King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 001 Class Test II
Textbook Sections: 1.1 to 2.5
Term 181
Time Allowed: 90 Minutes

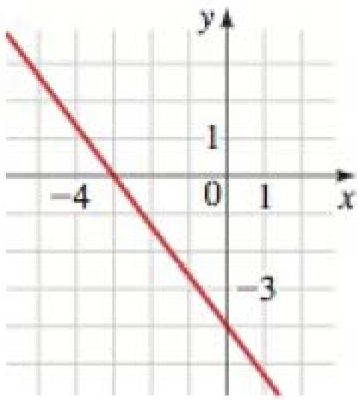
Student's Name: …………… ………………………………………………
ID #:……………………….. **Section:** ……. **Serial Number:** …….

Provide neat and complete solutions.
Show all necessary steps for full credit and write the answer in simplest form.

No Calculators, Cameras, or Mobiles are allowed during this exam.

Question	**Points**	Student's Score
1	4	
2	4	
3	4	
4	4	
5	5	
6	4	
7	4	
8	6	
9	4	
10	6	
11	5	
Total	**50**	___ / 50
		___ / 100

Question 1: (4 points) (1.3 Textbook Exercises 22): Find an equation for the line whose graph is sketched.



Solution:

We choose the two intercepts, $(0, -4)$ and $(-3, 0)$. So the slope is $m = \dfrac{0-(-4)}{-3-0} = -\frac{4}{3}$. Since the y-intercept is -4, the equation of the line is $y = mx + b = -\frac{4}{3}x - 4 \Leftrightarrow 4x + 3y + 12 = 0$.

Question 2: (4 points) (1.6 Textbook Example 4): Find the solution set of $2x = 1 - \sqrt{2-x}$

Solution:

EXAMPLE 4 ■ An Equation Involving a Radical

Solve the equation $2x = 1 - \sqrt{2-x}$.

SOLUTION To eliminate the square root, we first isolate it on one side of the equal sign, then square:

$2x - 1 = -\sqrt{2-x}$	Subtract 1
$(2x-1)^2 = 2 - x$	Square each side
$4x^2 - 4x + 1 = 2 - x$	Expand LHS
$4x^2 - 3x - 1 = 0$	Add $-2 + x$
$(4x+1)(x-1) = 0$	Factor
$4x + 1 = 0$ or $x - 1 = 0$	Zero-Product Property
$x = -\frac{1}{4}$ $\quad$ $x = 1$	Solve

The values $x = -\frac{1}{4}$ and $x = 1$ are only potential solutions. We must check them to see whether they satisfy the original equation. From *Check Your Answers* we see that $x = -\frac{1}{4}$ is a solution but $x = 1$ is not. The only solution is $x = -\frac{1}{4}$.

Now Try Exercise 43

$$SS = \left\{-\frac{1}{4}\right\}$$

Question 3: (4 points) (1.2 Textbook Exercises 90): Given $x^2 + y^2 + \frac{1}{2}x + 2y + \frac{1}{16} = 0$. Find the following

(a): $center = ?$

(b): $radius = ?$

(c): $Domain = ?$

(d): $Range = ?$

Solution: $x^2+y^2+\frac{1}{2}x+2y+\frac{1}{16}=0$

$$x^2+\frac{1}{2}x+\left(\frac{1}{4}\right)^2+y^2+2y+1^2=-\frac{1}{16}+\frac{1}{16}+1$$

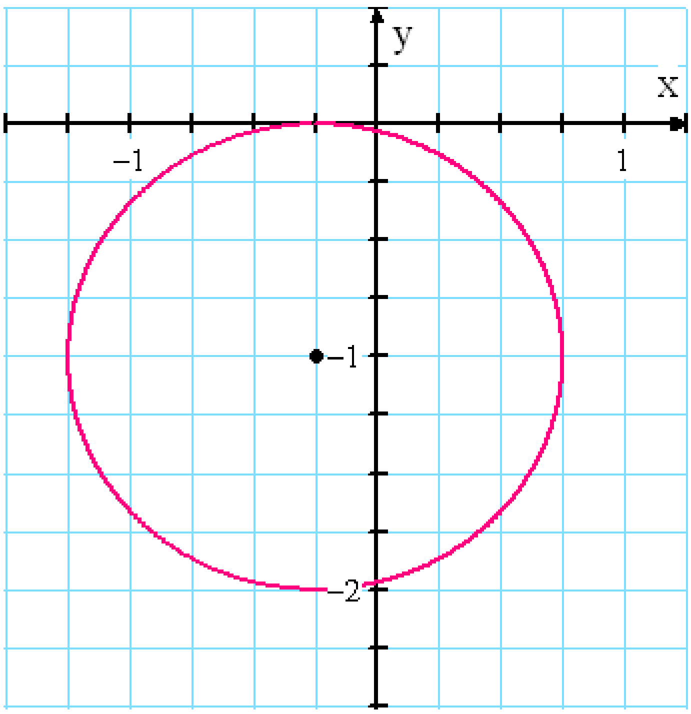
$$\left(x+\frac{1}{4}\right)^2+(y+1)^2=1$$

