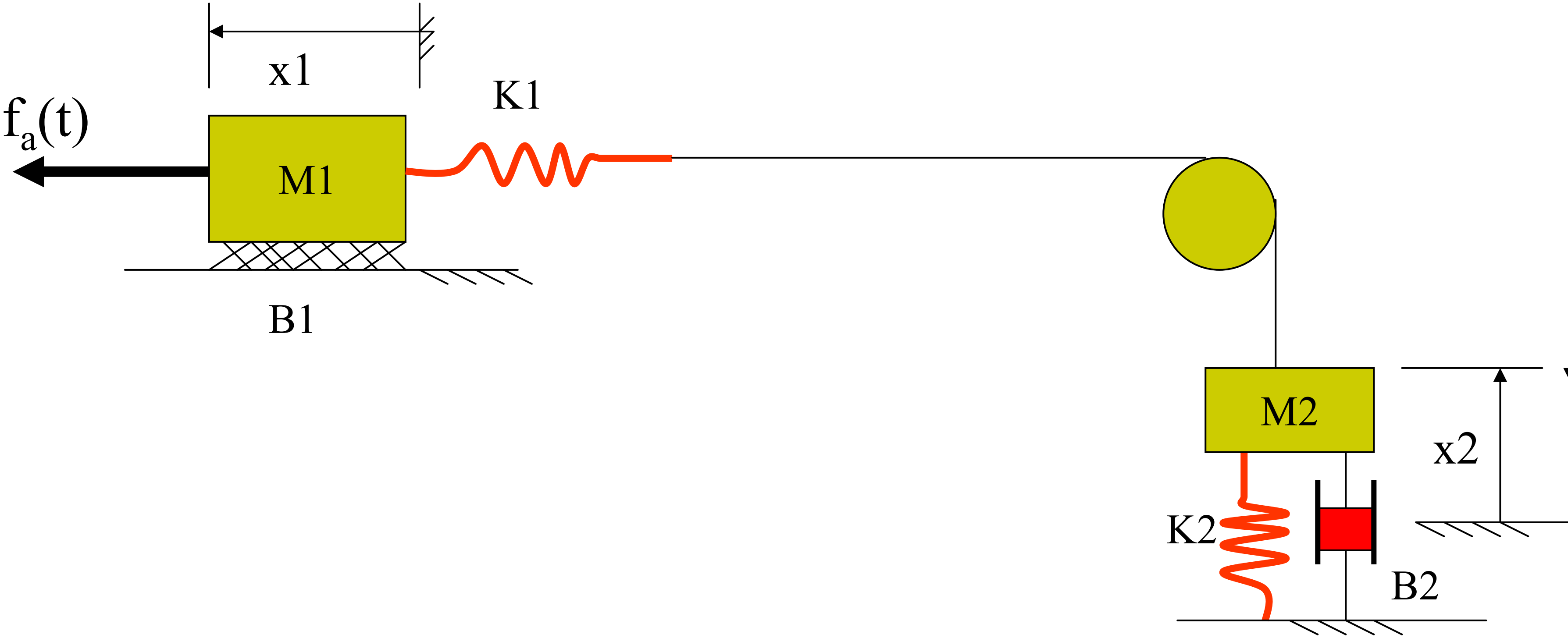
A realistic cable is allowed to stretch ,

