

Problem S4.5

Obtain the inverse Laplace transform of $F(s) = \frac{1}{s(s+1)^2}$

Solution:

$$\frac{1}{s(s+1)^2} = \frac{A}{s} + \frac{B}{(s+1)^2} + \frac{C}{s+1}$$

$$A = s \frac{1}{s(s+1)^2} \Big|_{s=0} = 1$$

$$B = (s+1)^2 \frac{1}{s(s+1)^2} \Big|_{s=-1} = \frac{1}{s} \Big|_{s=-1} = -1$$

$$C = \frac{d}{ds} \left((s+1)^2 \frac{1}{s(s+1)^2} \right) \Big|_{s=-1} = \frac{d}{ds} \left(\frac{1}{s} \right) \Big|_{s=-1} = -1$$

$$f(t) = 1 - te^{-t} - e^{-t}$$