Sec		Serial #:		ID:		Name:

Question #	1	2	3	4	5	6	7	8	9	10	Total:
Points	5	5	5	5	5	5	5	5	5	5	___ / 50 ___ / 100
Student's Score											

Q1. (5 points):

(a): Graph the function $y = -2\cos 3x$ where $-\frac{\pi}{3} \le x \le \pi$

(b): Find the intervals where the function is below the x-axis.

Solution: $0 \le 3x \le 2\pi \quad \Rightarrow \quad 0 \le x \le \frac{2\pi}{3}$

4) For $-\frac{\pi}{3} \le x \le \pi$, the graph of the function $y = -2\cos 3x$, lies below the x-axis in the interval

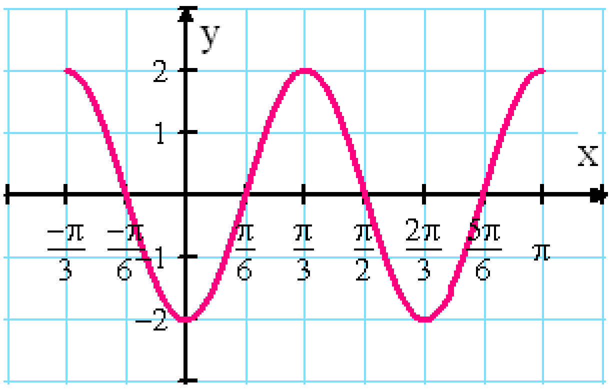
A) $\left[0, \frac{\pi}{3}\right] \cup \left[\frac{2\pi}{3}, \pi\right]$

B) $\left[-\frac{\pi}{3}, 0\right] \cup \left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$

C) $\left[-\frac{\pi}{3}, -\frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, \pi\right]$

✓ D) $\left[-\frac{\pi}{6}, \frac{\pi}{6}\right] \cup \left[\frac{\pi}{2}, \frac{5\pi}{6}\right]$

E) $\left[\frac{\pi}{6}, \frac{\pi}{2}\right]$



Question 2: (5 points): Given $y = -\csc(2x + \pi) + 2$, $-\frac{3\pi}{4} \le x \le \frac{3\pi}{4}$

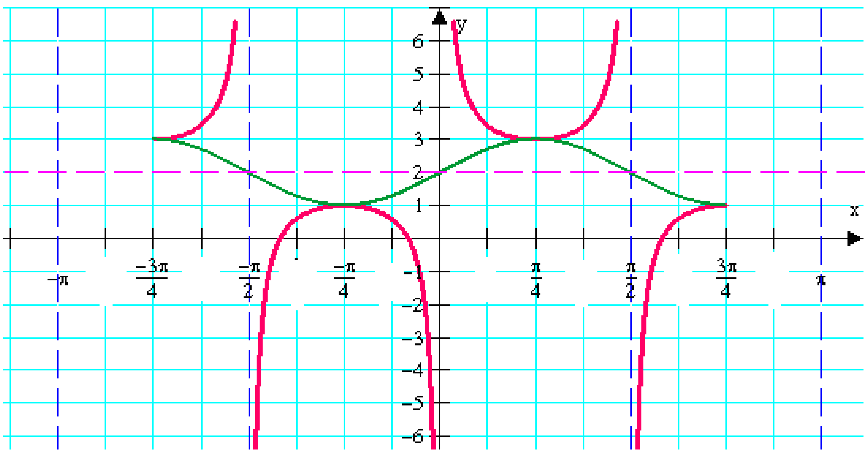
(a): Graph the function over the interval

(b): Find the equations of vertical asymptotes over the given interval.

(c): Find the intervals where the function over the given interval is decreasing.

(d): Find the intervals where the function over the given interval is increasing.

Solution: (a): $y = -\sin(2x + \pi) + 2 \Rightarrow 0 \le 2x + \pi \le 2\pi \Rightarrow -\pi \le 2x \le \pi \Rightarrow -\frac{\pi}{2} \le x \le \frac{\pi}{2}$



(b): vertical asymptotes: $x = -\frac{\pi}{2}$, $x = 0$, $x = \frac{\pi}{2}$

(c): decreasing on $\left(-\frac{\pi}{4}, 0\right) \cup \left(0, \frac{\pi}{4}\right)$

Q3. (5 points): Find all vertical asymptotes of $y = 2\tan\left(3\pi x + \dfrac{\pi}{4}\right)$ over the interval $\left[0, \dfrac{5}{12}\right]$.

