REPORT NO.
UCD/CGM-01/04

CENTER FOR GEOTECHNICAL MODELING

LINE SEARCH TECHNIQUES FOR ELASTO-PLASTIC FINITE ELEMENT COMPUTATIONS IN GEOMECHANICS

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SEPTEMBER 2001

University of Califorina at Davis, Center for Geotechnical Modeling Report UCD-CGM-00
Published in the Communications in Numerical Methods in Engineering, Vol. 17, issue 2, pages 115-125, 2001.

Line Search Techniques for Elasto–Plastic Finite Element Computations in Geomechanics

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Abstract

In this paper we present a globally convergent modification of Newton's method for integrating constitutive equations in elasto–plasticity of geomaterials. Newton's method is known to be q-quadratically convergent when the current solution approximation is adequate. Unfortunately, it is not unusual to expend significant computational time in order to achieve satisfactory results. We will present a technique which can be used when the Newton step is unsatisfactory. This scheme can be considered as a modified version of the traditional concept of backtracking along the Newton direction if a full Newton step provides unsatisfactory results. The method is also known as line search technique.

The technique is applied to the fully implicit Newton algorithm for a hardening or softening general isotropic geomaterials at the constitutive level. Various solution details and visualizations are presented, which emerge from the realistic modeling of highly nonlinear constitutive behavior observed in the analysis of cohesionless granular materials.

Key Words: Elasto–plasticity, Geomechanics, Constitutive integrations, Newton iterative method, Line search technique

1 Introduction

Problem of accurately following the equilibrium path in numerical modeling of geomaterials elasto–plasticity has been researched for some time now. It has been shown (Runesson and Samuelson [12], Simo and Taylor [14]) that consistent use of Newton iterative algorithm on the global, finite element and local constitutive levels provides for fast convergence. However, complexities of elasto–plastic constitutive models for geomaterials put high demand on the constitutive driver. In particular, low confinement region, usually found near the soil surface or during liquefaction behavior of sands, with highly nonlinear hardening and softening behavior, high dilatancy angles (non–associative behavior) and highly curved yield surface, can lead to the numerical failure (divergence) of the constitutive driver. This leads to the interruption of finite element computations and consequently requires rerun of the problem with smaller loading steps.

While the use of full Newton scheme improves rate of convergence, in some cases the iterative algorithm does not converge at all. The necessary condition for good convergence behavior of Newton methods is that current solution approximation is good enough, i.e. it is within the convergence region of Newton method. If the current solution approximation is not good enough, Newton method will not converge but rather bounce through solution space until erroneous iterations are interrupted. One of the strategies used to prevent failure of Newton methods is the line search technique. It should be noted that line search will not cure all of the problems associated with local, constitutive level Newton iterations. However, as long as the function (and their derivatives) used in iterations (yield function, potential function, hardening/softening functions) are analytic, line search will provide for global convergence in the sense defined by Dennis and Schnabel [3].

Equilibrium iterations for material nonlinear finite element computations can be in general separated in two levels. First iteration level is tied to the constitutive, elasto–plastic computations. On this level, constitutive driver, for a given strain increment[1] iterates in stress and internal variable space until convergence criteria is met. On the second level, nonlinear finite element system of equations solver is iterating until balance of internal and external forces is achieved. Global level iteration to achieve balance of internal and external forces have seen use of line search techniques (eg. Crisfield [2], Larsson et al. [10], Simo and Meschke [13]). In their recent paper, Dutko et al. [4],

[1]Assuming displacement based implementation of FEM.

used a variant of line search algorithm for constitutive level iterations. They applied this technique to the biaxial anisotropic yield criterion (Barlat et al. [1]) and presented interesting and useful results. Of particular interest are statistics on number of line search iterations for a sharply curved region of the yield line. Unfortunately, the actual line search technique used was just briefly described.

The paper is organized as follows. Section 2 briefly describes elastic–plastic algorithmic formulation based on the fully implicit, Newton procedure and the B Material Model used in computations. Section 3 describes theory behind the line search technique and section 4 describes application of the line search techniques to the constitutive integration problems. Section 5 presents numerical examples that illustrate described developments.

It is worthwhile noting that while the presented techniques is applied to small deformation elasto–plastic problems it has been used in the large deformation elasto–plastic algorithms as well. However, inherent complexities of LDEP algorithms developments might hide the basic ideas of using line search techniques so we restrict our presentation to small deformation elasto–plasticity. Although developments presented here are simplified to small deformation format, the generality of the approach is not lost.