it is modeled as an ideal cable connected to a spring

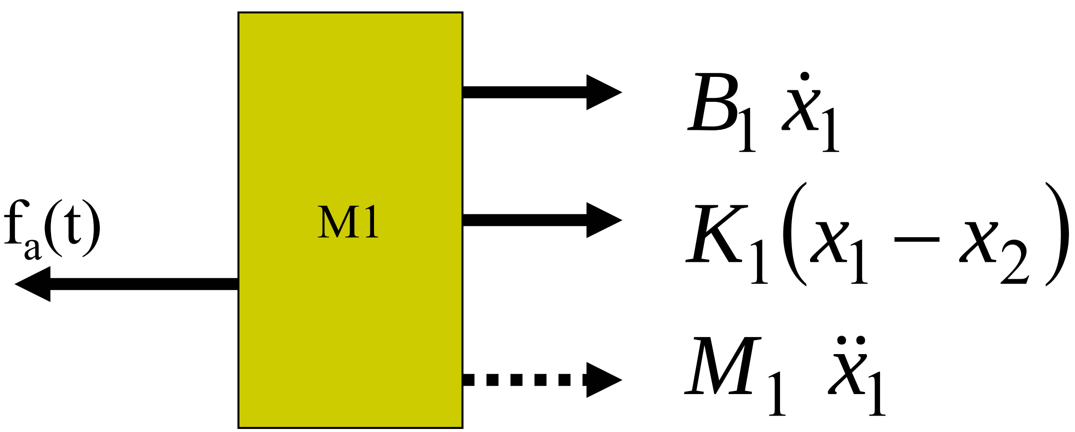
Ideal cable

realistic cable

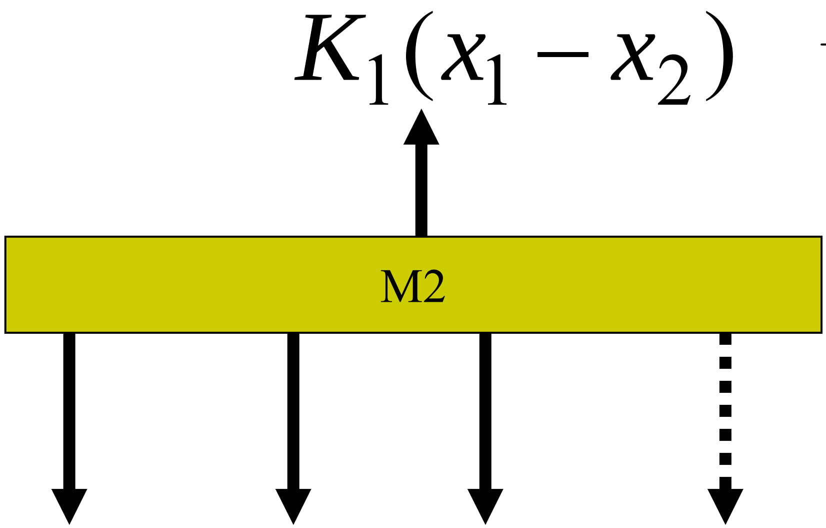
Example

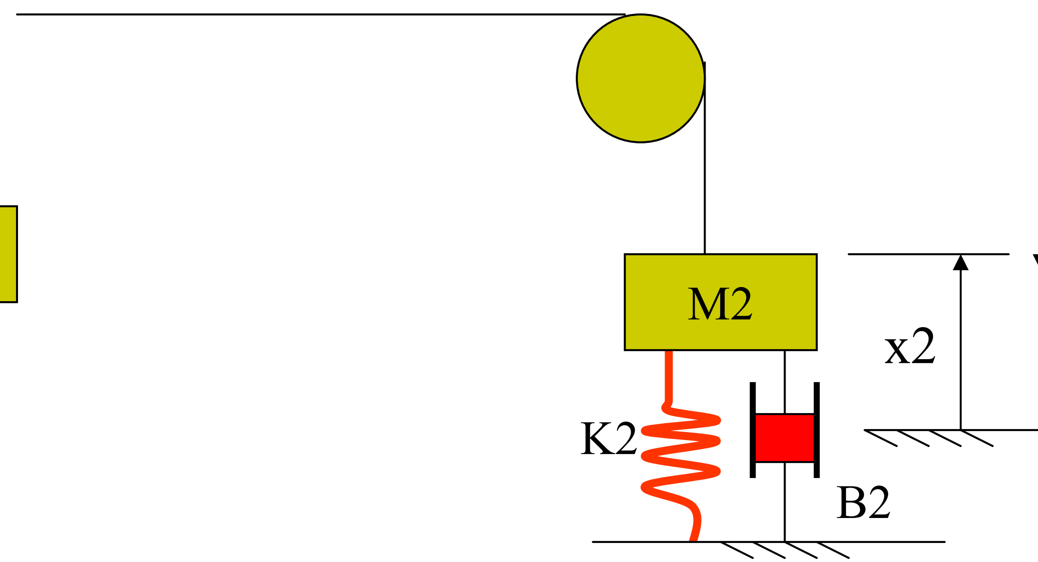


Example



Example





$$M_2 \ddot{x}_2 + B_2 \dot{x}_2 + K_2 x_2 + K_1 x_2 - K_1 x_1 + M_2 g = 0$$

Example Model

$$M_1 \ddot{x}_1 + B_1 \dot{x}_1 + K_1 x_1 - K_1 x_2 = f_a(t)$$

$$M_2 \ddot{x}_2 + B_2 \dot{x}_2 + K_2 x_2 + K_1 x_2 - K_1 x_1 + M_2 g = 0$$