SE311: Design of Digital Systems

Lecture 7: Gate Level Minimization

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(Term 041)

Outlines

- Introduction
- Algebraic Simplification
- K-Maps
  - Two variables
  - Three variables
  - Four variables
- Examples

Introduction

- The simplification of a logic circuit is used to reduce the number of gates used in the implementation.
- Simplification Methods:
  - Algebraic Simplification
  - K-Maps (Karnaugh Maps)

Algebraic Simplification

Use the postulates and theorems to simplify the expression

Example :

$$F = x'y'z' + x'yz' + xy'z' + xyz'$$

$$= x'z'(y'+y) \quad + xz'(y'+y)$$

$$= x'z' \quad + xz'$$

$$= z'(x'+x) = z'$$

Algebraic Simplification

Problem

- There are no specific rules that tell you what to do next to come up with the reduced expression.

Map Method

- Made up of **2^n** squares (n: the number of variables)
- Each square represent one **minterm** of the function
- A square is marked with one if the corresponding minterm is present in the expression of the function
- We can use the map to get all possible expressions of the function

K-map

Two variables

- 2 variables --> 4 squares

A \ B	0	1
0	AB=00	AB=01
1	AB=10	AB=11

A \ B	0	1
0	m0 A'B'	m1 A'B
1	m2 AB'	m3 AB

K-map

Two variables

A \ B	0	1
0	m0	m1
1	m2 1	m3 1

F= ∑(2,3) = m2+m3 = AB’+AB

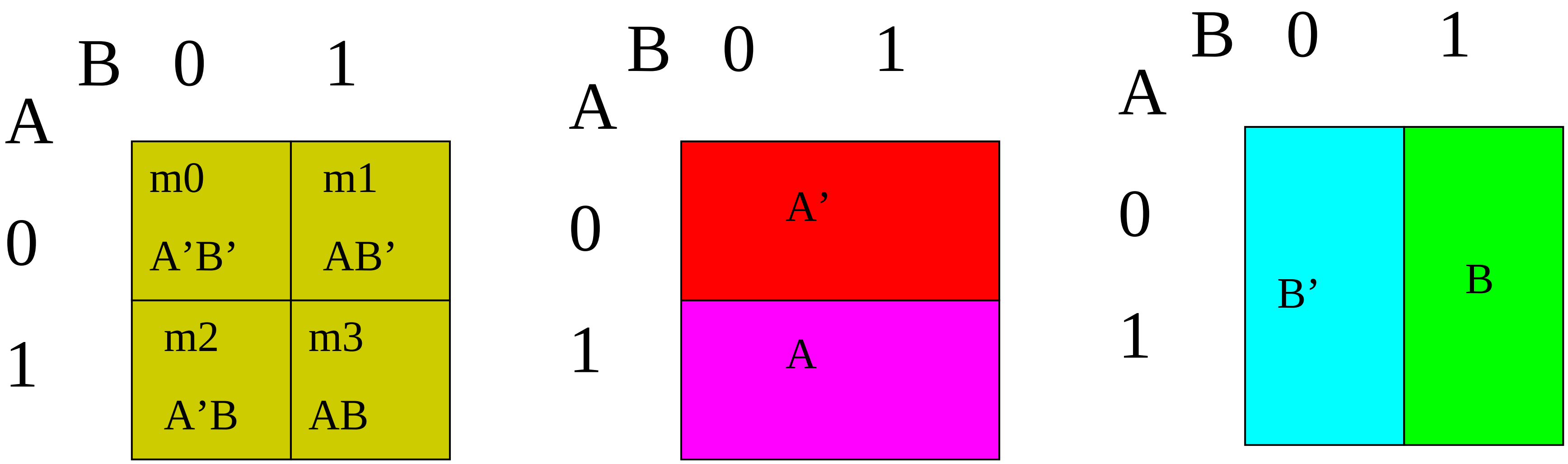
= A(B’+B) = A

A \ B	0	1
0	m0	m1
1	m2 1	m3 1

F=A

K-map

Two variables



In two variable case:

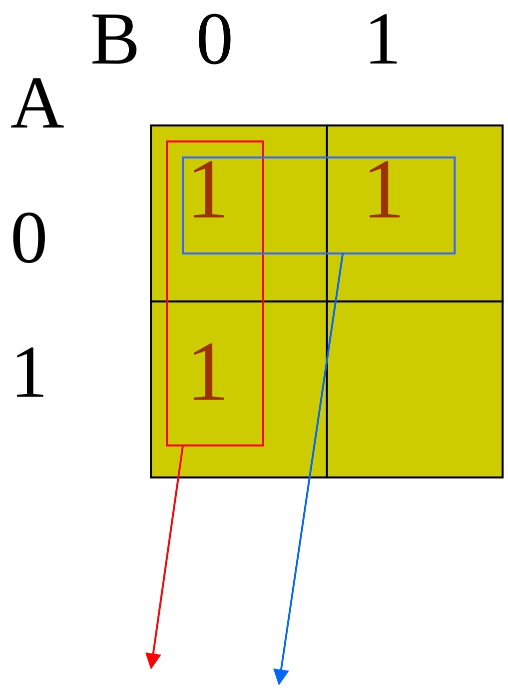
one square --> one minterm (two literals)

Two adjacent squares one literal

Simplification using K-map

Two variables

A \ B	0	1
0	m0 A’B’	m1 A’B
1	m2 AB’	m3 AB



F=A’B’+AB’+A’B

=B’+A’B = A’+AB’ = B’+A’

K-map

Two variables

A \ B	0	1
0	m0 A’B’	m1 A’B
1	m2 A B’	m3 AB

A \ B	0	1
0	1	
1	1	

F=A’B’+AB’ = B’

K-map

Three variables

B

A \ BC	00	01	11	10
0	m0 A’B’C’	m1 A’B’C	m3 A’BC	m2 A’BC’
A 1	m4 AB’C’	m5 AB’C	m7 ABC	m6 ABC’

C

Three variable case

1 square =3 literals

(minterm)

2 adjacent squares =2 literals

4 adjacent squares =1 literal

8 adjacent squares =0 literal

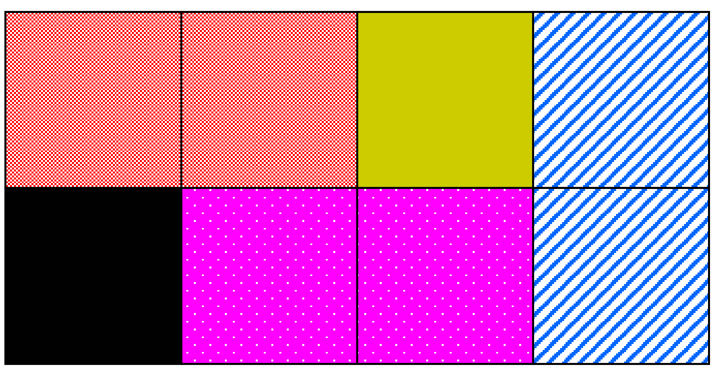
(function =1)