2 Elastic–Plastic Geomechanics

In this section we briefly present Backward Euler algorithms for the solutions of elastic–plastic problems in geomechanics. To this end, we use the additive decomposition of the strain increment into elastic and plastic parts together with flow theory of plasticity and Karush–Kuhn–Tucker conditions to formulate the elastic–plastic problem. Detailed description of the formulations can be found elsewhere (eg. Jeremić and Sture [8]). The fully implicit Backward Euler algorithm is developed in general stress tensor – internal variable tensor setting and is based on the following equations

$$
\begin{aligned}
{}^{n+1}\sigma_{ij} = E_{ijkl}\left({}^{n+1}\epsilon_{kl} - {}^{n+1}\epsilon^{p}_{kl}\right) \quad &; \quad {}^{n+1}\epsilon^{p}_{ij} = {}^{n}\epsilon^{p}_{ij} + \lambda\, {}^{n+1}m_{ij} \\
{}^{n+1}q_{*} = {}^{n}q_{*} + \lambda\, {}^{n+1}h_{*} \quad &; \quad F_{n+1} = 0
\end{aligned}
\tag{1}
$$

where, ϵ_{ij}, ϵ^{e}_{ij} and ϵ^{p}_{ij} are the total, elastic and plastic strain tensors respectively, σ_{ij} is the Cauchy stress tensor, q_{*} represents suitable set of internal variables, h_{*} is the plastic moduli and F_{n+1} is the yield surface function at the final position. The asterisk in the place of indices in q_* replaces n indices, so that for example in the case of isotropic hardening q_* is a scalar while for kinematic hardening q_* is second order tensor. Equations (1), are the nonlinear algebraic equations to be solved for the unknowns ${}^{n+1}\sigma_{ij}$, ${}^{n+1}\epsilon^p_{ij}$, ${}^{n+1}q_*$ and λ. Newton iterative scheme is used to solve for the single vector return in stress tensor – internal variable space. We can circumvent the problem of finding the solution in the softening region, which, in general stress tensor – internal variable space belongs to non–convex space, by defining the tensor of stress residuals. Then, by working on the problem of minimizing the stress residual we convert the non–convex problem to the convex one. Of course, as usual, by using Newton iterative method, we have to provide for continuation of iterative procedure even if the Newton step is not satisfactory. That is precisely the point where the line search technique shows it usefulness.

The Backward Euler algorithm is based on the elastic predictor – plastic corrector strategy:

$$ {}^{n+1}\sigma_{ij} = {}^{pred}\sigma_{ij} - \lambda\, E_{ijkl}\; {}^{n+1}m_{kl} \tag{2} $$

where ${}^{pred}\sigma_{ij} = E_{ijkl}\, \epsilon_{kl}$ is the elastic trial stress state and ${}^{n+1}m_{kl} = (\partial Q / \partial \sigma_{kl})|_{n+1}$ is the gradient to the plastic potential function in stress space at the final position. We define a tensor of residuals

$$ r_{ij} = \sigma_{ij} - \left({}^{pred}\sigma_{ij} - \lambda\, E_{ijkl}\; {}^{n+1}m_{kl} \right) \tag{3} $$

that represents the difference between the current stress state σ_{ij} and the Backward Euler stress state ${}^{pred}\sigma_{ij} - \lambda\, E_{ijkl}\; {}^{n+1}m_{kl}$. By using first order Taylor series expansion of the tensor of residuals r_{ij} and the first order Taylor expansion of the yield function F, and after some algebraic manipulations we obtain the change in consistency parameter λ

$$ d\lambda = \frac{ {}^{n+1}F^{old} - {}^{n+1}n_{mn}\; {}^{old}r_{ij}\, {}^{n+1}T^{-1}_{ijmn} }{ {}^{n+1}n_{mn} E_{ijkl}\; {}^{n+1}H_{kl} {}^{n+1}T^{-1}_{ijmn} - {}^{n+1}\xi_*\; h_* } \tag{4} $$

We have also introduced the fourth order tensors T_{ijmn} and H_{ijmn}:

$$ {}^{n+1}T_{ijmn} = \delta_{im}\delta_{nj} + \lambda\, E_{ijkl}\; \left.\frac{\partial m_{kl}}{\partial \sigma_{mn}}\right|_{n+1} \quad ; \quad {}^{n+1}H_{kl} = {}^{n+1}m_{kl} + \lambda \left.\frac{\partial m_{kl}}{\partial q_*}\right|_{n+1} h_* \tag{5} $$

where $n_{mn} = \partial F / \partial \sigma_{mn}$, $\xi_* = \partial F / \partial q_*$ and $dq_* = d\lambda\; h_*(\sigma_{ij}, q_*)$. With the solutions for $d\lambda$ we can write the iterative solution for $d\sigma_{mn}$ and dq_*, in the extended stress – internal variable space as:

$$ d\sigma_{mn} = - \left({}^{old}r_{ij} + \frac{ {}^{n+1}F^{old} - {}^{n+1}n_{mn}\; {}^{old}r_{ij} {}^{n+1}T^{-1}_{ijmn} }{ {}^{n+1}n_{mn} E_{ijkl}\; {}^{n+1}H_{kl} {}^{n+1}T^{-1}_{ijmn} - {}^{n+1}\xi_*\; h_* }\; E_{ijkl}\; {}^{n+1}H_{kl} \right) {}^{n+1}T^{-1}_{ijmn} \tag{6} $$

$$dq_* = \left(\frac{{}^{n+1}F^{old} - {}^{n+1}n_{mn} \; {}^{old}r_{ij} \, {}^{n+1}T^{-1}_{ijmn}}{{}^{n+1}n_{mn} E_{ijkl} \; {}^{n+1}H_{kl} \, {}^{n+1}T^{-1}_{ijmn} - {}^{n+1}\xi_* \; h_*} \right) h_* \tag{7}$$

Iterative procedure is continued until the objective function $\|r_{ij}\| = 0$ is satisfied given a certain tolerance.

