Optimal Process Targeting of Multi-Components Multi-Characteristics Products

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Abstract. In this paper a process targeting model for multi-components multi-characteristic products is developed. The cost function is based on a multivariable Taguchi type quadratic loss function for product uniformity, and a quadratic term for the cost of components. Closed form expressions for the optimal process targets are derived. We provided an illustrative example, and discussed the effect of the weights in the quadratic cost on the tradeoffs between the product quality and the production cost. Further extension of the work is also discussed.

Keywords: Targeting, filling processes, optimization, manufacturing, quality

1. Introduction

In a manufacturing environment, a product has to go across a number of processes, undergoing diverse operations before obtaining a final form. Due to the natural and technological inconsistencies, especially systems of mechanical and chemical in nature, it is bound to have some variations in the final product. In order to minimize this variation, and to improve the overall characteristics of the product, quality control became an essential part of manufacturing. Process Targeting is one of the areas in economics of quality control, which has received a lot of interest from the researchers in the recent times.

The Targeting problem was initiated by Springer [1] for the canning problem. The objective was to minimize the expected cost. Kartha [2] presented a similar model, for the case of maximization of profit, where under filled cans are sold in a secondary market. Bisgaard et al [3] extended the above case where the under filled cans are sold at a price proportional to the can fill. In Golhar [4], the model presented is for the case where the under filled cans are reprocessed at a fixed cost. Golhar and Pollock in [5] considered the case where the fill in expensive and an artificial upper limit is also determined alongside the process mean. Golhar and Pollock [6] studied the effect of variance reduction on the profit. Arcelus [7] introduced the product uniformity via a Taguchi quadratic loss function. In Min and Jang [8] a situation where inspection is based on three class screening is considered.

Several other directions of research have evolved e.g., use of various sampling plans instead of full inspection in Carlsson [9], Boucher and Jafari [10] and Sultan [11]. Others have considered problems having production processes with linear drift e.g., Rahim and Banerjee [12], Sultan and Pulak [13]. In this paper we present a unified process targeting model for multi-components multi-characteristic products, and propose a multivariable quadratic cost for both product uniformity, and components cost. This frame work leads to simple closed-form expressions for the optimal process targets.

[14] and [15] presented model for multi-class screening target problem with Taguchi quadratic loss function to introduce a penalty for uniformity and incorporation of unbiased error in measurement.

The paper is organized as follows. In the next section 2, the model developed in this paper is described. In Section 3, an illustrative example is presented. A brief summary of the sensitivity analysis is discussed in Section 4. Finally, extensions and conclusions are outlined in Section 5.

2. The Model

In this section, an economic targeting model is developed for a production process having multi-components and/or process parameters as inputs, and a product with multi-characteristics as an input. In this model, a Taguchi quadratic penalty, to infuse the uniformity count, and a quadratic components cost are incorporated. The object of this model is to find out the input settings for the process that minimizes the cost while also trying to achieve the target characteristics of the finished products. Model assumptions are as follows:

Consider a production system (Fig. 1) for instance a process plant (examples include: a food processing, a petrochemical process, or a pharmaceutical plant etc.), where in order to produce a product *n* components/processes are required, represented as $x_1, x_2, \ldots x_n$. These components/process parameters may represent a direct ingredient or a process variable such as temperature or a process mixing/heating time, or any other related factor. The ideal target values to be used in the Taguchi quadratic loss function are represented as $\hat{y}_1, \hat{y}_2, \ldots \hat{y}_m$.

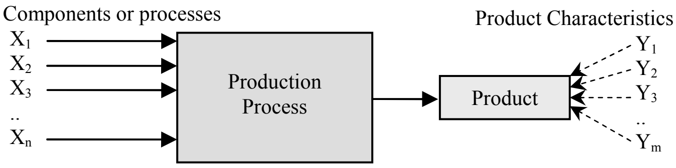

Fig. 1. A production process with *n* components/processes and a product with *n* characters.

The product quality is determined by the measurement of m characteristics $y_1, y_2, \ldots y_m$, e.g., weight, density, color, pH number, viscosity, etc. Ideal target values of the characteristics are represented as $\hat{y}_1, \hat{y}_2, \ldots \hat{y}_m$. If any characteristic is off target a quadratic (Taguchi) penalty is to be paid for it (see Fig.2).