Solution:

All vertical asymptotes are given by: $3\pi x + \dfrac{\pi}{4} = (2n+1)\dfrac{\pi}{2}$, $n = 0, \pm 1, , \pm 2, , \pm 3, \cdots$

$$3x = (2n+1)\frac{1}{2} - \frac{1}{4} = n + \frac{1}{2} - \frac{1}{4} = n + \frac{1}{4}$$

$$x = \frac{n}{3} + \frac{1}{12}$$

$$x = \frac{4n+1}{12}$$

If $n = 0$ then $x = \dfrac{1}{12} \in \left[0, \dfrac{5}{12}\right]$

If $n = 1$ then $x = \dfrac{5}{12} \in \left[0, \dfrac{5}{12}\right]$

If $n = 2$ then $x = \dfrac{9}{12} \notin \left[0, \dfrac{5}{12}\right]$

The vertical asymptotes are: $\boxed{x = \dfrac{1}{12}}$ and $\boxed{x = \dfrac{5}{12}}$

Question 4: (5 points) (7.2 Textbook Exercise 16): $(\tan x + \cot x)^2 - (\tan x - \cot x)^2 = ?$

Solution:

$$\begin{aligned}(\tan x + \cot x)^2 - (\tan x - \cot x)^2 &= \tan^2 x + 2\tan x \cot x + \cot^2 x - (\tan^2 x - 2\tan x \cot x + \cot^2 x) \\ &= +2\tan x \cot x + +2\tan x \cot x \\ &= 2 + 2 \\ &= 4\end{aligned}$$

Another Method:

16. $(\tan x + \cot x)^2 - (\tan x - \cot x)^2$

$$= [(\tan x + \cot x) + (\tan x - \cot x)] \cdot [(\tan x + \cot x) - (\tan x - \cot x)]$$

$$= (\tan x + \cot x + \tan x - \cot x) \cdot (\tan x + \cot x - \tan x + \cot x)$$

$$= (2\tan x)(2\cot x) = 4 \cdot \frac{\sin x}{\cos x} \cdot \frac{\cos x}{\sin x} = 4$$

Question 5: (5 points) (7.4 Textbook Exercise 15): Find $\sin\theta$, if $\cos 2\theta = -\dfrac{5}{12}$, with $90° < \theta < 180°$.

Solution: $\cos 2\theta = 1 - 2\sin^2\theta \Rightarrow -\dfrac{5}{12} = 1 - 2\sin^2\theta$

$$2\sin^2\theta = 1 + \frac{5}{12} \Rightarrow 2\sin^2\theta = \frac{17}{12} \Rightarrow \sin^2\theta = \frac{17}{24} \Rightarrow \sin\theta = +\sqrt{\frac{17}{24}} = \frac{\sqrt{17}}{\sqrt{24}} = \frac{\sqrt{17}}{2\sqrt{6}} = \frac{\sqrt{17}\sqrt{6}}{12} = \frac{\sqrt{102}}{12}$$

Another Method:

15. $\cos 2\theta = -\frac{5}{12}, 90° < \theta < 180°$

$\cos 2\theta = 2\cos^2\theta - 1 \Rightarrow$

$2\cos^2\theta = \cos 2\theta + 1 = -\frac{5}{12} + 1 = \frac{7}{12} \Rightarrow$

$\cos^2\theta = \frac{7}{24}$

Since $90° < \theta < 180°, \cos\theta < 0$. Thus,

$$\cos\theta = -\sqrt{\frac{7}{24}} = -\frac{\sqrt{7}}{\sqrt{24}} = -\frac{\sqrt{7}}{2\sqrt{6}}$$
$$= -\frac{\sqrt{7}}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = -\frac{\sqrt{42}}{12}$$

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(-\sqrt{\frac{7}{24}}\right)^2$$
$$= 1 - \frac{7}{24} = \frac{17}{24}$$

Since $90° < \theta < 180°, \sin\theta > 0$. Thus,

$$\sin\theta = \sqrt{\frac{17}{24}} = \frac{\sqrt{17}}{\sqrt{24}} = \frac{\sqrt{17}}{2\sqrt{6}}$$
$$= \frac{\sqrt{17}}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{102}}{12}$$

