Term 081

CISE302: Linear Control Systems

8. Transfer Functions

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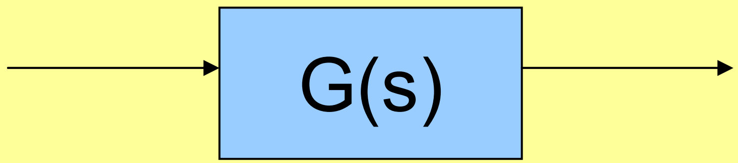
Reading Assignment :

Learning Objective

- To state definitions of Transfer function
- To obtain transfer functions from ODE models
- To obtain transfer function if the input and output functions are given
- To draw pole-zero plot of a system
- To compute zero-state response for a given input

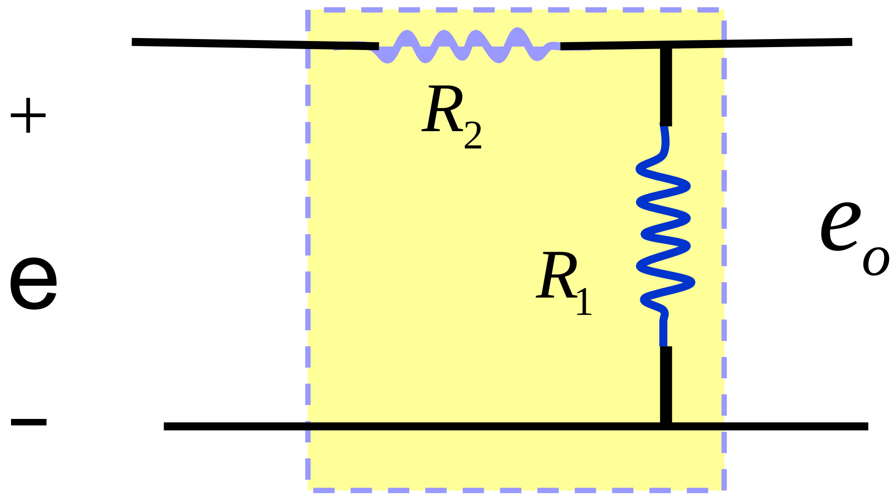
The Transfer function

- The Transfer function is a mathematical model widely used in linear control systems



- The Transfer function is defined as the ratio of Laplace transform of the output over the Laplace transform of the input

Voltage Divider



$$e_o = \frac{R_1}{R_1 + R_2} e,$$

input

e

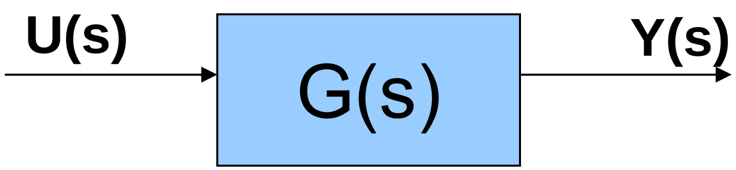
$$G = \frac{R_1}{R_1 + R_2}$$

output

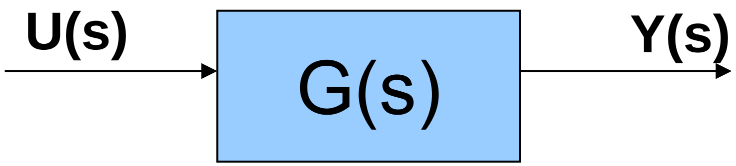
$$e_o = G\, e$$

Transfer function

- Static systems such as the voltage divider, are represented by block diagram with constant gain.
- Linear time-invariant dynamic systems, are represented by a transfer function



Transfer function



$$Y(s) = G(s)U(s)$$

- Input u(t), L{u(t)}=U(s)
- Output y(t), L{y(t)}=Y(s)

Transfer function G(s)

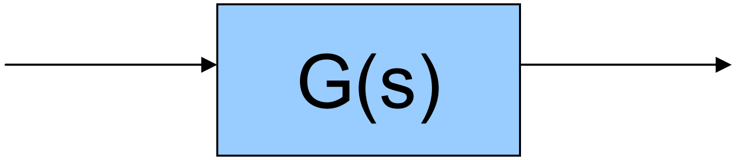
$$G(s) = \frac{L\{y(t)\}}{L\{u(t)\}} = \frac{Y(s)}{U(s)}$$

Properties

- Transfer function is defined for linear time invariant systems only
- Transfer function Transfer function   is the ratio of Laplace transform of output over the Laplace transform of the input
- Transfer function   is the Laplace transform of the impulse response
- All conditions are assumed to be zero.