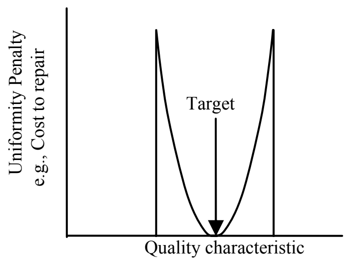

Fig. 2. Taguchi quadratic penalty for non-uniformity of a product

We assume that the measurement is related to the process components by the relation:

$$Y = CX + W \tag{1}$$

Where *C* is an *m x n* relational matrix (contribution factor of each x on y), and *W* is a vector of *m* normally distributed random noises with zero mean and covariance matrix Π_w. The dispensed components are described by the relation:

$$X = \mu + V \tag{2}$$

Where $\mu = (\mu_1, \mu_2, \ldots, \mu_n)$ the process set point vector, and V is is a vector of *n* normally distributed random variables with zero mean and covariance matrix Π_v representing natural variance of the production process. We further assume that *W* and *V* are independent.

It is now desired to find the optimal set point vector which minimizes the loss function given as:

$$J = E\{(Y-\hat{Y})^T Q(Y-\hat{Y}) + X^T RX\} \quad (3)$$

Where the weighting matrix Q reflects the cost of product non-uniformity, and R is the cost matrix of the product components/processes. Both Q an R are symmetric, and R is positive definite.

2.1 Solution To The Problem. For simplicity, let us assume that there is no measurement noise, i.e. W=0; Expanding Eq. 3

$$\begin{aligned} J &= E\{(C\mathbf{\mu} + CV - \hat{Y})^T Q(C\mathbf{\mu} + CV - \hat{Y}) + (\mathbf{\mu}+V)^T R(\mathbf{\mu}+V)\} \\ J &= E\{\mathbf{\mu}^T C^T QC\mathbf{\mu} + V^T C^T QCV + \hat{Y}^T Q\hat{Y} + \mathbf{\mu}^T C^T QCV - \mathbf{\mu}^T C^T Q\hat{Y} + V^T C^T QC\mathbf{\mu} - \\ & V^T C^T Q\hat{Y} - \hat{Y}^T QC\mathbf{\mu} - \hat{Y}^T QCV + \mathbf{\mu}^T R\mathbf{\mu} + \mathbf{\mu}^T RV + V^T R\mathbf{\mu} + V^T RV\} \end{aligned} \quad (4)$$

As V is a zero mean random vector, Eq. 3 can be simplified to

$$\begin{aligned} J &= E\{\mathbf{\mu}^T C^T QC\mathbf{\mu} + V^T C^T QCV + \hat{Y}^T Q\hat{Y} - \mathbf{\mu}^T C^T Q\hat{Y} \\ & - \hat{Y}^T QC\mathbf{\mu} + \mathbf{\mu}^T R\mathbf{\mu} + V^T R\mathbf{\mu} + V^T RV\} \\ J &= E\{\mathbf{\mu}^T (C^T QC + R)\mathbf{\mu} + V^T (C^T QC + R)V + \hat{Y}^T Q\hat{Y} - \mathbf{\mu}^T C^T Q\hat{Y} - \hat{Y}^T QC\mathbf{\mu} + V^T R\mathbf{\mu}\} \end{aligned} \quad (5)$$

Differentiating Eq. 5 with respect to $\mathbf{\mu}$ and equating to zero

$$\begin{aligned} &(C^T QC + R)\mathbf{\mu} - C^T Q\hat{Y} = 0 \\ &\mathbf{\mu}_{opt} = (C^T QC + R)^{-1} C^T Q\hat{Y} \end{aligned} \quad (6)$$

Eq. 6 gives the optimal process set points for the input components/processes vector X. When W is not equal to zero, it can be shown that the expression for optimal set points is still given by Eq. 6, however the expression for the expected value of the minimum cost is given by:

$$J_{\min} = \hat{y}^T Q\hat{y} - \mu_{opt}^T C^T Q\hat{y} + Tra((C^T QC + R)\Pi_v) + Tra(Q\Pi_w) \quad (7)$$

Eqs. 6 and 7 show that the optimal process targets does not depend on the characteristic of the process uncertainty and the measurement noise. However, the minimum quadratic cost is directly affected by the covariance matrices of the process uncertainty and the measurement noise.