Question 6 (5 points)(7.4 Classroom Example 3): If $p = \sin 165° \cos 165°$ and $q = \cos^2\frac{\pi}{8} - \frac{1}{2}$ then $p + q = ?$

Solution:

$$p = \sin 165° \cos 165° = \frac{1}{2}(2\sin 165° \cos 165°) = \frac{1}{2}\sin 2(165°) = \frac{1}{2}\sin 330° = \frac{1}{2}(-\sin 30°) = -\frac{1}{4}$$

$$q = \cos^2\frac{\pi}{8} - \frac{1}{2} = \frac{1}{2}\left(2\cos^2\frac{\pi}{8} - 1\right) = \frac{1}{2}\cos 2\left(\frac{\pi}{8}\right) = \frac{1}{2}\cos\frac{\pi}{4} = \frac{1}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}$$

$$p + q = -\frac{1}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{2} - 1}{4}$$

Question 7: (5 points) (Reduction ID Ex 7): Given $f(x) = -\frac{1}{2}\cos 2x + \frac{\sqrt{3}}{2}\sin 2x - 4$. Find the following

(a): the range of f **(b):** the period of f **(c):** the amplitude of f

(d): Write $f(x) = -\frac{1}{2}\cos 2x + \frac{\sqrt{3}}{2}\sin 2x - 4$ in the form $y = k\sin(2x + \alpha) - 4$ where the measure of α is in radian **(e):** Find the phase shift of f.

Solution: $a = \frac{\sqrt{3}}{2}, b = -\frac{1}{2} \Rightarrow \alpha$ is in Quadrant IV because $(a,b) = \left(\sqrt{3}/2, -1/2\right)$ is in Quadrant IV.

$$k = \sqrt{a^2 + b^2} = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$$

$$\left.\begin{array}{l}\sin\alpha=\dfrac{b}{k}=-\dfrac{1}{2}\\[2mm] \cos\alpha=\dfrac{a}{k}=\dfrac{\sqrt{3}}{2}\end{array}\right\}\Rightarrow \quad \begin{array}{l}\alpha\in \text{IV}\\ \alpha=-\dfrac{\pi}{6}\ \text{ OR }\ \alpha=-\dfrac{\pi}{6}+2\pi=\dfrac{11\pi}{6}\end{array}$$

$$f(x)=\frac{\sqrt{3}}{2}\sin 2x-\frac{1}{2}\cos 2x-4=k\sin(2x+\alpha)-4=\sin(2x-\frac{\pi}{6})-4$$

OR $f(x)=\dfrac{\sqrt{3}}{2}\sin 2x-\dfrac{1}{2}\cos 2x-4=k\sin(2x+\alpha)-4=\sin(2x+\dfrac{11\pi}{6})-4$

(a): $R_f=[-5,-3]$ **(b):** $P=\dfrac{2\pi}{2}=\pi$ **(c):** Amp $=1$

(d): $f(x)=\sin(2x-\dfrac{\pi}{6})-4$, $f(x)=\sin(2x+\dfrac{11\pi}{6})-4$

(e): The phase shift of $f(x)=\dfrac{\sqrt{3}}{2}\sin 2x-\dfrac{1}{2}\cos 2x-4=\sin(2x-\dfrac{\pi}{6})-4$ is: $\dfrac{\pi}{12}$, ($\dfrac{\pi}{12}$ to the right)

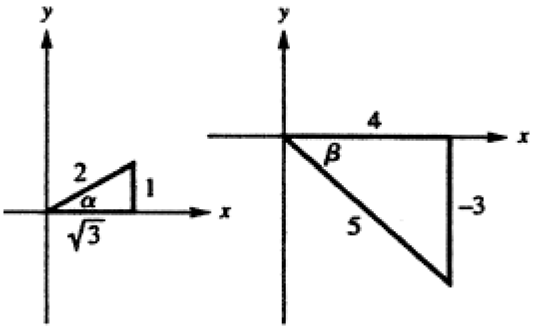
The phase shift of $f(x)=\dfrac{\sqrt{3}}{2}\sin 2x-\dfrac{1}{2}\cos 2x-4=\sin(2x+\dfrac{11\pi}{6})-4$ is: $-\dfrac{11\pi}{12}$, ($\dfrac{11\pi}{12}$ to the left)

