

Serial #: _____ ID _____ NAME _____

Show all necessary steps for full marks.**Question 1: (5 points):** Simplify the following expressions: (Assume $a \neq 0$, $b \neq 0$, and $c \neq 0$)

$$(a): |-2^{-3^2}| \quad (b): \frac{(a+b)^{-1}}{a^{-1}+b^{-1}} \quad (c): (a^{-1}-b^{-1})^2(a-b)^{-2} \quad (d): \frac{(-2a^{-1}bc^{-2})^{-2}}{(3^{-1}b)(2^{-1}ac^{-2})^3}$$

Solution:

$$(a): |-2^{-3^2}| = 2^{-3^2} = 2^{-9} = \frac{1}{2^9}$$

$$(b): \frac{(a+b)^{-1}}{a^{-1}+b^{-1}} = \frac{\frac{1}{a+b}}{\frac{1}{a} + \frac{1}{b}} = \frac{\frac{1}{a+b}}{\frac{b+a}{ab}} = \frac{1}{a+b} \cdot \frac{ab}{b+a} = \frac{ab}{(a+b)^2}$$

$$(c): (a^{-1}-b^{-1})^2(a-b)^{-2} = \frac{(a^{-1}-b^{-1})^2}{(a-b)^2} = \frac{\left(\frac{1}{a}-\frac{1}{b}\right)^2}{(a-b)^2} = \frac{\left(\frac{b-a}{ab}\right)^2}{(a-b)^2} = \frac{(b-a)^2}{a^2b^2} = \frac{(b-a)^2}{a^2b^2} \cdot \frac{1}{(a-b)^2} = \frac{1}{a^2b^2}$$

$$(d): \frac{(-2a^{-1}bc^{-2})^{-2}}{(3^{-1}b)(2^{-1}ac^{-2})^3} = \frac{(-2)^{-2} \cdot a^2 \cdot b^{-2} \cdot c^4}{3^{-1} \cdot b \cdot 2^{-3} \cdot a^3 \cdot c^{-6}} = \frac{3 \cdot 2^3 \cdot a^2 \cdot c^4 \cdot c^6}{(-2)^2 \cdot a^3 \cdot b \cdot b^2} = \frac{3 \cdot 8 \cdot c^{4+6}}{4 \cdot a^{3-2} \cdot b^{1+2}} = \frac{6 \cdot c^{10}}{a \cdot b^3}$$

Question 2: (5 points): (P.4 Textbook Exercises 39, 62 and 84): Simplify the following expressions (Assume that all letters denote positive numbers.)

$$(a): \sqrt{32} + \sqrt{18} \quad (b): \frac{x^{3/4}x^{7/4}}{x^{5/4}} \quad (c): \frac{(2y^{4/3})^2y^{-2/3}}{y^{7/3}} \quad (d): \sqrt{s}\sqrt{s^3}$$

Solution:

$$(a): \sqrt{32} + \sqrt{18} = \sqrt{16(2)} + \sqrt{9(2)} = 4\sqrt{2} + 3\sqrt{2} = 7\sqrt{2}$$

$$(b): \frac{x^{3/4}x^{7/4}}{x^{5/4}} = x^{\frac{3}{4} + \frac{7}{4} - \frac{5}{4}} = x^{\frac{5}{4}}$$

$$(c): \frac{(2y^{4/3})^2y^{-2/3}}{y^{7/3}} = \frac{4y^{8/3}y^{-2/3}}{y^{7/3}} = 4y^{\frac{8}{3} - \frac{2}{3} - \frac{7}{3}} = 4y^{-1/3} = \frac{4}{y^{1/3}} \cdot \frac{y^{2/3}}{y^{2/3}} = \frac{4y^{2/3}}{y}$$

$$(d): \sqrt{s}\sqrt{s^3} = (s \cdot s^{3/2})^{1/2} = \left(s^{1+\frac{3}{2}}\right)^{1/2} = (s^{5/2})^{1/2} = s^{5/4}$$

Question 3: (5 points): If the coefficient of x^4 in the product $(bx^2 + x)(3x^4 - 2x^3 + x^2 + 1)$ is equal to zero, then find the value of b **Solution:** The product of bx^2 and x^2 is bx^4 . The product of x and $-2x^3$ is $-2x^4$.

$$bx^4 - 2x^4 = (b-2)x^4$$

$$b-2=0$$

$$b=2$$

Question 4: (5 points): If $M = \frac{2}{1 + \sqrt{3} - \sqrt{12}}$ and $N = 8 \cdot \sqrt[3]{\frac{3}{16}}$, then $M + N = ?$

Solution:

21) If $M = \frac{2}{1 + \sqrt{3} - \sqrt{12}}$ and $N = 8 \cdot \sqrt[3]{\frac{3}{16}}$, then $M + N =$

✓ A) $-1 - \sqrt{3} + 2 \sqrt[3]{12}$

B) $-2 - \sqrt{3} + \sqrt[3]{6}$

C) $2 - \sqrt{3} + 2 \sqrt[3]{12}$

D) $-1 - \sqrt{3} + \sqrt[3]{3}$

E) $2 - \sqrt{3} + \sqrt[3]{6}$

$$M = \frac{2}{1 + \sqrt{3} - 2\sqrt{3}}$$

$$= \frac{2}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}} = \frac{2(1 + \sqrt{3})}{1 - 3}$$

$$= \frac{2(1 + \sqrt{3})}{-2} = -(1 + \sqrt{3})$$

$$N = 8 \frac{\sqrt[3]{3}}{\sqrt[3]{8 \cdot 2}} = \frac{8 \sqrt[3]{3}}{2 \sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}}$$

$$= \frac{8 \sqrt[3]{12}}{(2)(2)} = 2 \sqrt[3]{12}$$

$$\therefore M + N = -1 - \sqrt{3} + 2 \sqrt[3]{12}$$