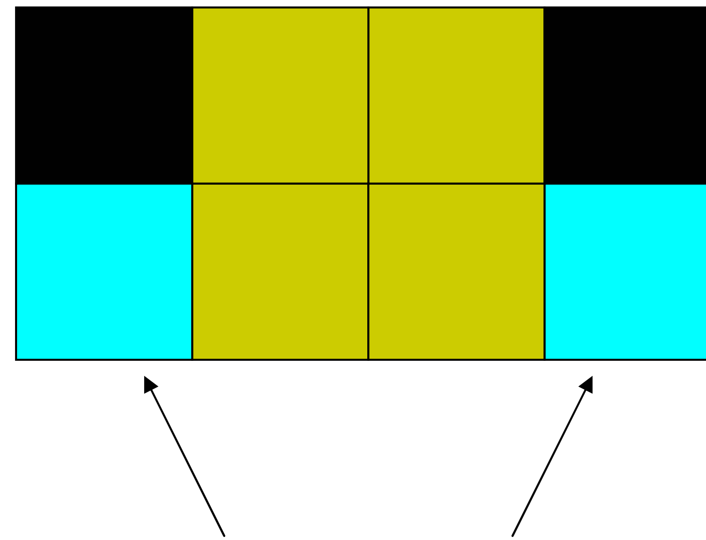
Adjacent Squares

Examples of two adjacent squares

<u>Adjacent squares share common edges!!!!</u>

same colors -->Adjacent squares

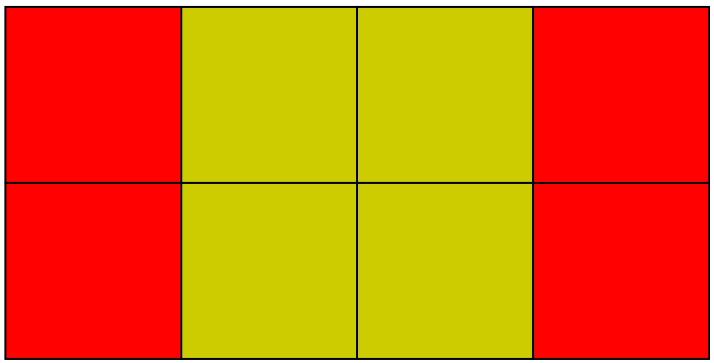


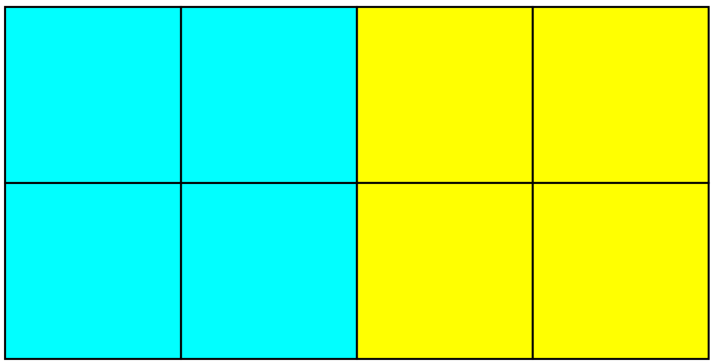


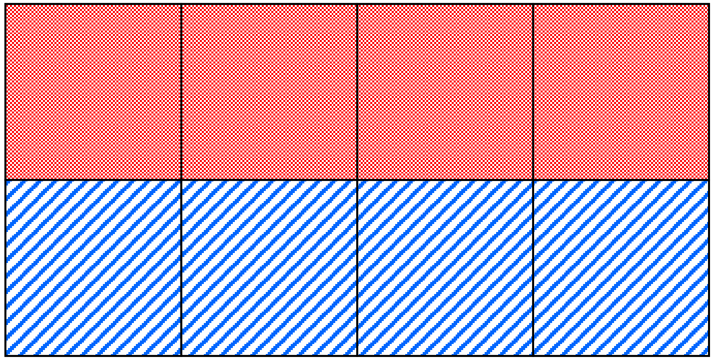
<u>Adjacent squares</u>

Adjacent Squares

Examples of four adjacent squares







4 adjacent squares

one literal

Four Variable Maps

C (over columns 11, 10)

AB \ CD				
00	m0	m1	m3	m2
01	m4	m5	m7	m6
11	m12	m13	m15	m14
10	m8	m9	m11	m10

A (rows 11, 10); B (rows 01, 11); D (columns 01, 11)

Four variable case

1 square =4 literals

(minterm)