3. Illustrative Example

In this section, an illustrative example is presented. The problem is as follows: Consider a carbonated cocktail juice which is to be manufactured to a target volume of 900 ml and a target calorific level of 150 calories. The juice is made up of: 1. the orange juice concentrate 2. the apple juice concentrate 3. the peach juice concentrate and 4. the carbonated water. The cost per ml of each of the component is assumed to be 0.004, 0.0042, 0.006, and 0.002 units respectively, The input settings are to be found so that cost is minimized while achieving these target values. In Case I, both volume and calorific values are considered equally important. The weights of their values are set high to reduce the deviation from the target requirement. While in Case II the target values are considered to be equal but of very low importance as compared to the cost of production. In case III and IV individual target weights are considered.

CASE I and II: Taguchi penalty factor, in Case I, is assumed to be set at 50 each when target values are more important than cost of production, while in case II it is set at 0.1 each so as to represent a situation where achieving targets is of low concern as compared to the cost of production. The optimal solution was found using Eq. 6 as in Table 1.

Table 1. Solution of the Illustrative example for case I and II

	Optimal mean set points of inputs				Mean Output		Price
Taguchi Pen. fac. (Vol. – Cal.)	Juice Conc. orange [ml]	Juice Conc. apple [ml]	Juice Conc. Peach [ml]	Carb. water. [ml]	Volume [ml]	Calories [Kcal]	Per 900 ml bottle
50 – 50	122.72	96.00	110.73	570.53	900.0	150.0	2.70
0.1-0.1	124.32	99.43	111.21	526.76	861.7	152.3	2.64

In Case I, Vector representing Target values, the Taguchi penalty matrix Q, the product cost matrix R and the input/output factors relationship matrix are given in equation 8

$$\hat{Y}=\begin{bmatrix}900\\150\end{bmatrix},\quad Q=\begin{bmatrix}50&0\\0&50\end{bmatrix},\quad R=\begin{bmatrix}0.004&0&0&0\\0&0.0042&0&0\\0&0&0.006&0\\0&0&0&0.002\end{bmatrix},\quad C=\begin{bmatrix}1&1&1&1\\0.43&0.55&0.35&0.01\end{bmatrix} \tag{8}$$

Therefore the optimal target mean output matrix solution can be calculated as (as shown in Eq. 9:)
μ=

$$\left(\begin{bmatrix}1&1&1&1\\0.43&0.55&0.35&0.01\end{bmatrix}^T\begin{bmatrix}50&0\\0&50\end{bmatrix}\begin{bmatrix}1&1&1&1\\0.43&0.55&0.35&0.01\end{bmatrix}+\begin{bmatrix}0.004&0&0&0\\0&0.0042&0&0\\0&0&0.006&0\\0&0&0&0.002\end{bmatrix}\right)^{-1}\begin{bmatrix}1&1&1&1\\0.43&0.55&0.35&0.01\end{bmatrix}^T\begin{bmatrix}50&0\\0&50\end{bmatrix}\begin{bmatrix}900\\150\end{bmatrix}=\begin{bmatrix}122.72\\96.0\\110.73\\570.53\end{bmatrix} \tag{9}$$

Case II is similar to Case I except for Matrix Q with updated penalties of 0.1, 0.1 to deemphasize the target values as compared to the cost of production.

For the Cases III and IV where one of the characteristics is far more important i.e., assuming penalty to be set to zero for the non important while keeping the other at 50, the results are as follows in Table 2:

Table 2. Solution of the Illustrative example for case III and IV

	Optimal mean set points of inputs				Mean Output		Price
Taguchi Pen. fac. (Vol. – Cal.)	Juice Conc. orange [ml]	Juice Conc. apple [ml]	Juice Conc. Peach [ml]	Carb. water. [ml]	Volume [ml]	Calories [Kcal]	Per 900 ml bottle
50 – 0	194.84	185.56	129.90	389.68	900.0	235.2	3.12
0 – 50	116.23	141.59	63.07	5.41	326.3	150.0	1.45

It can be seen clearly that the model is giving optimal values from economic stand point while taking care of the uniformity of the product according to the given target values and the weight of the target values to be achieved as compared to the cost of production.