Transfer function from input-output

$u(t) = 1$



$y(t) = 3 - 2e^{-3t}$

The transfer function can be obtained as the ratio of the Laplace transform of the output over the Laplace transform of the input

$$G(s) = \frac{L\{y(t)\}}{L\{u(t)\}} = \frac{L\{3 - 2e^{-3t}\}}{L\{1\}} = \frac{\frac{3}{s} - \frac{2}{s+3}}{\frac{1}{s}} = \frac{s+9}{s+3}$$

Transfer Function From a ODE model

- An ODE model is a differential equation with two variables: the input and the output
- Procedure:
  - Apply Laplace transform to the differential equation
  - Assume zero initial conditions
  - Solve for the ratio of Laplace transforms of output and input

Only defined for linear time invariant systems

Example

Determine the transfer function of the system described by

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 4y(t) = 2u(t)$$

Solution Procedure:

- **Apply Laplace transform to the differential equation**
- **Assume zero initial conditions**
- **Solve for the ratio Y(s) / U(s)**

Example

Applying Laplace transform assuming zero initial conditions

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 4y(t) = 2u(t)$$

$$\downarrow \qquad \downarrow \qquad \downarrow \qquad \downarrow$$

$$s^2Y(s) + 3sY(s) + 4Y(s) = 2U(s)$$

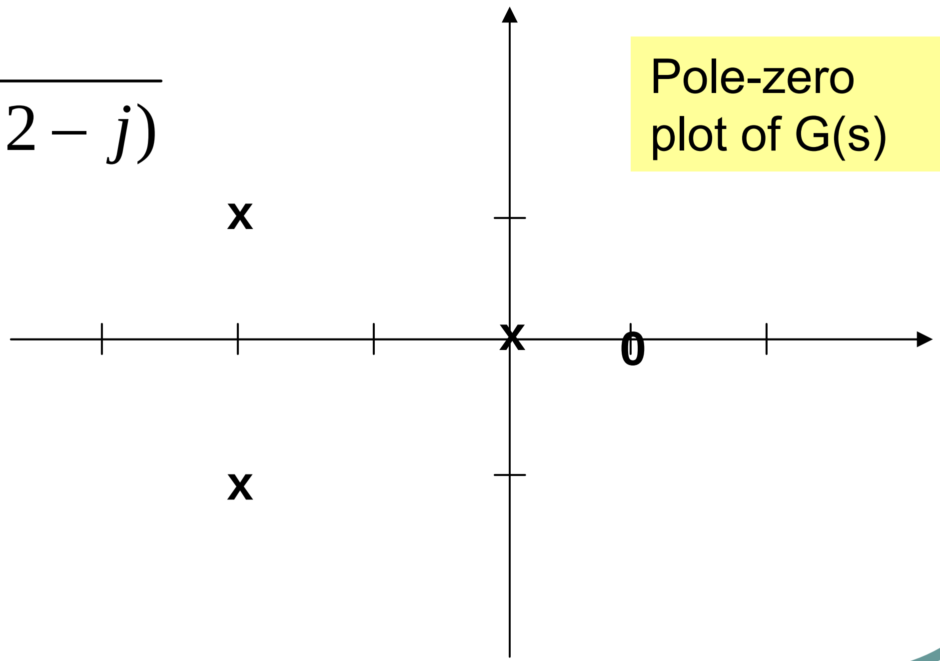
$$(s^2 + 3s + 4)Y(s) = 2U(s)$$

$$G(s) = \frac{Y(s)}{U(s)} = \frac{2}{s^2 + 3s + 4}$$

Pole-Zero plot

- The pole-zero plot is a plot showing the poles and zeros of the system transfer function
- "x" is used to mark Poles, Zeros are marked with 'o'

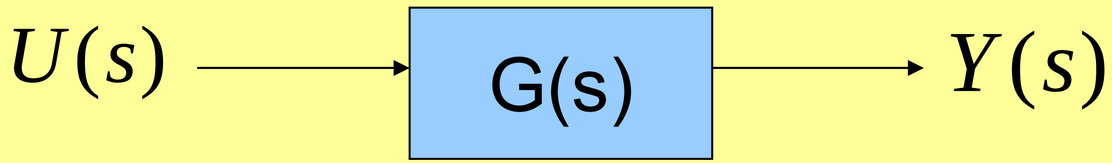
$$G(s) = \frac{2(s-1)}{s(s+2+j)(s+2-j)}$$

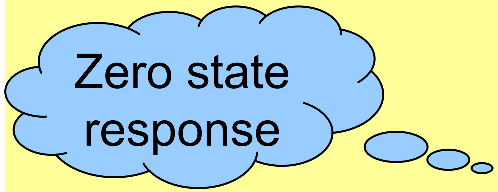


Pole-zero plot of G(s)

Zero State Response of a Transfer function

- The zero state response of a system is the response of the system to a given input with all initial condition equal to zero.





$$Y(s) = G(s)U(s)$$

$$y(t) = L^{-1}\{G(s)U(s)\}$$

Summary

- Transfer function is defined for linear time invariant systems only
- Transfer function Transfer function is the ratio of Laplace transform of output over the Laplace transform of the input
- Transfer function is the Laplace transform of the impulse response
- All conditions are assumed to be zero.