(a): $center=\left(-\frac{1}{4},-1\right)$

(b): $radius=1$

(c): $Domain=\left[-\frac{1}{4}-1,-\frac{1}{4}+1\right]=\left[-\frac{5}{4},\frac{3}{4}\right]$

(d): $Range=[-1-1,-1+1]=[-2,0]$



Question 4: (4 points) (1.4 Textbook Exercises 54): Solve the equation $A=2\pi r^2+2\pi r\,h$ for $r=?$

Solution:

54. $A=2\pi r^2+2\pi rh \Leftrightarrow 2\pi r^2+2\pi rh-A=0$. Using the Quadratic Formula,

$$r=\frac{-(2\pi h)\pm\sqrt{(2\pi h)^2-4(2\pi)(-A)}}{2(2\pi)}=\frac{-2\pi h\pm\sqrt{4\pi^2h^2+8\pi A}}{4\pi}=\frac{-\pi h\pm\sqrt{\pi^2h^2+2\pi A}}{2\pi}.$$

Question 5: (5 points) (1.5 Textbook Exercises 46):

If $A+Bi=\frac{(1+2i)(3-i)}{2+i}+i^{2001}$, then find $A=?$ and $B=?$

Solution:

$$\frac{(1+2i)(3-i)}{2+i}=\frac{3-i+6i-2i^2}{2+i}=\frac{5+5i}{2+i}\cdot\frac{2-i}{2-i}=\frac{10-5i+10i-5i^2}{4-i^2}=\frac{(10+5)+(-5+10)i}{5}$$

$$=\frac{15+5i}{5}=\frac{15}{5}+\frac{5}{5}i=3+i$$

$$i^{2001}=i^{2000}i=\left(i^2\right)^{1000}i=i$$

$A+Bi=3+2i \quad\Rightarrow\quad \boxed{A=3}$ and $\boxed{B=2}$

Question 6: (4 points) (1.6 Textbook Exercises 22):

Find the solution set of $2(x-4)^{7/3}-(x-4)^{4/3}-(x-4)^{1/3}=0$

Solution:

Let $u=x-4$; then $0=2(x-4)^{7/3}-(x-4)^{4/3}-(x-4)^{1/3}=2u^{7/3}-u^{4/3}-u^{1/3}=u^{1/3}(2u+1)(u-1)$. So $u=x-4=0 \Leftrightarrow x=4$, or $2u+1=2(x-4)+1=2x-7=0 \Leftrightarrow 2x=7 \Leftrightarrow x=\frac{7}{2}$, or $u-1=(x-4)-1=x-5=0$ $\Leftrightarrow x=5$. The solutions are 4, $\frac{7}{2}$,and 5.

Question 7: (4 points) (1.7 Textbook Exercises 69):

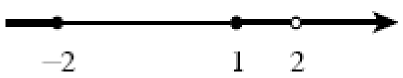
Find the solution set of the following inequality in interval notation. $\dfrac{(x-1)(x+2)}{(x-2)^2}\geq 0$

Solution: CRV: -2, 1 and 2

Interval	$(-\infty,-2)$	$(-2,1)$	$(1,2)$	$(2,\infty)$
Sign of $x-1$	$-$	$-$	$+$	$+$
Sign of $x+2$	$-$	$+$	$+$	$+$
Sign of $(x-2)^2$	$+$	$+$	$+$	$+$
Sign of $\dfrac{(x-1)(x+2)}{(x-2)^2}$	$+$	$-$	$+$	$+$

the solution set is $\{x \mid x\leq -2 \text{ or } x\geq 1 \text{ and } x\neq 2\}$. Interval: $(-\infty,-2]\cup[1,2)\cup(2,\infty)$.

