

Show all necessary steps for full marks.

Question 1: (4 points): If $f(x)$ is an even function such that $f(-3) = 4$, then find the coordinates of the two points that must lie on the graph of $y = 3f(2x)$.

Solution: Since the function f is an even function, then $f(-x) = f(x)$
 $\Rightarrow f(-3) = f(3) = 4 \Rightarrow$ The points $(-3,4)$ and $(3,4)$ satisfy $y = f(x)$.

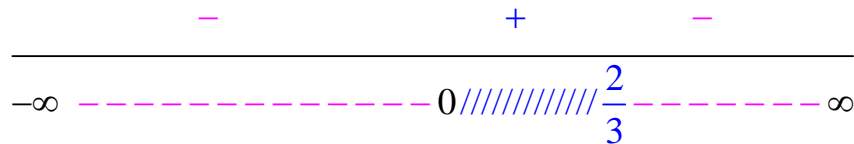
$y = f(x)$	$y = f(2x)$	$y = 3f(2x)$
	$\left(\frac{x}{2}, y\right)$	$\left(\frac{x}{2}, 3y\right)$
$(-3,4)$	$\left(\frac{-3}{2}, 4\right)$	$\left(\frac{-3}{2}, 12\right)$
$(3,4)$	$\left(\frac{3}{2}, 4\right)$	$\left(\frac{3}{2}, 12\right)$

Question 2: (4 points): If $f(x) = \sqrt{x-3}$ and $g(x) = \frac{2}{x}$, then find the domain of $(f \circ g)(x)$.

Solution: $(f \circ g)(x) = f(g(x))$
 $= f\left(\frac{2}{x}\right)$
 $= \sqrt{\frac{2}{x} - 3} = \sqrt{\frac{2-3x}{x}}$

$$\frac{2-3x}{x} \geq 0$$

CRV: $0, \frac{2}{3}$



$$D = \left(0, \frac{2}{3}\right]$$

$$D_g = (-\infty, 0) \cup (0, \infty)$$

$$D_{f \circ g} = D \cap D_g = D = \left(0, \frac{2}{3}\right]$$

Question 3: (4 points): Find the range of the function $f(x) = -2x^2 + 4x - 5$

Solution: $h = -\frac{b}{2a} = -\frac{4}{2(-2)} = 1, k = f(h) = -2 + 4 - 5 = -3 \Rightarrow$ The vertex $= (h, k) = (1, -3)$

Since $a = -2$ is a negative number, so the parabola opens down. Therefore y takes value from $-\infty$ to -3 . Thus $R_f = (-\infty, -3]$

Question 4: (4 points): If $f(x) = 2x^2 + 5, g(x) = 2x + m$ and the graph of the function $(f \circ g)(x)$ has y -intercept 23, then $m = ?$

Solution: The graph of the function $(f \circ g)(x)$ has y -intercept 23

$\Rightarrow (0, 23)$ is on the graph of $f \circ g$

$$(f \circ g)(0) = 23$$

$$f(g(0)) = 23$$

$$f(m) = 23$$

$$2m^2 + 5 = 23$$

$$2m^2 = 18$$

$$m^2 = 9$$

$$\boxed{m = \pm 3}$$

Q8.

If $f(x) = 2x^2 + 5, g(x) = 2x + m$ and the graph of the function $(f \circ g)(x)$ has y -intercept 23, then $m =$

- A) -5
- B) $\pm\sqrt{7}$
- C) 5
- D) ± 7
- E) ± 3

$$f(x) = 2x^2 + 5$$

$$g(x) = 2x + m$$

$$f \circ g(x) = 2(2x + m)^2 + 5$$

$$= 2(4x^2 + 4mx + m^2) + 5$$

$$= 8x^2 + 8mx + 2m^2 + 5$$

y -intercept if $x=0$

$$\Rightarrow 2m^2 + 5 = 23$$

$$2m^2 = 18$$

$$m^2 = 9 \Rightarrow m = \pm 3.$$

Question 5: (4 points): Find the maximum of the product $(3 - 2x)(x + 2)$

Solution: $(3 - 2x)(x + 2) = -2x^2 - x + 6$

$$h = -\frac{b}{2a} = -\frac{-1}{2(-2)} = -\frac{1}{4}$$

$$\text{Maximum} = f(h) = f\left(-\frac{1}{4}\right) = -2\left(-\frac{1}{4}\right)^2 - \left(-\frac{1}{4}\right) + 6 = -\frac{1}{8} + \frac{1}{4} + 6 = \frac{-1 + 2 + 48}{8} = \frac{49}{8}$$