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Operator Scheduling Using Queuing Theory and Mathematical Programming Models

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The modeling and solution is presented for a real-life operator scheduling problem at a call center. Queuing and integer programming models are combined to minimize the total weekly labor cost while providing an acceptable service level for each hour of each day of the week. The models determine optimum staffing levels and employee weekly work schedules for meeting a varying workload for every hour of the week. First, queuing analysis is applied to data on the number and duration of calls in order to estimate minimum hourly labor demands. Next, an integer programming model is formulated and used to determine optimum operator schedules. The proposed schedule reduces the waiting time, while minimizing the cost and the number of operators.

Key words: Operator Scheduling; Queuing Theory; Mathematical Programming Models

1. Introduction

The modeling and solution are presented for an actual operator scheduling problem at a call center. Queuing and optimization models are used to minimize labor cost by determining the optimum number of operators and their schedules to meet a fluctuating demand. The call center operators provide help 24 hours a day, seven days a week to all employees of a large company. They respond to employees' technical enquiries, problems, and complaints. The call center has three groups of operators, with different pay scales and different work schedules. The call center's management received many complaints about long waiting time (callers placed on hold) during peak periods. These complaints indicated the need to reschedule operators in order to maintain an adequate staffing level for all hours of every day of the week.

In the current practice, operators were assigned to different weekly work schedules based only on management judgment and experience. In order to determine the optimum work schedules, extensive data was collected and analyzed on the number and duration of incoming calls during all hours of the week. Using queuing theory, this data was converted into hourly labor demands for a typical work week. Subsequently, an Integer Programming (IP) model of the operator scheduling problem was formulated and solved.

The remainder of this paper is organized as follows. Related literature is reviewed in Section 2, and then the problem and its context are described in Section 3. Queuing analysis for calculating hourly operator demands is presented in Section 4. In Section 5, the scheduling assumptions and alternatives are presented, and then the operator scheduling IP model is formulated and solved. Finally, some conclusions and summary are presented in Section 6.

2. Literature Review

This review focuses on queuing theory and mathematical programming techniques for call center staffing and telephone operator scheduling published since the year 2000. One of the most popular techniques is linear programming (LP) and integer programming (IP). Brusco and Jacobs (2000) formulated a compact implicit IP model for flexible tour scheduling of employees at a Motorola call center. Çezik at al. (2001) developed an IP model to schedule operators in a call center by combining days-off and shift scheduling constraints into a network flow structure.

Many LP-based approaches combine LP with other techniques. Caprara et al. (2003) used IP, dynamic programming, and heuristics to determine the minimum-workforce days-off schedule of emergency call center employees. Lin et al. (2000) combined regression, simulation, and IP models to determine hourly staffing levels and schedules of operators at a 24-hour hotline service. Bard (2004) used IP followed by several post-processors to determine the full-time staffing level for a service facility. Harrison and Zeevi (2005) used stochastic fluid models to reduce call center staffing problems to a multidimensional newsboy problem, which they numerically solved by LP and simulation.

Simulation has been widely used in scheduling call center operators. Saltzman and Mehrotra (2001) used a simulation model for staffing a technical support call center in a software company. As mentioned earlier, Harrison and Zeevi (2005) combined simulation with LP for call center staffing. Atlason et al. (2004) combined simulation and IP in an iterative cutting plane algorithm to determine the minimum-cost schedule of a call center's employees. AbdelMalek and Allahverdi (2005) used simulation to analyze the ($G/G/c$) queuing model in order to determine the optimum number of technicians for the help desk of a telecommunications company.

Queuing models have been primarily used to determine call center staffing levels to satisfy specific service-level criteria. Duder and Rosenwein (2001) used queuing-based "rule-of-thumb" formulas to estimate the cost of abandonment and to determine the optimum number of operators. Green et al. (2003) used queuing models and heuristic rules to determine staffing levels for a call center. Whitt (2005a) used approximations of Markovian queuing models to estimate staffing levels needed to achieve abandonment rate and waiting time targets. Whitt (2005b) also uses queuing analysis for staffing a call center, considering the proportion of servers present as a random variable.

