Serial #: _________ ID ______________ NAME ____

Show all necessary steps for full marks.

Question 1: (4 points): If $f(x)$ is an even function such that $f(-3)=4$, then find the coordinates of the two points that must lie on the graph of $y=3f(2x)$.

Solution: Since the function f is an even function, then $f(-x)=f(x)$

$\Rightarrow$ $f(-3)=f(3)=4$ $\Rightarrow$ The points $(-3,4)$ and $(3,4)$ satisfy $y=f(x)$.

$y=f(x)$	$y=f(2x)$ $\left(\frac{x}{2},y\right)$	$y=3f(2x)$ $\left(\frac{x}{2},3y\right)$
$(-3,4)$	$\left(\frac{-3}{2},4\right)$	$\left(\frac{-3}{2},12\right)$
$(3,4)$	$\left(\frac{3}{2},4\right)$	$\left(\frac{3}{2},12\right)$

Question 2: (4 points): If $f(x)=\sqrt{x-3}$ and $g(x)=\frac{2}{x}$, then find the domain of $(f\circ g)(x)$.

Solution:
$$
\begin{aligned}
(f\circ g)(x)&=f(g(x))\\
&=f\left(\frac{2}{x}\right)\\
&=\sqrt{\frac{2}{x}-3}=\sqrt{\frac{2-3x}{x}}
\end{aligned}
$$

$$\frac{2-3x}{x}\geq 0$$

CRV: 0 , $\frac{2}{3}$

$$
\begin{array}{ccccc}
 & - & & + & & - \\
\hline
-\infty & ------------- & 0 & ////////////// & \frac{2}{3} & -------- & \infty
\end{array}
$$

$$D=\left(0,\frac{2}{3}\right]$$

$$D_g=(-\infty,0)\cup(0,\infty)$$

$$D_{f\circ g}=D\cap D_g=D=\left(0,\frac{2}{3}\right]$$

Question 3: (4 points): Find the range of the function $f(x) = -2x^2 + 4x - 5$

Solution: $h = -\frac{b}{2a} = -\frac{4}{2(-2)} = 1$, $k = f(h) = -2+4-5 = -3 \Rightarrow$ The vertex $= (h,k) = (1,-3)$

Since $a = -2$ is a negative number, so the parabola opens down. Therefore y takes value from $-\infty$ to -3. Thus $R_f = (-\infty, -3]$

Question 4: (4 points): If $f(x) = 2x^2 + 5$, $g(x) = 2x + m$ and the graph of the function $(f \circ g)(x)$ has y-intercept 23, then $m = ?$

Solution: The graph of the function $(f \circ g)(x)$ has y-intercept 23

$\Rightarrow (0, 23)$ is on the graph of $f \circ g$

$$(f \circ g)(0) = 23$$
$$f(g(0)) = 23$$
$$f(m) = 23$$
$$2m^2 + 5 = 23$$
$$2m^2 = 18$$
$$m^2 = 9$$
$$\boxed{m = \pm 3}$$

Q8.

If $f(x) = 2x^2 + 5$, $g(x) = 2x + m$ and the graph of the function $(f \circ g)(x)$ has y-intercept 23, then $m =$

A) -5
B) $\pm\sqrt{7}$
C) 5
D) ± 7
E) ± 3

$f(x) = 2x^2 + 5$

$g(x) = 2x + m$

$f \circ g(x) = 2(2x+m)^2 + 5$

$= 2(4x^2 + 4mx + m^2) + 5$

$= 8x^2 + 8mx + 2m^2 + 5$

y-intercept if $x = 0$

$\Rightarrow 2m^2 + 5 = 23$

$2m^2 = 18$

$m^2 = 9 \Rightarrow m = \pm 3.$

Question 5: (4 points): Find the maximum of the product $(3-2x)(x+2)$

Solution: $(3-2x)(x+2) = -2x^2 - x + 6$

$$h = -\frac{b}{2a} = -\frac{-1}{2(-2)} = -\frac{1}{4}$$

$$\text{Maximum} = f(h) = f\left(-\frac{1}{4}\right) = -2\left(-\frac{1}{4}\right)^2 - \left(-\frac{1}{4}\right) + 6 = -\frac{1}{8} + \frac{1}{4} + 6 = \frac{-1+2+48}{8} = \frac{49}{8}$$