In order to start the Newton iterative procedure, we use forward Euler solution as a starting point

$${}^{start}\sigma_{mn} = E_{mnpq} \, d\epsilon_{pq} - E_{mnpq} \frac{{}^{cross}n_{rs} \; E_{rstu} \; d\epsilon_{tu}}{{}^{cross}n_{ab} \; E_{abcd} \; {}^{cross}m_{cd} - \xi_* h_*} \; {}^{cross}m_{pq} \tag{8}$$

$${}^{start}q_* = {}^{previous}q_* + \left(\frac{{}^{cross}n_{mn} \; E_{mnpq} \; d\epsilon_{pq}}{{}^{cros}n_{mn} \; E_{mnpq} \; {}^{cros}m_{pq} - \xi_* h_*} \right) h_* \tag{9}$$

where ${}^{cross}()$ denotes the point where trial state crosses yield surface.

Consistent, Newton iterations on the constitutive level are reflected on the global, finite element level through use of algorithmic tangent stiffness (ATS) tensor ${}^{ATS}E^{ep}_{pqmn}$ defined as

$$d\sigma_{pq} = {}^{ATS}E^{ep}_{pqmn} d\epsilon_{mn} \quad \text{with} \quad {}^{ATS}E^{ep}_{pqmn} = R_{pqmn} - \frac{R_{pqkl} \, {}^{n+1}H_{kl} \, {}^{n+1}n_{ij} R_{ijmn}}{{}^{n+1}n_{ot} R_{otpq} \; {}^{n+1}H_{pq} + {}^{n+1}\xi_* \; h_*} \tag{10}$$

where we have used reduced stiffness tensor $R_{mnkl} = \left({}^{n+1}T_{ijmn}\right)^{-1} E_{ijkl}$. It is important to note that the line search technique described later *does not* alter the ATS tensor ${}^{ATS}E^{ep}_{pqmn}$. A detailed derivation is given in Jeremić and Sture [6].

We use small deformation version of the B material model (Jeremić et al. [5]) for our computations. The model relies on the development behind the so called MRS–Lade model (Sture et al. [16]). The B–Model is a single surface model, with uncoupled cone portion and cap portion hardening. Very low confinement region was carefully modeled and the yield surface was shaped in such a way to mimic recent findings obtained during Micro Gravity Mechanics tests aboard Space Shuttle (Sture et al. [15]). Figure (1) depicts meridian and deviatoric traces and a full yield surface. It also depicts cone and cap hardening/softening functions. Detailed description of the model is given by Jeremić et al. [5].

3 Line Search Technique

In this section we develop theory behind the line search techniques. We base our developments on concept of minimization theory for a scalar function $f(x_*)$. We show