3. Current Schedule and Problem

The call center is staffed by operators 24 hours a day, seven days a week. The call center has 47 operators, classified into three categories: (1) shift-schedule company employees, (2) non-shift company employees on regular day schedules, and (3) shift-schedule contractors. It should be noted that the local weekend is Thursday and Friday, i.e., regular workdays are Saturday to Wednesday. The current weekly work schedules of operators in these three categories, shown in Table 1, are described as follows:

Table 1 Current assignment of 47 operators to 11 weekly schedules

#	Operator Type	Shift Time	Break	Off Days	Number
01	Shift employees	3 shifts, 24 hours	None	Varying	19
02	Day employees	7:00 am-4:00 pm	12:00 am-1:00 pm	Thu-Fri	1
03	Contractors	6:00 am-3:00 pm	11:00am-1:00 pm	Thu-Fri	16
04	Contractors	9:00 am-6:00 pm	12:00 pm-2:00 pm	Tue-Wed	4
05	Contractors	3:00pm-12:00 pm	7:00 pm-9:00 pm	Mon-Tue	1
06	Contractors	3:00pm-12:00 pm	7:00 pm-9:00 pm	Wed-Thu	1
07	Contractors	3:00pm-12:00 pm	7:00 pm-9:00 pm	Fri-Sat	1
08	Contractors	3:00pm-12:00 pm	7:00 pm-9:00 pm	Tue-Wed	1
09	Contractors	3:00pm-12:00 pm	7:00 pm-9:00 pm	Sun-Mon	1
10	Contractors	3:00pm-12:00 pm	7:00 pm-9:00 pm	Thu-Fri	1
11	Contractors	3:00pm-12:00 pm	7:00 pm-9:00 pm	Sat-Sun	1

3.1. Company shift-schedule employees

This category has 19 operators divided into four groups. Groups A, B, and C have five operators each, while group D has four operators. The operators work in three 8-hour shifts with no meal breaks: (1) from 7:00 am to 3:00 pm, (2) from 3:00 pm to 11:00 pm, and (3) from 11:00 pm to 7:00 am. Over a four-week cycle, employees work seven consecutive days on each of the three shifts, using the days-on/days-off pattern 7/2-7/2-7/3. Ideally, this schedule is carried out by four equally-sized operator groups, such that on any given day, three groups are working (one on each shift), while one group is off.

3.2. Company day-schedule employees

One operator is on the regular day schedule working five days a week, Saturday to Wednesday, taking the weekend off on Thursday and Friday. The work times are 7:00 a.m. to 4:00 p.m., with a one-hour lunch break from 12:00 noon to 1:00 pm.

3.3. Contractors

There are 27 contractors assigned to three shift types as follows: (a) 16 contractors on the 6:00 am to 3:00 pm shift from Saturday to Wednesday, taking one-hour lunch breaks any time from 11:00 am to 1:00 pm., (b) four contractors on the 9:00 am to 6:00 pm shift from Thursday to Monday, taking one-hour lunch breaks anytime from 12:00 noon to 2:00 pm, and (c) seven contractors on the 3:00 pm to 12:00 midnight shift, taking one-hour dinner breaks any time between 7:00 pm and 9:00 pm.

The call center's management received many complaints from callers expressing serious frustration with being placed on hold for several minutes during peak periods, although the first-come, first-served (FCFS) rule is used to queue incoming calls. Another problem with the current schedule is varying times of one-hour breaks for contractors within predetermined two-hour intervals as shown in Table 1. This policy makes it too difficult to control the staffing levels in order to satisfy the varying workload.

Based on these concerns, management aimed to scientifically find the minimum-cost staffing levels and weekly work schedules of the operators, to efficiently meet the workload demands in each work period and minimize callers' waiting time. To achieve these objectives, the approach was divided into the following stages: data collection and analysis to determine hourly labor requirements, model formulation, and model solution.

4. Queuing analysis to estimate labor demands

4.1. Call frequency and duration data

Data was collected on the number of calls per hour and also the duration of each call for five typical months. The average number of calls in each hour of the week is shown in Figure 1. The average number of calls per hour is 79.21 during workdays (hours 1-120) and 29.10 during weekends (hours 121-168). All five regular workdays have the same pattern, while the two weekend days have another common pattern. The service time (call duration) has a mean of 4.033 minutes and a standard deviation of 1.741 minutes.

4.2. Using queuing to determine staffing levels

A first-come, first-served (FCFS) policy is applied in answering incoming calls. Calls that cannot be answered immediately because all operators are busy join a queue and wait for operators to be available. Therefore, an $(M/M/c)$:$(GD/\infty/\infty)$ queuing model was used to represent the system. The intervals with similar arrival rates were grouped and analyzed together using a separate queuing model. Since the day of the week has no effect on the service time (call duration), the same service time mean μ (number of calls processed per time unit) was used for every day of the week. In