4. Sensitivity Analysis

In this section, sensitivity analysis is presented. Here two important issues were considered. Firstly, the effect of price on the optimal target input means were assessed. Secondly, the effects of quadratic uniformity penalties on the output values, for which the target is set, were evaluated.

The results are presented in Table 3. Similar case, as presented in the section 3 (Illustrated example) was taken for sensitivity analysis. Target Volume was taken to be fixed at 900 ml. While two levels (High=250 cal and Low=150 cal) of target calorific values were studied. Various levels of 'price factors' were taken to evaluate the effect of cost on optimal input settings. Various combinations of Taguchi penalties are also taken to asses the effect on output target values. The combination were taken to evaluate various levels of Taguchi penalties as well as the effects on target output if one or both are important with higher or lower penalties as compared to the costs at the same time.

The model performed as expected and following behavior was observed. Higher values of the Taguchi penalty factors as compared to the price factors forces mean optimal output to be very close to the target values. The model shows sensitivity to the price factor as well, i.e., showing variation in mean optimal input values with the variation in price factor. One important aspect evident from the result is that the higher values of Taguchi penalty factors as compared to price factors are required and these penalties cannot be set to zero. This is because the optimal settings will tend towards zero as the Taguchi penalty factor tends towards zero. This behavior is obvious as the model does not have any constraints to limit optimal mean settings within some predefined acceptable limits. As can be seen from the results, the model will also provide the optimal target input means settings that will always give mean optimal output values less (or approximately equal, in case of very high penalty factors) than the target values.

Table 3. Results of sensitivity analysis

Target *volume* is set at 900 ml and *calorific values* levels at 1=150 calories and 2=250 calories

Target	**Price of concentrates**				**Tag. Penalties**		**Mean Output**		**Price**
Cal. [cal]	Conc. orange	Conc. apple	Conc. Peach	Carbonated water	Vol.	Cal.	Vol. [ml]	Cal. [cal]	$/900 ml
1	0.004	0.0042	0.006	0.002	50	0	900.0	235.2	3.12
1	0.004	0.0042	0.006	0.002	0	50	326.3	150.0	1.45
1	0.004	0.0042	0.006	0.002	10	10	899.9	150.1	2.70
1	0.004	0.0042	0.006	0.002	10	1	899.9	151.4	2.70
1	0.004	0.0042	0.006	0.002	1	10	898.9	150.1	2.70
1	0.004	0.0042	0.006	0.002	1	1	898.9	151.4	2.70
1	0.004	0.0042	0.006	0.002	1	0.5	898.9	152.7	2.71
1	0.004	0.0042	0.006	0.002	0.5	1	897.7	152.7	2.71
1	0.004	0.0042	0.006	0.002	10	10	758.9	187.2	2.57
2	0.004	0.0042	0.006	0.002	10	1	899.9	250.0	3.19
2	0.004	0.0042	0.006	0.002	1	10	899.9	249.8	3.19
2	0.004	0.0045	0.006	0.002	10	10	899.3	250.0	3.19
2	0.004	0.0045	0.006	0.002	10	1	899.9	250.0	3.20
2	0.004	0.0045	0.006	0.002	1	10	899.9	249.7	3.20
2	0.0045	0.004	0.006	0.002	10	10	899.3	250.0	3.20
2	0.0045	0.004	0.006	0.002	10	1	899.9	250.0	3.19
2	0.0045	0.004	0.006	0.002	1	10	899.9	249.7	3.19
2	0.004	0.0042	0.006	0.002	50	0	899.3	250.0	3.19
2	0.004	0.0042	0.006	0.002	0	50	775.3	213.5	2.74

5. Conclusion and Suggestions

The paper presented a model and a quadratic cost criterion for solving the multi-characteristic process targeting problem. One key advantage of this model is its simplicity where a closed form solution is obtained. Previous approaches to this problem are mathematically complex and require search algorithm to solve. However, in order to use this model effectively careful estimation of the Taguchi penalty factors are required to achieve the best compromise between the process quality and production cost. The model can be applied to a wide spectrum of applications in process industry such as food, beverages, petrochemicals, pharmaceuticals, cement, paints, chemicals industry etc. Various extensions to this work are possible. An immediate extension is to include constraints on the acceptable limits for output values. Other possible extensions include systems having more complex relations between the quality characteristics and the process variables.

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