2 adjacent squares =3 literals

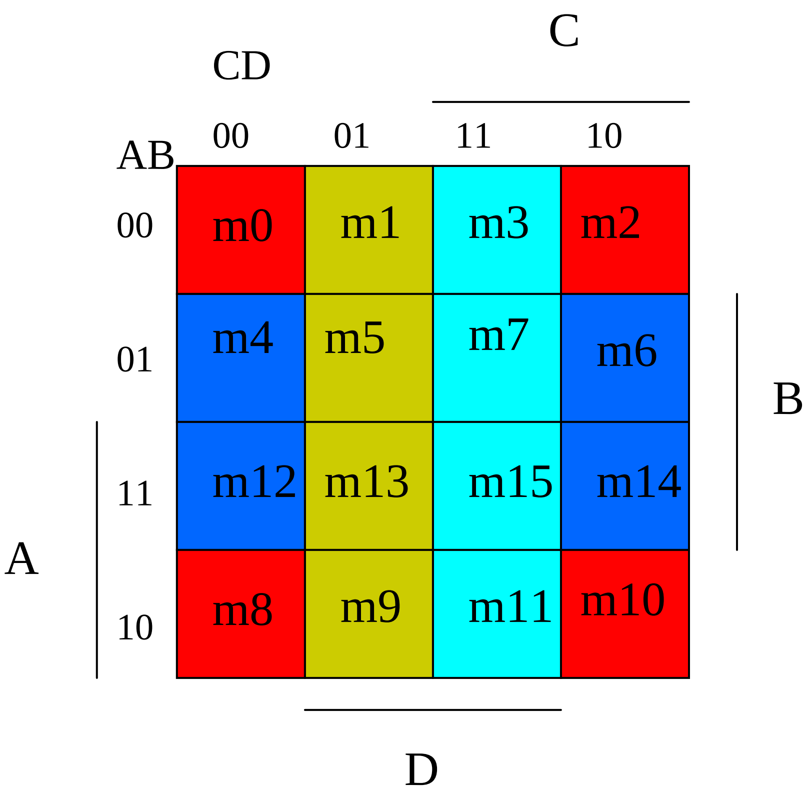
4 adjacent squares =2 literal

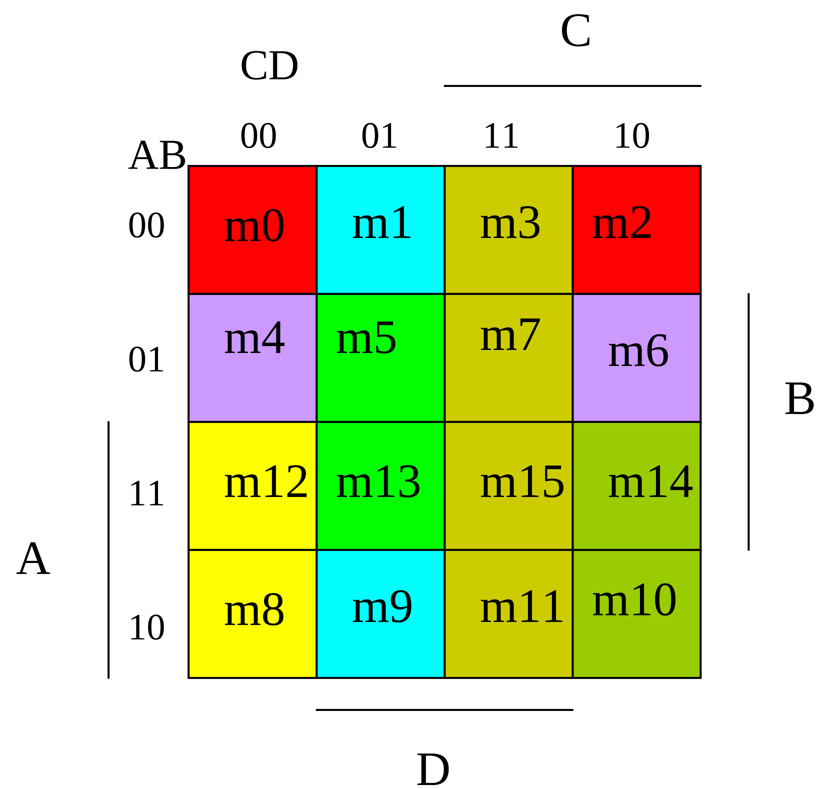
8 adjacent squares =1 literal

16 adjacent squares =0 literal

(function =1)