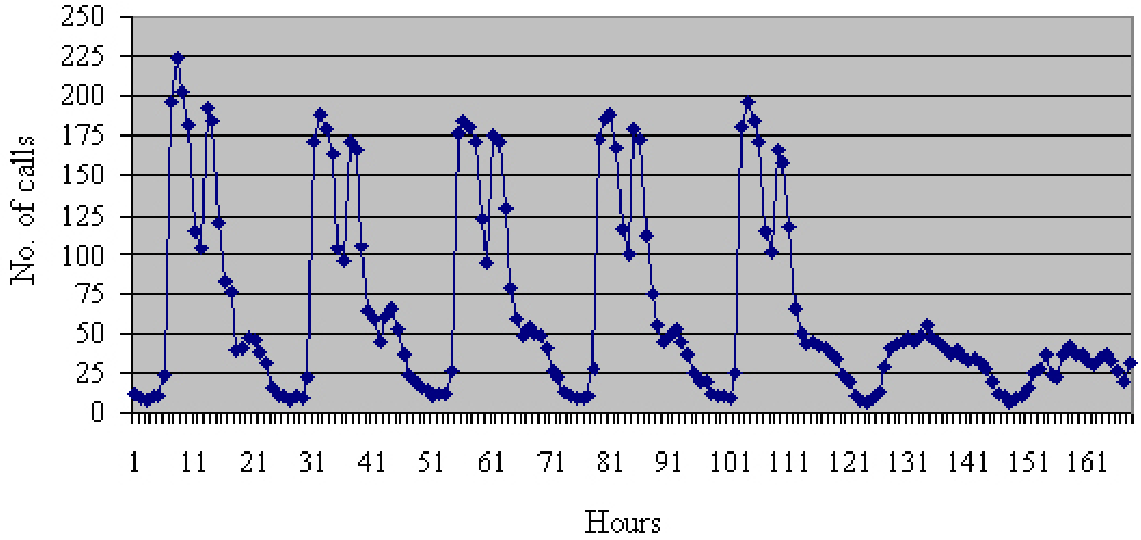

Figure 1 Average number of arriving calls in one week using the historical data

order to calculate the effective service rate, we excluded non-productive hours per shift. Out of any eight-hour work shift, roughly one hour is lost due to unscheduled breaks, personal needs, and so on. Therefore, the net work time averages $7/8 \times 60 = 52.5$ minutes per hour. Since the average call duration is 4.033 minutes, the effective service rate is given by:

$$\mu = \frac{52.5}{4.033} \cong 13 \; calls \; per \; hour \tag{1}$$

Management's primary concern was excessive waiting which has led to user frustration and complaints. Therefore, the call center's management wanted to limit the average waiting time. For the $(M/M/c)$:$(GD/\infty/\infty)$ queue, the expected waiting time $W_{i,q}$ if c_i operators are assigned during hour i is calculated by the following equation (Taha, 2003):

$$p_{0,j} = \left\{ \sum_{n=0}^{c_i - 1} \frac{\rho^n}{n!} + \frac{\rho^{c_i}}{c_i!} \left(\frac{1}{1 - \frac{\rho}{c_i}} \right) \right\}^{-1}, \quad \frac{\rho}{c_i} < 1 \tag{2}$$

$$W_{q,i} = \frac{\rho^{c_i+1} p_0}{\lambda_i (c_i - 1)!(c_i - \rho)^2} \tag{3}$$

where

ρ = $\lambda i/\mu$
$p_{0,i}$ = probability of no waiting (zero queue length) during hour i
λi = arrival rate (number of calls received during hour i)
μ = service rate (number of calls processed per hour)
c_i = number of operators working during hour i

Table 2 Minimum number of operators needed in hour i, r_i

Time Interval	Hours i	Minimum number of operators Workdays	Weekends
midnight-6:00 am	1, .., 6	4	3
6:00 am-10:00 am	7, ..., 10	19	6
10:00 am-noon	11, ..., 12	13	5
noon-3:00 pm	13,..., 15	18	6
3:00 pm-8:00 am	16, ..., 20	8	6
8 pm-midnight	21, ..., 24	6	5

By setting a limit on the maximum expected waiting time ($W_{q,i} = W_{max}$), the minimum staffing level required for each hour c_i can be determined by equations (2) and (3). The call center's management specified a value of $W_{max} = 2$ minutes, in order to obtain a feasible optimum schedule that could be satisfied by the existing workforce size. Applying equations (2) and (3) for $W_{max} =$ 2 minutes, and then adding a standard 10% allowance for employee absenteeism, the results are shown in Table 2.

5. IP scheduling model and assumptions

5.1. Assumptions

1. Shifts start and finish only on the hour.
2. The number of company employees on shift schedule (all 4 groups) must be between 4 and 20. Thus, the size of each group must be between 1 and 5.
3. The total monthly cost per employee, including benefits and overhead, is given for the three employee categories as: (i) shift employees: \$5,781.39, (ii) non-shift employees: \$4,680.19, and (iii) contractors on any shift: \$7,978.31. For the current workforce of 47 operators (19 shift employees, 1 day employee, and 27 contractors), the total monthly labor cost is \$329,940.97.