Question 8: (5 points) (7.5 Textbook Exercise 96): $\tan\left(\cos^{-1}\dfrac{\sqrt{3}}{2}-\sin^{-1}\left(-\dfrac{3}{5}\right)\right)=?$ (Simplify your answer)

Solution:

94. $\tan\left(\cos^{-1}\dfrac{\sqrt{3}}{2}-\sin^{-1}\left(-\dfrac{3}{5}\right)\right)$

Let $\alpha=\cos^{-1}\dfrac{\sqrt{3}}{2}$, $\beta=\sin^{-1}\left(-\dfrac{3}{5}\right)$. Sketch angle α in quadrant I and angle β in quadrant IV.



We have $\tan\alpha=\dfrac{1}{\sqrt{3}}$ and $\tan\beta=-\dfrac{3}{4}$.

$$\tan\left(\cos^{-1}\frac{\sqrt{3}}{2}-\sin^{-1}\left(-\frac{3}{5}\right)\right)$$
$$=\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$
$$=\frac{\frac{1}{\sqrt{3}}-\left(-\frac{3}{4}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)\left(-\frac{3}{4}\right)}=\frac{\frac{4+3\sqrt{3}}{4\sqrt{3}}}{\frac{4\sqrt{3}-3}{4\sqrt{3}}}$$
$$=\frac{4+3\sqrt{3}}{4\sqrt{3}-3}=\frac{4+3\sqrt{3}}{-3+4\sqrt{3}}\cdot\frac{-3-4\sqrt{3}}{-3-4\sqrt{3}}$$
$$=\frac{-12-25\sqrt{3}-36}{9-48}=\frac{-48-25\sqrt{3}}{-39}$$
$$=\frac{48+25\sqrt{3}}{39}$$

Question 9: (5 points): Whenever possible, find the value of each of the following:

(a): $\cos^{-1}\left(-\dfrac{1}{2}\right)$

(b): $\csc^{-1}\left(-\dfrac{1}{2}\right)$

(c): $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

(d): $\sin^{-1}\left(\sin\frac{5\pi}{9}\right)$

(e): $\cos\left(\sin^{-1}\frac{5}{13}\right)$

Solution:

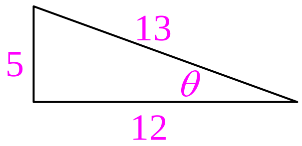
(a): $\cos\left(-\frac{1}{2}\right)=\frac{2\pi}{3}=120°$

(b): $\csc^{-1}\left(-\frac{1}{2}\right)=\text{undefined}$

(c): $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)=\frac{\pi}{6}=30°$

(d): $\sin^{-1}\left(\sin\frac{5\pi}{9}\right)=\sin^{-1}\left(\sin\left(\pi-\frac{5\pi}{9}\right)\right)=\sin^{-1}\left(\sin\frac{4\pi}{9}\right)=\frac{4\pi}{9}=80°$

(e): $\cos\left(\sin^{-1}\frac{5}{13}\right)=\cos(\theta)=\frac{12}{13}$ Because



Let $\theta=\sin^{-1}\frac{5}{13}$ then $\sin\theta=\frac{5}{13}$

Question 10: (5 points):

Find solutions of the equation $-2\cos 2x\sin 3x+2\cos 3x\sin 2x=\sqrt{3}$ over the interval $[-2\pi,2\pi]$ is:

Solution: $-2\cos 2x\sin 3x+2\cos 3x\sin 2x=\sqrt{3}$

$$\cos 2x\,\sin 3x-\cos 3x\,\sin 2x=-\frac{\sqrt{3}}{2}$$

$$\sin 3x\,\cos 2x-\cos 3x\,\sin 2x=-\frac{\sqrt{3}}{2}$$

$$\sin(3x-2x)=-\frac{\sqrt{3}}{2}$$

$\sin x=-\frac{\sqrt{3}}{2}$ x is in quadrant III or in quadrant IV

$x=-\frac{2\pi}{3}$, $\frac{4\pi}{3}$, $x=\frac{5\pi}{3},-\frac{\pi}{3}$

$x=$ are in the interval $[-2\pi,2\pi]$

$$SS=\left\{-\frac{\pi}{3},\ -\frac{2\pi}{3},\ \frac{4\pi}{3},\ \frac{5\pi}{3}\right\}$$