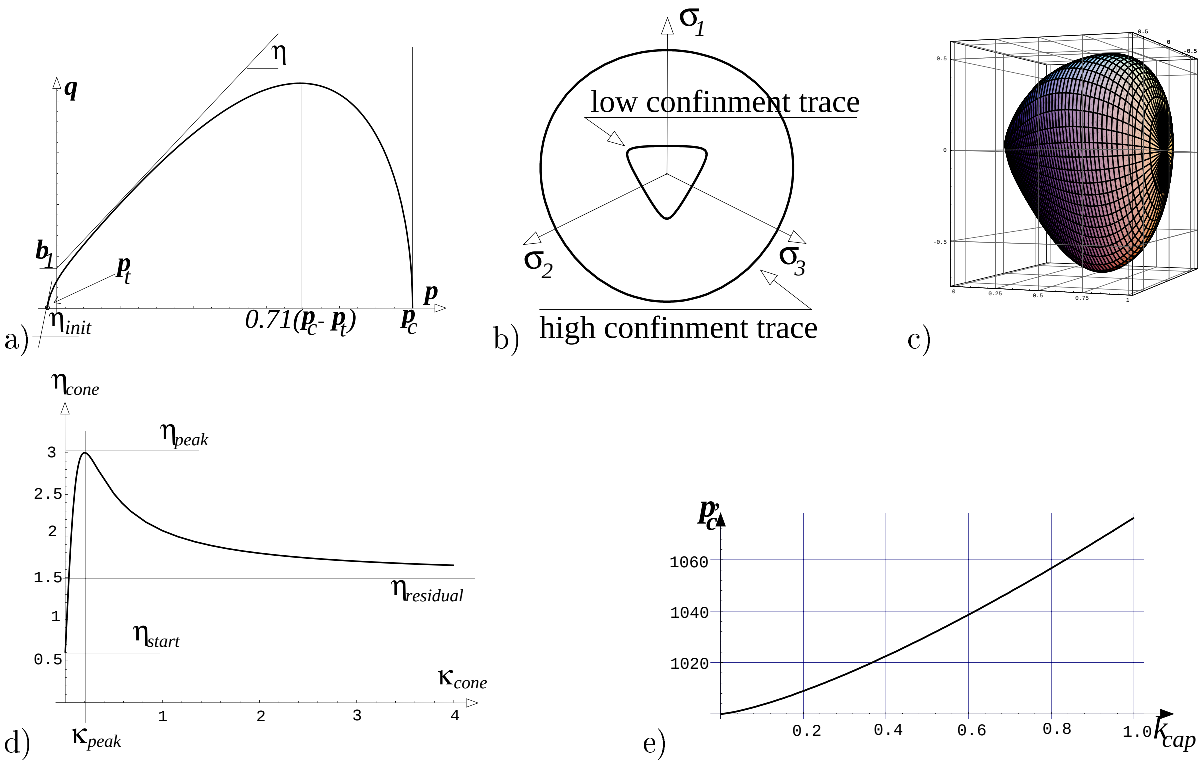


Figure 1: (a) Meridian trace of yield, ultimate or potential surface. (b) Change of the deviatoric trace of yield surface changes along the mean stress axes. (c) Yield and/or potential surface in principal stress space. (d) Cone hardening function. (e) Cap hardening function.

later that in our case, function to be minimized will be the energy norm of the vector of residuals r_{ij} (Eq. 3).

The minimization problem is defined as (eg. Dennis and Schnabel [3].):

$$\min_{x_* \in \mathcal{R}^n} f(x_*) : \mathcal{R}^n \longrightarrow \mathcal{R} \tag{11}$$

The basic idea of a global method for minimization is to take the step that lead downhill for the function $f(x_*)$. One chooses a direction p_* from the current point x_*^c in which $f(x_*)$ decreases initially and a new point x_*^+ in this direction from x_*^c is such that $f(x_*^+) < f(x_*^c)$. Such a direction p_* is called a descent direction. From the mathematical point of view, p_* is a descent direction from x_*^c if the directional derivative of $f(x_*)$ at x_*^c in the direction p_* is negative:

$$\frac{\partial f(x_*^c)}{\partial x_*} p_* < 0 \tag{12}$$

If (12) holds, then it is guaranteed that for a small positive ζ, $f(x_*^c + \zeta p_*) < f(x_*^c)$. The idea of line search algorithm can be described as follows:

- At iteration k do:
  - calculate a descent direction p_*^k,
  - set $x_*^{k+1} \longleftarrow (x_*^k + \zeta^k p_*^k)$ for some ζ^k that makes x_*^{k+1} an acceptable next iterate.

Figure (2) shows the basic concept: select x_*^{k+1} by considering the half of a one–dimensional cross section of $f(x_*)$ in which $f(x_*)$ decreases initially from x_*^k.

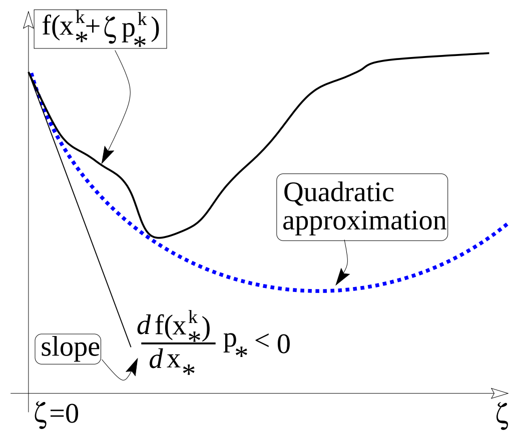


Figure 2: A cross section of $f(x_*)$ from x_*^k in the direction p_*^k.

The term "line search" refers to the procedure of choosing the acceptable ζ^k. The so called "exact line search" accounts for finding the exact solution of the one–dimensional minimization problem, i.e. finding the exact ζ^k so that $f(x_*^k + \zeta^k p_*^k)$ attains minimum. This was the preferred approach to the problem until mid 1960s. More careful computational testing has led to the use of "slack line search" which has a weak acceptance criteria for ζ^k as a more computationally efficient procedure. The common procedure now is to try the full Newton step first (with $\zeta^k = 1$) and then, if $\zeta^k = 1$ fails to satisfy criterion, to reduce ζ^k in a systematic way, along the direction defined by that step.