5.2. New scheduling options

New schedules were introduced (bringing the total to 44) to better match the varying demand pattern. Aiming to minimize disruption to the current work practices, new weekly schedules were created by introducing the following combinations of days off, shift start/finish times, and lunch (dinner) hours.

1. Non-shift employees work hours will be from 6:00 am to 3:00 pm instead of 7:00 am to 4:00 pm to match the start of the peak period at 6:00 am. The one-hour lunch break is still taken from 12:00 noon to 1:00 pm.
2. For contractors who work from 6:00 am to 3:00 pm with Thursday and Friday off, 14 variations were introduced by considering all seven pairs of consecutive days-off and two possible lunch hours (either 11:00 am-12:00 noon or 12:00 noon-1:00 pm).
3. For contractors who work from 9:00 am to 6:00 pm with Tuesday and Wednesday off, the work hours were delayed to 10:00 am-7:00 pm. Also, 14 variations were introduced by considering all seven pairs of consecutive days-off and two possible lunch hours (either 1:00 pm-2:00 pm or 2:00 pm-3:00 pm).
4. For contractors who work from 3:00 pm to midnight with different days off, 14 versions were introduced by considering all seven pairs of consecutive days-off and two possible dinner hours (either 7:00 pm-8:00 pm or 8:00 pm-9:00 pm).

5.3. Integer programming model

An IP model was constructed to optimize employee weekly work schedules. The objective of the IP model shown below is to minimize labor cost, subject to meeting the hourly staffing demands and satisfying all applicable scheduling rules and constraints.

$$MinimizeZ = \sum_{j=1}^{44} k_j x_j \tag{4}$$

subject to

$$\sum_{j=1}^{44} a_{ij} x_j \geq r_i, i = 1, 2, ..., 168 \tag{5}$$

$$1 \leq x_1 \leq 5 \tag{6}$$

where

Z = total labor cost of assigned operators
x_j = number of operator on weekly work schedule $j, x_j \geq 0$ and integer, $j = 1, ..., 44$
k_j = cost of weekly work schedule j
$a_{i,j}$ = 1 if hour i is a work period for schedule j, 0 otherwise
r_i = minimum number of employees required in hour i

5.4. Optimum solution

Using the above inputs and assumptions, the integer program was solved using LINDO. Table 3 shows the optimum schedule assignments, while Table 4 compares the feature of the current and proposed schedules.

Table 3 **Weekly work assignments in the proposed schedule**

#	Operator Type	Shift Time	Break	Off Days	Number
1	Shift Employees	3 shifts, 24 hours	None	Varying	20
2	Day employees	6:00 am-3:00pm	12:00 pm-1:00 pm	Thu-Fri	5
3	Contractors	6:00 am-3:00 pm	11:00 am-12:00 pm	Sun-Mon	1
4	Contractors	6:00 am-3:00 pm	11:00 am-12:00 pm	Thu-Fri	8
5	Contractors	6:00 am-3:00 pm	11:00 am-12:00 pm	Fri-Sat	1
6	Contractors	10:00 am-7:00 pm	1:00 pm-2:00 pm	Thu-Fri	4
6	Contractors	3:00 pm-12:00 pm	7:00 pm-8:00 pm	Thu-Fri	1
8	Contractors	3:00 pm-12:00 pm	8:00 pm-9:00 pm	Tue-Wed	1
9	Contractors	3:00 pm-12:00 pm	8:00 pm-9:00 pm	Thu-Fri	2
10	Contractors	3:00 pm-12:00 pm	8:00 pm-9:00 pm	Fri-Sat	1

Table 4 **Comparison of schedules**

Solution property	Current Schedule	Optimum Schedule
Shift employees: $4x_1$	19	20
Non-shift employees:x_2	1	5
Contractors: $x_3 + x_4 + \ldots + x_{44}$	27	19
Total workforce	47	44
Total monthly cost ($)	329,941	290,617
Workforce utilization (%)	76.1	81.1

The call center's management liked the proposed schedule because it: (1) decreases labor cost by $39,324 per month, (2) reduces the workforce size by three operators, (3) improves workforce utilization by 5%, and (4) reduces the average waiting time to 2 minutes. Utilization is the ratio of required man-hours to assigned man-hours per week.

6. Conclusion

This paper described operator staffing and scheduling for a call center. Queuing models were used to analyze the data in order to determine the minimum number of operators required for each hour. New weekly schedules were proposed to improve flexibility in meeting hourly varying labor demands. An integer programming model was constructed to find the optimum employee work schedules that satisfy labor requirements with the minimum cost. The optimum operator schedule is expected to save $470,000 a year while reducing average waiting time and satisfying all management objectives.

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