Graph:



Question 8: (6 points) (1.8 Textbook Exercises 41-43): Find the solution set of the following inequalities

(a): $8-|2x-1|\geq 6$

(b): $7|x+2|+5>4$

(c): $\frac{1}{2}\left|4x+\frac{1}{3}\right|>\frac{5}{6}$

Solution:

41. $8-|2x-1|\geq 6 \Leftrightarrow -|2x-1|\geq -2 \Leftrightarrow |2x-1|\leq 2 \Leftrightarrow -2\leq 2x-1\leq 2 \Leftrightarrow -1\leq 2x\leq 3 \Leftrightarrow -\frac{1}{2}\leq x\leq \frac{3}{2}$.
Interval: $\left[-\frac{1}{2},\frac{3}{2}\right]$.

42. $7|x+2|+5>4 \Leftrightarrow 7|x+2|>-1 \Leftrightarrow |x+2|>-\frac{1}{7}$. Since the absolute value is always nonnegative, the inequality is true for all real numbers. In interval notation, we have $(-\infty,\infty)$.

43. $\frac{1}{2}\left|4x+\frac{1}{3}\right|>\frac{5}{6} \Leftrightarrow \left|4x+\frac{1}{3}\right|>\frac{5}{3}$, which is equivalent to either $4x+\frac{1}{3}>\frac{5}{3} \Leftrightarrow 4x>\frac{4}{3} \Leftrightarrow x>\frac{1}{3}$; or $4x+\frac{1}{3}<-\frac{5}{3} \Leftrightarrow$ $4x<-2 \Leftrightarrow x<-\frac{1}{2}$. Interval: $\left(-\infty,-\frac{1}{2}\right)\cup\left(\frac{1}{3},\infty\right)$.

Question 9: (4 points) (2.1 Textbook Exercise 46): If $f(x)=\dfrac{1}{x+1}$, then find the difference quotient.

$\dfrac{f(x+h)-f(x)}{h}$, $h \neq 0$

Solution:

$$\frac{f(x+h)-f(x)}{h}=\frac{1}{h}\left[f(x+h)-f(x)\right]=\frac{1}{h}\left[\frac{1}{x+h+1}-\frac{1}{x+1}\right]=\frac{1}{h}\cdot\frac{x+1-x-h-1}{(x+h+1)(x+1)}$$

$$=\frac{1}{h}\cdot\frac{-h}{(x+h+1)(x+1)}$$

$$=\frac{-1}{(x+1)(x+h+1)}$$

Question 10: (6 points) (2.2 Textbook Exercises 45): Given $f(x)=\begin{cases} 4 & \text{if } x<-2 \\ x^2 & \text{if } -2\le x\le 2 \\ -x+6 & \text{if } x>2 \end{cases}$

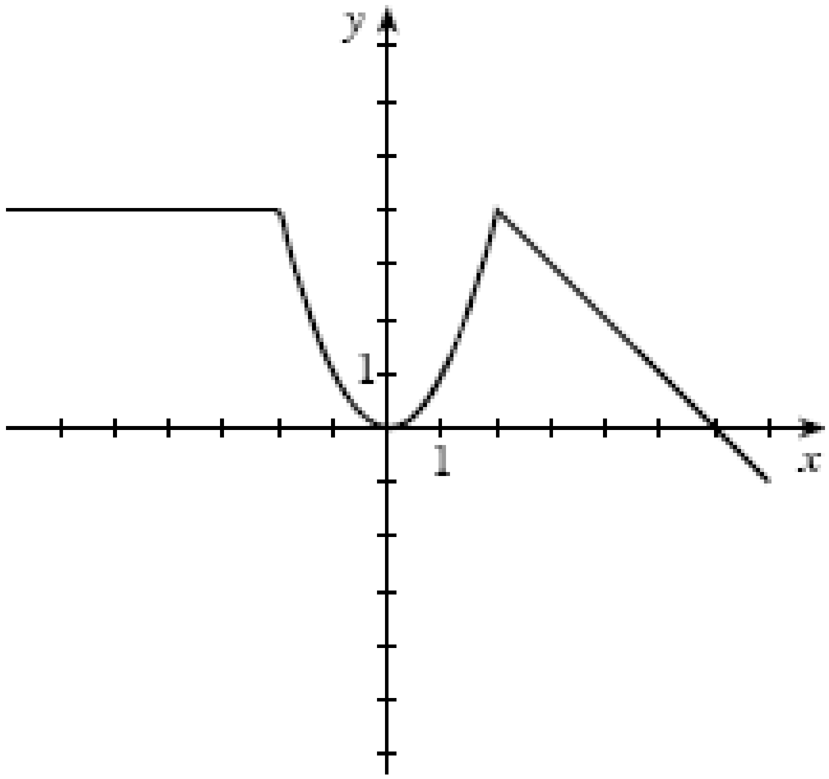
(a): Sketch the graph of the function..