Systematic reduction of ζ^k along the descent direction can be achieved by applying the line search techniques through backtracking algorithm:

Given $\alpha \in (0, \frac{1}{2})$:

$\zeta^k = 1$;

while $f(x_*^k + \zeta^k p_*^k) > f(x_*^k) + \alpha \zeta^k \frac{\partial f(x_*^k)}{\partial x_*} p_*^k$ do:

$\zeta^k \longleftarrow \rho \zeta^k$ where, usually $\rho = \frac{1}{2}$;

$x_*^{k+1} \longleftarrow x_*^k + \zeta^k p_*^k$;

Dennis and Schnabel [3] provide rather powerful convergence results for properly chosen steps.

4 Application to Elasto–Plastic Computations in Geomechanics

In this section we describe the application of line search technique to the elasto–plastic constitutive level equilibrium iterations. The objective function that is followed is the Euclidean norm of tensor of residuals r_{ij}. To this end we rewrite the elastic predictor plastic corrector Eq. (2) as:

$$ {}^{n+1}\sigma_{ij} = {}^{pred}\sigma_{ij} - \zeta \; \lambda \; E_{ijkl} \; {}^{n+1}m_{kl} \tag{13} $$

It is important to note that the addition of scalar line search parameter ζ does not change the initial equations. Moreover, as ζ is only used in line search improvement and is not a function of any state variable (stress, internal variables or displacements) first order Taylor expansions used to come up with the iterative steps (Eq. (4) – (7) are not changed. The only difference is that now solution for the change in consistency parameter $d\lambda$ reads:

$$ \zeta \; d\lambda = \frac{{}^{n+1}F^{old} - {}^{n+1}n_{mn} \; {}^{old}r_{ij} \, {}^{n+1}T^{-1}_{ijmn}}{{}^{n+1}n_{mn} E_{ijkl} \; {}^{n+1}H_{kl} \, {}^{n+1}T^{-1}_{ijmn} - {}^{n+1}\xi_* \; h_*} \tag{14} $$

with changed tensors T_{ijmn} and H_{kl}

$$ {}^{n+1}T_{ijmn} = \delta_{im}\delta_{nj} + \zeta \; \lambda \; E_{ijkl} \; \left. \frac{\partial m_{kl}}{\partial \sigma_{mn}} \right|_{n+1} \quad ; \quad {}^{n+1}H_{kl} = {}^{n+1}m_{kl} + \zeta \; \lambda \left. \frac{\partial m_{kl}}{\partial q_*} \right|_{n+1} h_* \tag{15} $$

With this changes the line search algorithm can now be specialized to elasto–plastic implicit computations. In each iterative step k we do:

- Given $\beta \in (0, \frac{1}{2})$:
- $\zeta^k = 1$; (Full Newton step)

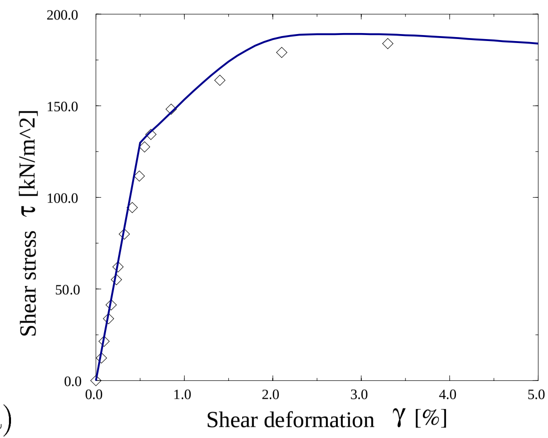
- while
$$\|r_{ij}({}^k\sigma_{ij} + \zeta^k d\,{}^k\sigma_{ij}, {}^kq_* + \zeta^k d\,{}^kq_*)\| > \|r_{ij}({}^k\sigma_{ij}, {}^kq_*)\| + \beta\zeta^k \left\| \left(\frac{\partial\|r_{ij}\|}{\partial\sigma_{mn}} {}^k\sigma_{mn} + \frac{\partial\|r_{ij}\|}{\partial q_*} {}^kq_* \right) \right\|$$
do:
  - $\zeta^k \longleftarrow \rho\zeta^k$ where, usually $\rho = \frac{1}{2}$;
  - ${}^{k+1}\sigma_{ij} \longleftarrow {}^k\sigma_{ij} + \zeta^k d\,{}^k\sigma_{ij}$ and ${}^{k+1}q_* \longleftarrow {}^kq_* + \zeta^k d\,{}^kq_*$

We always start with $\zeta^k = 1$ (that is, a full Newton step) and only if that iteration fail, we apply the backtrack algorithm. As we are following value of the Euclidean norm of tensor of residuals r_{ij}, failure of Newton step is defined as divergence of that scalar variable. In other words, increase the value of Euclidean norm of tensor of residuals r_{ij} means that our Newton step in the extended stress – internal variable space is not valid. It also means that convergence of the Newton iterative procedure is questionable and that it will most probably fail in this iterative step.

