Serial #: ________ ID ______________ NAME ____

Show all necessary steps for full marks.

Question 1: (5 points): If $f(x)=\frac{1-3x}{2+5x}$. If $f^{-1}(x)$ is written in the form $\frac{Ax+B}{x+C}$, then $A+B+C=?$

Q14. Let $f(x)=\frac{1-3x}{2+5x}$. If $f^{-1}(x)$ is written in the form $\frac{Ax+B}{x+C}$, then $A+B+C=$

A) $\frac{2}{5}$

B) $\frac{4}{5}$

C) $-\frac{4}{5}$

D) $\frac{6}{5}$

E) $-\frac{3}{5}$

Sec# Polynomial and Rational Functions - Inverse Functions

$y=\frac{1-3x}{2+5x}$

$2y+5xy=1-3x$

$5xy+3x=1-2y$

$x(5y+3)=1-2y$

$x=\frac{1-2y}{5y+3}$, $y=\frac{1-2x}{5x+3}$

$f^{-1}(x)=\frac{-2x+1}{5(x+3/5)}=\frac{-\frac{2}{5}x+\frac{1}{5}}{x+3/5}$

$A=-\frac{2}{5}$, $B=\frac{1}{5}$, $C=\frac{3}{5}$

$A+B+C=-\frac{2}{5}+\frac{1}{5}+\frac{3}{5}$

$=\frac{2}{5}$

Solution:

$$y=\frac{1-3x}{2+5x}$$

$$x=\frac{1-3y}{2+5y}$$

$$2x+5xy=1-3y$$

$$5xy+3y=-2x+1$$

$$y(5x+3)=-2x+1$$

$$y=\frac{-2x+1}{5x+3}=\frac{\frac{1}{5}(-2x+1)}{\frac{1}{5}(5x+3)}=\frac{-\frac{2}{5}x+\frac{1}{5}}{x+\frac{3}{5}}=\frac{Ax+B}{x+C}\Rightarrow A=-\frac{2}{5},\ B=\frac{1}{5},\ C=\frac{3}{5}$$

$$A+B+C=\frac{-2+1+3}{5}=\frac{2}{5}$$

Question 2: (5 points): Given $f(x)=\frac{1}{5}x^2-\frac{4}{25}x+k$, where $x\ge 2$. If $f^{-1}(2)=5$, then $k=?$

Solution:

$$f^{-1}(2)=5$$

$$f\left(f^{-1}(2)\right)=f(5)$$

$$2=\frac{1}{5}(5)^2-\frac{4}{25}(5)+k$$

$$2=5-\frac{4}{5}+k$$

$$-3+\frac{4}{5}=k\quad\Rightarrow\quad\boxed{k=-\frac{11}{5}}$$

21. Given $f(x)=\frac{1}{5}x^2-\frac{4}{25}x+k$, where $x\ge 2$. If $f^{-1}(2)=5$, then k is equal to

(a) $-\frac{11}{5}$

(b) $-\frac{31}{5}$

(c) $\frac{31}{5}$

(d) $\frac{11}{5}$

(e) $\frac{1}{5}$

$f^{-1}(2)=5\Rightarrow f(5)=2$

$\Rightarrow\frac{1}{5}\cdot 25-\frac{4}{25}\cdot 5+k=2$

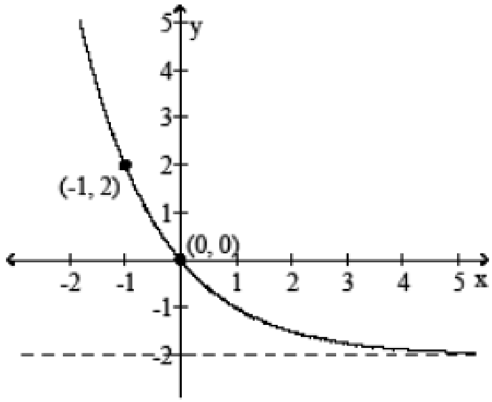
$\Rightarrow 5-\frac{4}{5}+k=2$

$\Rightarrow k=2-5+\frac{4}{5}$

$=-3+\frac{4}{5}$

$=\frac{-11}{5}$

Question 3: (5 points): If the function $f(x) = 2^{(ax+b)} + c$ represents the graph below, then $a+b+c=?$



Solution: Given $\boxed{c=-2}$ and the points $(0,0)$ and $(-1,2)$ are on the graph.