(b): Find the range.

(c): Find the interval(s) where the function is increasing, decreasing or constant.

Solution:

(a):



(b): $Range=(-\infty,4]$

(c): constant on $(-\infty,-2]$

dereasing on $[-2,0]$ and on $(2,\infty)$

increasing on $[0,2]$

Question 11: (5 points): (1.1 Textbook Exercises 12, 14, 16, 18 and 20): Sketch the regions given by each set

(a): $\{(x, y) | y < 3\}$

(b): $\{(x, y) | 0 \le y \le 2\}$

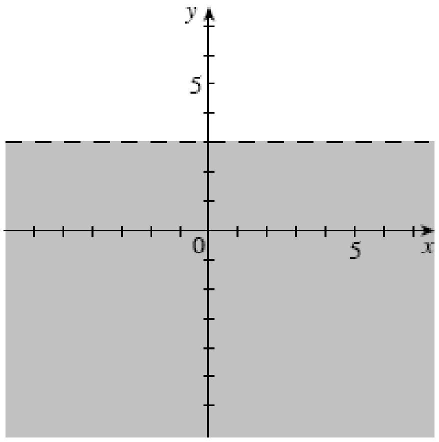
(c): $\{(x, y) | xy > 0\}$

(d): $\{(x, y) | x < 2 \text{ and } y \ge 1\}$

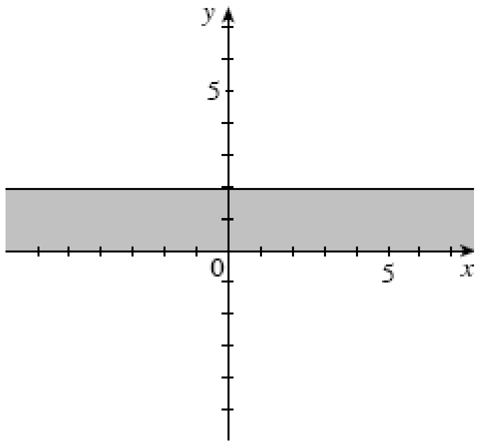
(e): $\{(x, y) | -3 \le x \le 3 \text{ and } -1 \le y \le 1\}$

Solution:

(a): $\{(x, y) | y < 3\}$

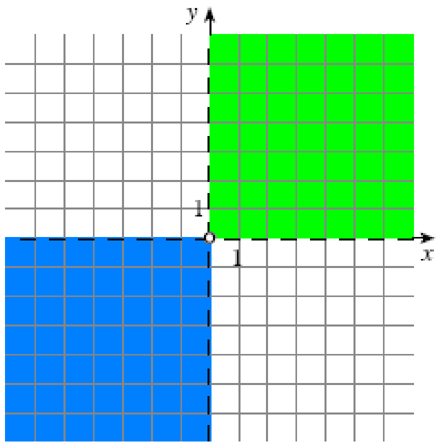


(b): $\{(x, y) | 0 \le y \le 2\}$

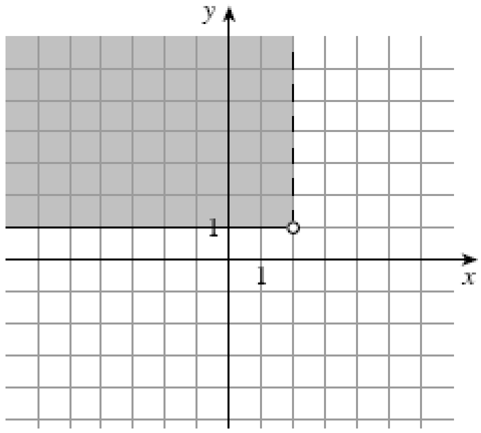


(c): $\{(x, y) | xy > 0\}$

$\{(x, y) | x < 0 \text{ and } y < 0 \text{ or } x > 0 \text{ and } y > 0\}$



(d): $\{(x, y) | x < 2 \text{ and } y \ge 1\}$



(e): Sketch the regions given by each set

$\{(x, y) | -3 \le x \le 3 \text{ and } -1 \le y \le 1\}$