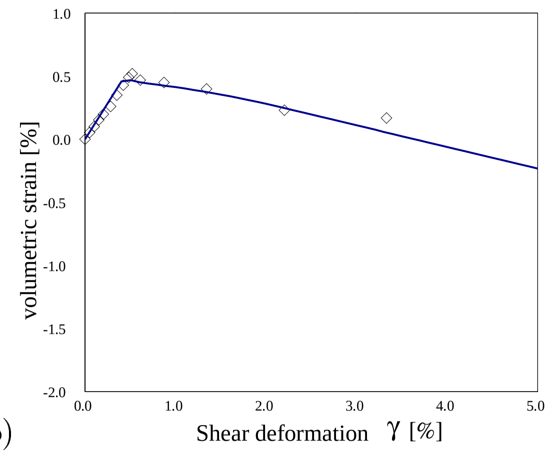
5 Numerical Example

Previous theoretical developments has been implemented in FEI (Finite Element Interpreter) finite element program. FEI is our experimental software platform, written in C++ with some Fortran modules. It is build on top of **nDarray** and **FEMtools** class libraries (Jeremić and Sture [9]). The line search algorithm is implemented on both global, finite element level as well as on the local, constitutive level, described in this paper. We follow nonlinear finite element iterations in analyzing directional shear cell (DSC) experiments done at the University of Colorado at Boulder. For example McFadden ([11]) used DSC apparatus in order to investigate shear behavior of various sands. Here we present modeling of particular plane strain experiment $K_0 14.5 - 30$ in figure (3).

Loading is divided in two stages. The first stage comprises isotropic compression to target confinement state (in this case $p = 180.0$ kPa). The second stage is a shear loading until instability occurs (instability in the experimental test, numerically we can follow the specimen beyond limit point). It is not clear if the instabilities in DSC experiments (loss of control over loading process) were due to the bifurcation phenomena inside the specimen or to the global rotation of the specimen. Since DSC is a load controlled device, only shear deformation of $\gamma_{xy} \simeq 3.5$ % was reached in the laboratory experiment.



a)



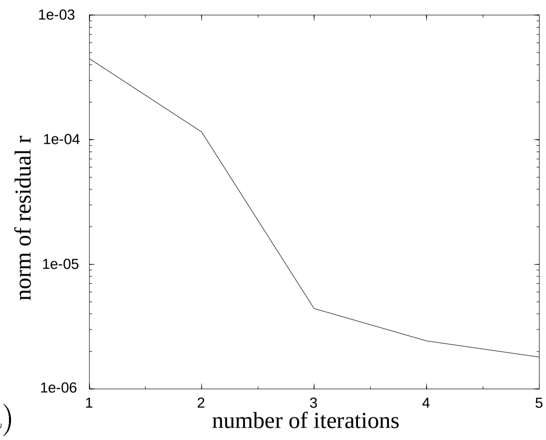
b)

Figure 3: Numerical modeling of a shear test $K_0 14.5 - 30$ a) Shear stress – axial strain curves. b) Volumetric strain – axial strain curves.

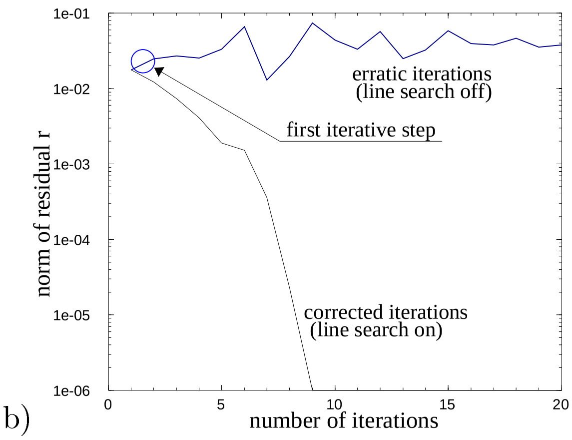
As the numerical experiment is proceeding, the length of the loading steps is controlled by using the variable hyperspherical arc–length constraint ([7]). In this particular case, the Newton iterative algorithm failed at constitutive level after stress state advanced into elastic–plastic regime. This part of the elastic–plastic computations is sometimes problematic, since there is a sudden activation of the hardening mechanism. In this particular case, we have stopped the computations at the beginning of the problematic incremental step. Then, we resumed iterations by manually decreasing the step size from 0.5% to 0.1%. Figure 4(a) shows values of the norm of the tensor of residual $\|r_{ij}\|$ in such a successful iteration. It can be seen from Fig. 4(a) that convergence is quite fast, leading to the conclusion that the initial estimate of the stress and internal variable state was within the Newton convergence region.