$0 = 2^{a(0)+b} - 2 \quad \Rightarrow \quad 2^b = 2 \quad \Rightarrow \quad \boxed{b=1}$

$2 = (2)^{a(-1)+b} - 2 \quad \overset{b=1}{\Rightarrow} \quad (2)^{-a+1} = 4 \quad \Rightarrow \quad (2^{-1})^{-a+1} = 2^2 \quad \Rightarrow \quad -a+1 = 2 \quad \Rightarrow \quad \boxed{a=-1}$

$a+b+c = -1+1+(-2) = -2$

Question 4: (5 points): Consider the function $f(x) = -2^{-2x+3} + 4$

(a): Find the domain and the range of f in interval notation.

(b): Sketch the graph of f .

(c): Sketch the graph of $g(x) = \left|-2^{-2x+3} + 4\right|$.

Solution: (a): $D_f = (-\infty, \infty)$

$-2^{-2x+3} < 0 \quad \Rightarrow \quad -2^{-2x+3} + 4 < 4 \quad \Rightarrow \quad y < 4 \quad \Rightarrow \quad R_f = (-\infty, 4)$

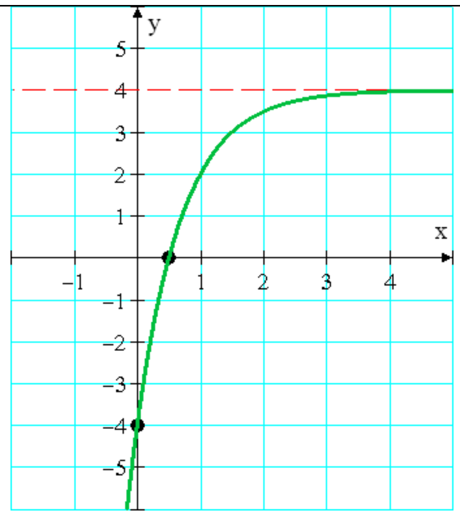
(b): The line $y = 4$ is the horizontal asymptote.

Let $x = 0$, then $f(0) = -2^{-0+3} + 4 = -2^3 + 4 = -4 \quad \Rightarrow$ the y-intercept is $(0,-4)$

Let $f(x) = 0$, then $0 = -2^{-2x+3} + 4 \quad \Rightarrow \quad 2^{-2x+3} = 2^2 \quad \Rightarrow \quad -2x+3 = 2 \quad \Rightarrow \quad x = \dfrac{1}{2}$

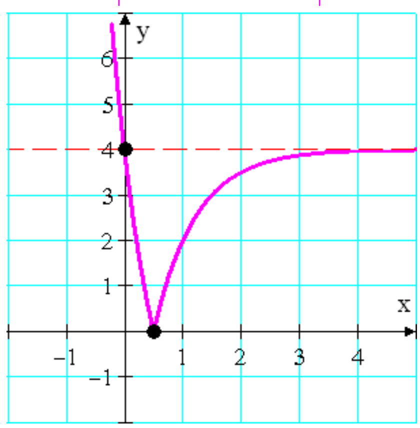
The x-intercept is $(1/2, 0)$

x	0	1/2	1
$y = f(x)$	-4	0	2



$D_f = (-\infty, \infty) \quad R_f = (-\infty, 4]$

(C): $g(x) = \left|-2^{-2x+3} + 4\right|$



$D_g = (-\infty, \infty) \quad R_g = [0, \infty)$