

Serial #: _____ ID _____ NAME _____

Show all necessary steps for full marks.

Question 1: (4 points): Textbook Exercise 87: Given $f(x) = \log_2 x$ and $g(x) = x - 2$. Find the functions $f \circ g$ and $g \circ f$ and their domains.

Solution:

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \text{ where } x \in D_g = (-\infty, \infty) \text{ and } g(x) \in D_f = (0, \infty) \\ &= f(x - 2) \\ &= \log_2(x - 2) \end{aligned}$$

Domain of $f \circ g = D_{f \circ g} = (2, \infty)$ because $x \in D_g = (-\infty, \infty)$ and $x - 2 > 0$

$$\begin{aligned} (g \circ f)(x) &= g(f(x)) \text{ where } x \in D_f = (0, \infty) \text{ and } f(x) \in D_g = (-\infty, \infty) \\ &= g(\log_2 x) \\ &= (\log_2 x) - 2 \end{aligned}$$

Domain of $g \circ f = D_{g \circ f} = (0, \infty)$ because $x \in D_f = (0, \infty)$ and $f(x) \in D_g = (-\infty, \infty)$

Question 2: (6 points):

(a): Sketch the graph of f .

(b): Find the interval on which the graph of f lies completely above the x -axis.

(d): Find the inverse of f .

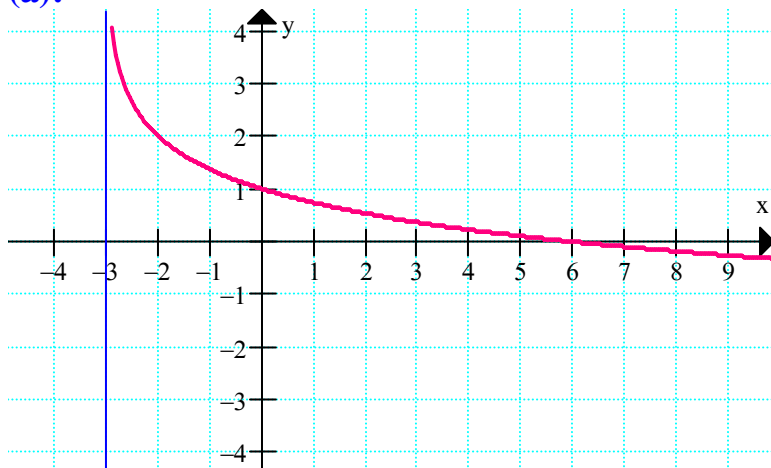
Solution: $x = 0 \Rightarrow f(0) = 2 - \log_3 3 = 2 - 1 = 1$

The y -intercept is $(0, 1)$.

To find x -intercept, let $f(x) = 0 \Rightarrow \log_3(x + 3) = 2 \Rightarrow x + 3 = 9 \Rightarrow x = 6$

The x -intercept is $(6, 0)$

(a):



$$x + 3 > 0 \Rightarrow x > -3$$

$$\text{Domain} = (-3, \infty) \text{ , } \text{Range} = (-\infty, \infty)$$

(b): $2 - \log_3(x + 3) > 0$

$$-\log_3(x + 3) > -2$$

$$\log_3(x + 3) < 2$$

$$0 < x + 3 < 9$$

$$-3 < x < 6$$

$$x \in (-3, 6) = D_f$$

Question 3: (6 points): If x and y are positive real numbers, then write the logarithmic expression $5\log_3 x - 8\log_9 y + \log_{\sqrt{3}} z + 1$ as a single logarithm of base 3.

Solution:

$$\begin{aligned}
 5\log_3 x - 8\log_9 y + \log_{\sqrt{3}} z + 1 &= 5\log_3 x - 8\frac{\log_3 y}{\log_3 9} + \frac{\log_3 z}{\log_3 \sqrt{3}} + \log_3 3 \\
 &= \log_3 x^5 - \frac{8}{2}\log_3 y + \frac{\log_3 z}{\frac{1}{2}} + \log_3 3 \\
 &= \log_3 x^5 - 4\log_3 y + 2\log_3 z + \log_3 3 \\
 &= \log_3 x^5 + \log_3 y^{-4} + \log_3 z^2 + \log_3 3 \\
 &= \log_3 (x^5 y^{-4} z^2 3) \\
 &= \log_3 \frac{3x^5 z^2}{y^4}
 \end{aligned}$$

Question 4: (5 points): (Textbook Exercise 105): Solve each equation $(x-1)^{\log(x-1)} = 100(x-1)$

Solution:

$$\log(x-1)^{\log(x-1)} = \log[100(x-1)]$$

$$[\log(x-1)]\log(x-1) = \log 100 + \log(x-1)$$

$$[\log(x-1)]^2 - \log(x-1) - 2 = 0$$

$$u^2 - u - 2 = 0, \quad u = \log(x-1)$$

$$(u-2)(u+1) = 0$$

$$[\log(x-1) - 2][\log(x-1) + 1] = 0$$

$$\log(x-1) = 2 \quad \text{or} \quad \log(x-1) = -1$$

$$x-1 = 100 \quad \text{or} \quad x-1 = 10^{-1}$$

$$\boxed{x = 101} \quad \text{or} \quad \boxed{x = \frac{11}{10}}$$

$$SS = \left\{ \frac{11}{10}, 101 \right\}$$