Similar to the previous numerical experiment, we resumed computations at the beginning of the problematic step, but this time with larger incremental deformation of 0.5% and with the line search algorithm *turned off*. Figure 4(b) shows values of the norm of the tensor of residual $\|r_{ij}\|$ for the first incremental step. We can observe that the initial, uncorrected iteration produces non–converging set of values for the norm of the tensor of residual $\|r_{ij}\|$. It is interesting to note that the very first iterative step takes the value of $\|r_{ij}\|$ to higher than initial values. Moreover, Figure 5(a) shows that although value of evaluated yield surface F initially point downward from the initial predictor, it is eventually diverging too.

Finally, once again we resume computations at the beginning of the problematic incremental step, now with the line search algorithm *turned on*. It can be seen from



a)



b)

Figure 4: (a) Residual norm $\|r_{ij}\|$ values for a successful iteration with small incremental steps (one step of 0.1% shear deformation). (b) Residual norm values $\|r_{ij}\|$ values for erratic and corrected iterations (one step of 0.5% shear deformation).

Fig. 4(b) that not only the algorithm helps advance the iterations without any signs of divergence, it also produces a very good convergence rate. This, of course, is the result of our algorithm always first trying the full Newton step (for which we set $\zeta^k = 1$).

Figure 5(b) depicts the problematic, first iterative step from Figure 4(b) in some more details. We follow the value of the objective function $\|r_{ij}\|$ through 10 subincrements within the problematic iterative step. At this point it is important to remember that Newton iterative algorithm makes a quadratic approximation (in multidimensional space) of the objective function, here the Euclidean norm $\|r_{ij}\|$. In this particular step, the approximation for $\|r_{ij}\|$ does a poor job, and although the objective function initially decreases, eventually its final value is higher than that it had at the initial state. It is interesting to note that at 5th subincrement, objective function $\|r_{ij}\|$ actually does attain a minimum value, but full Newton step (at 10th subincrement) leads to a higher than initial values for $\|r_{ij}\|$.

Figure (6) depicts failed and corrected iterations in the space of stress invariants p, q and θ

$$p = -\frac{1}{3}\sigma_{kk} \quad q = \sqrt{3\frac{1}{2}s_{ij}s_{ij}} \quad \cos 3\theta = \frac{3\sqrt{3}}{2}\frac{\frac{1}{3}s_{ij}s_{jk}s_{ki}}{\sqrt{(\frac{1}{2}s_{ij}s_{ij})^3}} \quad s_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij} \qquad (16)$$

In this particular view, we are looking at the B model yield surface from the tensile region (negative p). Divergent iteration is oscillating through the stress space and there is no sign of recovery. On the other hand, corrected oscillation, after modifying one

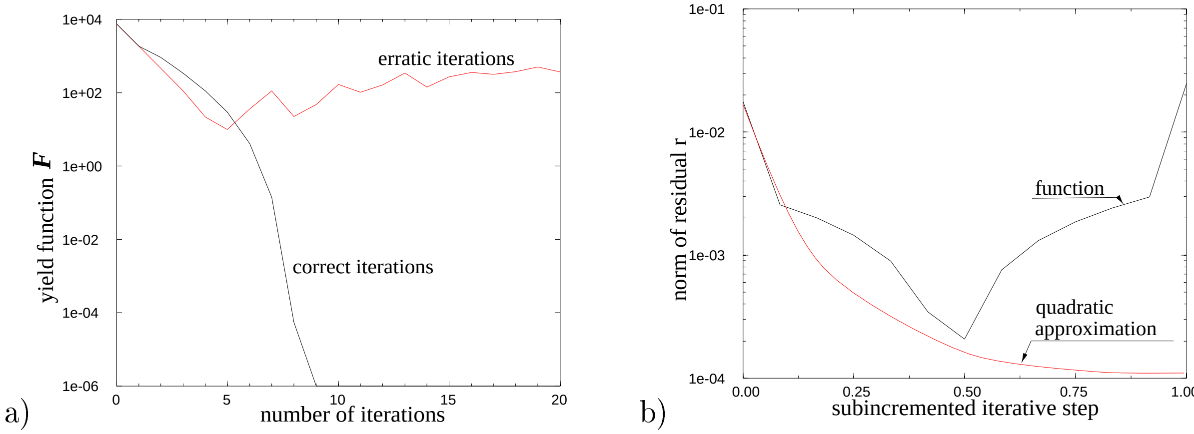


Figure 5: (a) Yield surface values F for erratic and corrected iterations (one step of 0.5% shear deformation). (b) Dissection of failed iteration.

iterative step by the line search algorithm, converges successfully.

6 Summary

In this paper we have presented a globally convergent modification of Newton's method for integrating constitutive equations in elasto–plasticity of geomaterials. The method has been developed in rigorous mathematical framework and implemented in experimental finite element program FEI. The practical use of method was illustrated in details. The application of line search techniques in numerical modeling of elasto–plasticity of geomaterials should provide for robust following of equilibrium path on both, global, finite element and local, constitutive levels.