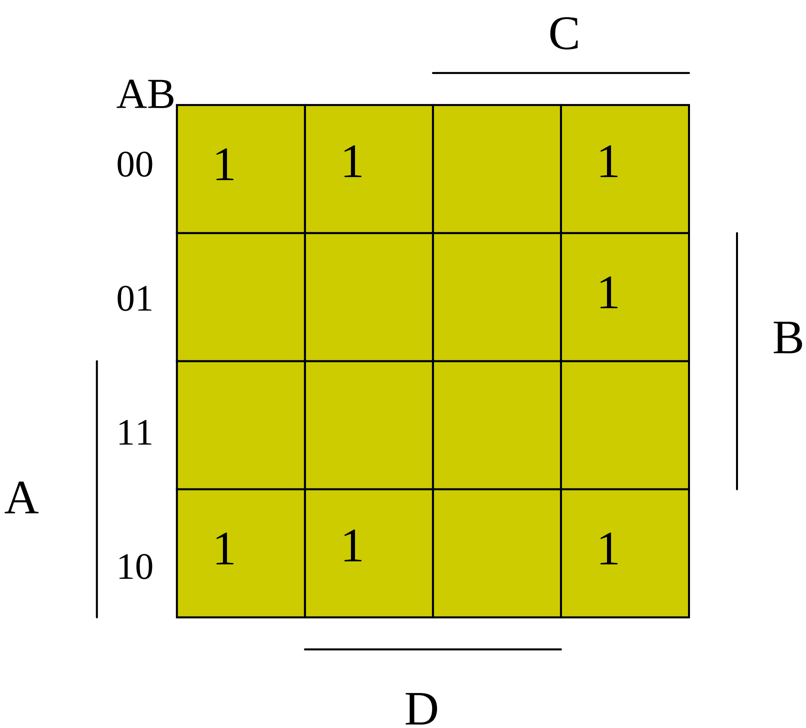
Examples of adjacent squares





Simplify

$f(A,B,C,D)=A'B'C'+B'CD+A'BCD'+AB'C'$



Four variable case

1 square =4 literals

(minterm)

2 adjacent squares =3 literals

4 adjacent squares =2 literal

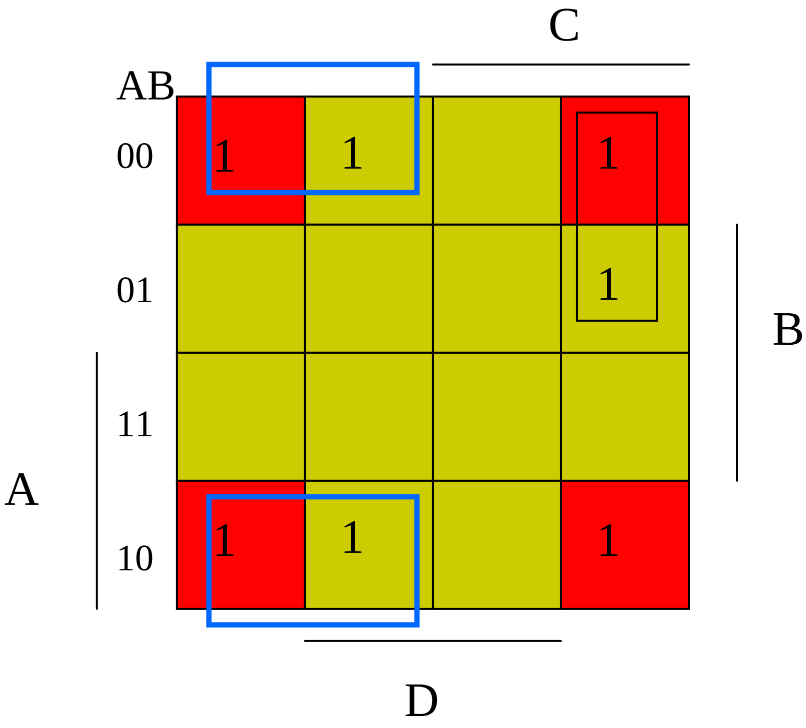
8 adjacent squares =1 literal

16 adjacent squares =0 literal

(function =1)

Simplify

f(A,B,C,D)=A'B'C'+B'CD+A'BCD'+AB'C'



f= D'B' +C'B' +A'CD'

Summary

- We can use K-maps to write all possible expressions of a function
- 1 square represents a minterm (all literals are present)
- Adjacent squares share edges
- For >2 variables think of the left most edge as being adjacent to right most edge & top-bottom edges)
- Two adjacent squares one literal is missing
- Four adjacent squares Two literals are missing