SE311: Design of Digital Systems

Lecture 3: Complements and Binary arithmetic

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(Term 041)

Outlines

- Complements
- Signed Numbers
  - Representations
  - Arithmetic
- Binary Codes
- Binary Storage
- Binary logic

Complements

- Complements are used to represent negative numbers
- They make subtraction easier
- Subtraction is replaced by addition of a number to the complement of the second A - B = A+ (- B)
- X = The complement of (complement of X) X= - (-X)
- Two types of complement
  - (r-1)'s complement (Diminished radix complement)
  - r's complement (Radix complement)
- Complements of numbers with fractions:

  remove the radix point, obtain the complement and return the radix point to its position.

Complements

(r-1)'s complement

(Diminished radix complement)

BINARY: 1's complement

DECIMAL: 9's complement

$$Complement(X) = (r^n - 1) - X$$

$$1's\ Complement(X) = (2^n - 1) - X$$

r's complement

(Radix complement)

BINARY : 2's complement

DECIMAL: 10's complement

$$Complement(X) = r^n - X$$

$$2's\ Complement(X) = 2^n - X$$

n : the number of digits used in the representation r : the base

r's complement of X =1+ (r-1)'s complement of X

2's complement of X = 1+ 1's complement of X

Complement

1's complement

- digits are used in the representation

To obtain the 1's complement:

Replace 0 by 1 and 1 by 0

2's complement

- digits are used in the representation

To obtain the 2's complement:

Add one to the 1's complement

Example: 8 digits are used (n=8)

X = 0 0 0 0 1 0 0 1 — 9

1's Complement (X) = 1 1 1 1 0 1 1 0 — -9 in 1's complement

2's Complement (X) = 1 1 1 1 0 1 1 1 — -9 in 2's complement

Subtraction with Complement

M-N=M+ 2's complement (N)

- Discard the carry
- MSB indicates the sign
  - 1 : Negative M<N
  - 0 : Positive M>N
- Negative numbers are represented in 2's complement

Example: (n=8)

M= 0 0 0 0 1 1 0 0

N = 0 0 0 0 1 0 0 1

2's Complement (N) = 1 1 1 1 0 1 1 1

M-N = 0 0 0 0 1 1 0 0

\+ 1 1 1 1 0 1 1 1

1 0 0 0 0 0 0 1 1

Carry (positive) answer

Subtraction with Complement

Example: (n=8)

M= 0 0 0 0 1 1 0 0

N = 0 0 0 1 1 0 0 1

2's Complement (N) = 1 1 1 0 0 1 1 1

M-N =	0 0 0 0 1 1 0 0	12
	+ 1 1 1 0 0 1 1 1	- 25
	1 1 1 1 0 0 1 1	-13

No Carry (negative) answer

Binary Codes

Binary Codes

A binary code is a rule used to assign a binary number for each discrete element.

- With n-bit code we can represent up to 2^n discrete elements
- Important Binary Codes
  - BCD (Binary Coded Decimal)
  - ASCII (American Standard code for information interchange)
  - Gray code
  - Excess 3 code

Binary Coded Decimal

Each decimal digit is represented by four-digit binary code.

Decimal	BCD	Decimal	BCD
0	0000	5	0101
1	0001	6	0110
2	0010	7	0111
3	0011	8	1000
4	0100	9	1001

- (125)D (0001 0010 0101)BCD
- one needs 8-bits to represent the binary equivalent of 125 and 12-bits to represent the BCD equivalent.

Gray Code

- Useful in applications like analog to digital conversion.
- The code of a number differ from the code of the next number in exactly one bit

Decimal	Gray code	Decimal	Gray code
0	0000	5	0111
1	0001	6	0101
2	0011	7	0100
3	0010	8	1100
4	0110	9	1101

ASCII Code

American Standard Code for Information Interchange

- Used in communicating information between a computer and its peripherals or other computers.
- Each letter, digit or character is represented by 7-bit code

Character	ASCII	Character	ASCII
A	100 0001	0	011 0000
a	110 0001	1	011 0001
ESC	0011010	BS	000 1000

- ASCII code (128 discrete elements: 94 printable characters and 34 control characters)

Binary Storage and Registers

- **Binary Cell**
  - a device that has two possible states.
  - can be used to store a single bit

- **Registers**: a group of binary cells
- **n-bit registers**: can store n-bit binary numbers
- Question:

  How many different possible states can n-bit registers have?

Binary Logic

Binary Logic

Binary Logic involves binary variables and Logic operations

Binary variables

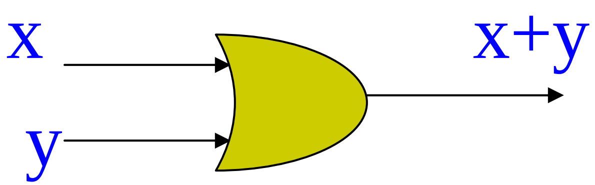
- Can take one of two possible values (0 or 1) .
- Designated by letters A, B,C, x, y, z

Logic Operations

- Three basic operations **AND**, **OR**, **NOT** .

OR

x OR y



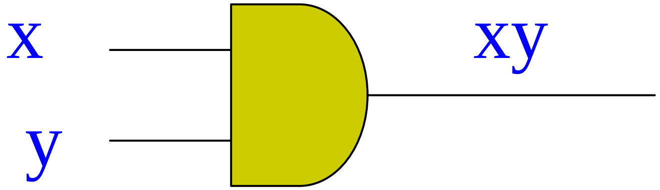
Truth Table

x	y	x+y
0	0	0
0	1	1
1	0	1
1	1	1

The number of possible combinations of n binary inputs is 2^n

AND

x AND y

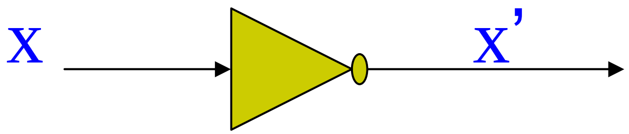


Truth Table

x	y	xy
0	0	0
0	1	0
1	0	0
1	1	1

NOT

NOT x



Truth Table

X	X'
0	1
1	0

Summary

- Complements
  - r's complement 2's complement (Binary)
  - (r-1)'s complement 1's complement (Binary)
- Signed Numbers
  - Representations
  - Arithmetic
- Binary Codes (BCD, Gray, ASCII)
- Binary Storage (Binary cell, register)
- Binary logic (binary variables, logic operations)

Learning Objectives

- Be able to calculate the 2's complement of a given n-bit binary number.
- Be able to explain the differences between signed-magnitude, signed-1's complement and signed-2's complement representations of negative binary numbers.
- Be able to perform subtraction problems in base 2 using the 2's complement method.
- Be able to determine the minimum number of bits needed to represent a given set of items.
- Be able to name and compare binary codes.
- Be able to explain the purpose and special feature of Gray codes.
- Be able to translate a message from ASCII code into standard English characters. (assuming that an ASCII code table is available)
- Know the definition of a *binary cell*.
- Know the definition of a *register*.
- Know the AND, OR and NOT truth tables.
- Know the logic gate and operator symbols for the AND, OR and NOT.