Acknowledgment

The author gratefully acknowledge partial support by National Science Foundation's Pacific Earthquake Engineering Research Center Grant NSF-PEER 2132000.3.

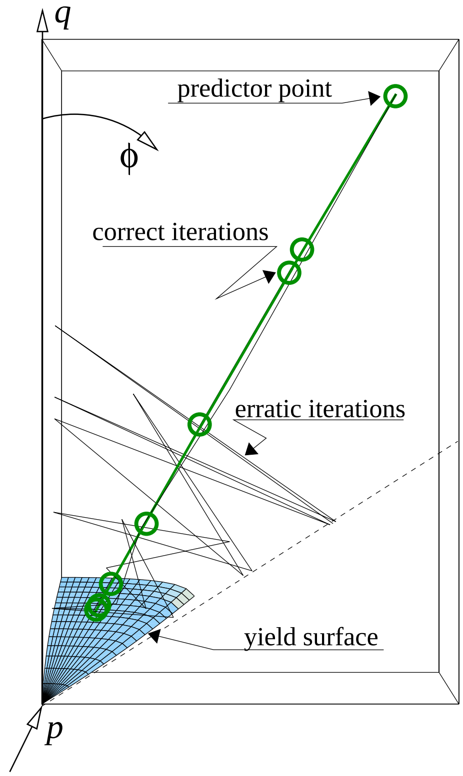


Figure 6: Illustration of erratic and correct iterations in stress space.

References

[1] Barlat, F., Lege, D. J., and Brem, J. C. A six component yield function for anisotropic materials. *International Journal for Plasticity 7* (1991), 693–712.

[2] Crisfield, M. A. *Non - Linear Finite Element Analysis of Solids and Structures Volume 1: Essentials.* John Wiley and Sons, Inc. New York, 1991.

[3] Dennis, Jr., J. E., and Schnabel, R. B. *Numerical Methods for Unconstrained Optimization and Nonlinear Equations.* Prentice Hall , Engelwood Cliffs, New Jersey 07632., 1983.

[4] Dutko, M., Perić, D., and Owen, D. R. J. Universal anisotropic yield criterion based on superquadratic functional representation: Part1. algorithmic issues and accuray analysis. *Computer methods in applied mechanics and engineering 109* (1993), 73–93.

[5] Jeremić, B., Runesson, K., and Sture, S. A model for elastic–plastic pressure sensitive materials subjected to large deformations. *International Journal of Solids and Structures 36*, 31/32 (1999), 4901–4918.

[6] Jeremić, B., and Sture, S. Implicit integration rules in plasticity: Theory and implementation. Report to NASA Marshall Space Flight Center, Contract: NAS8-38779, University of Colorado at Boulder, June 1994.

[7] Jeremić, B., and Sture, S. Refined finite element analysis of geomaterials. In *Proceedings of 11th Engineering Mechanics Conference* (Fort Lauderdale, Florida, May 1996), Y. K. Lin and T. C. Su, Eds., Engineering Mechanics Division of the American Society of Civil Engineers, pp. 555–558.

[8] Jeremić, B., and Sture, S. Implicit integrations in elasto–plastic geotechnics. *International Journal of Mechanics of Cohesive–Frictional Materials 2* (1997), 165–183.

[9] Jeremić, B., and Sture, S. Tensor data objects in finite element programming. *International Journal for Numerical Methods in Engineering 41* (1998), 113–126.

[10] Larsson, R., Runesson, K., and Sture, S. Embedded localization band in undrained soil based on regularized strong discontinuity–theory and FE–analysis. *International Journal of Solids and Structures 33*, 20-22 (1996), 3081–3101.

[11] McFadden, J. J. Experimental response of sand during pricipal stress rotations. Master of Science thesis, University od Colorado at Boulder, December 6 1988.

[12] Runesson, K., and Samuelsson, A. Aspects on numerical techniques in small deformation plasticity. In *NUMETA 85 Numerical Methods in Engineering, Theory and Applications* (1985), G. N. P. J. Middleton, Ed., AA.Balkema., pp. 337–347.

[13] Simo, J. C., and Meschke, G. A new class of algorithms for classical plasticity extended to finite strain. application to geomaterials. *Computational Mechanics 11* (1993), 253–278.

[14] Simo, J. C., and Taylor, R. L. Consistent tangent operators for rate-independent elastoplasticity. *Computer Methods in Applied Mechanics and Engineering 48* (1985), 101–118.

[15] Sture, S., Costes, N., Batiste, S., Lankton, M., AL-Shibli, K., Jeremić, B., Swanson, R., and Frank, M. Mechanics of granular materials at low effective stresses. *ASCE Journal of Aerospace Engineering 11*, 3 (July 1998), 67–72.

[16] Sture, S., Runesson, K., and Macari-Pasqualino, E. J. Analysis and calibration of a three invariant plasticity model for granular materials. *Ingenieur Archiv 59* (1989), 253–266.