

Show all necessary steps for full marks.

Question 1: (5 points): Given $A = \left\{ 3, -2\frac{3}{4}, \sqrt{3}, \pi, 4\bar{2}, 3.1415, -\frac{2}{3}, 0.1\bar{4}, 1.31331333133331\dots, \sqrt{9}, \frac{3}{\sqrt{2}} \right\}$

(a): List the rational numbers of the set A

(b): List the irrational numbers of the set A

Solution: (a): Rational Numbers: $3, -2\frac{3}{4}, 4\bar{2}, 3.1415, -\frac{2}{3}, 0.1\bar{4}, \sqrt{9}$

(b): Irrational numbers: $\sqrt{3}, \pi, .31331333133331\dots, \frac{3}{\sqrt{2}}$

Question 2: (5 points): List the 4 smallest element of each infinite set

$$A = \{2x \mid x \text{ is a positive integer}\}$$

$$B = \{y \mid y = 2x + 1, x \text{ is a natural number}\}$$

$$C = \{y \mid y = x^2 - 1, x \text{ an integer}\}$$

$$D = \{n^2 \mid n \text{ is a integer}\}$$

Solution: 4 smallest elements of A are : 2, 4, 6, 8

4 smallest elements of B are : 3, 5, 7, 9

4 smallest elements of C are : -1, 0, 3, 8

4 smallest elements of D are : 0, 1, 4, 9

Because:

$$A = \{2x \mid x \text{ is a positive integer}\} = \{2, 4, 6, 8, 10, \dots\}$$

$$B = \{y \mid y = 2x + 1, x \text{ is a natural number}\} = \{3, 5, 7, 9, 11, 13, \dots\}$$

$$C = \{y \mid y = x^2 - 1, x \text{ an integer}\} = \{0^2 - 1, (\pm 1)^2 - 1, (\pm 2)^2 - 1, (\pm 3)^2 - 1, (\pm 4)^2 - 1, (\pm 5)^2 - 1, \dots\}$$

$$= \{0 - 1, 1 - 1, 4 - 1, 9 - 1, 16 - 1, 25 - 1, \dots\} = \{-1, 0, 3, 8, 15, 24, \dots\}$$

$$D = \{n^2 \mid n \text{ is a integer}\} = \{0^2, (\pm 1)^2, (\pm 2)^2, (\pm 3)^2, (\pm 4)^2, \dots\}$$

$$= \{0, 1, 4, 9, 16, 25, \dots\}$$

Question 3: (5 points): Write the following without absolute value symbols:

$$|2x - 12| - |x - 2|, \quad 2 < x < 3$$

Solution: $|2x - 12| - |x - 2| = -(2x - 12) - (x - 2) = -2x + 12 - x + 2 = -3x + 14$

Question 4: (5 points): TRUE / FALSE?

(a): The set of irrational numbers is closed with respect to addition.

(b): The set $\{-1, 0, 1\}$ is closed with respect to multiplication.

(c): If x is any integer and y is any irrational number then x/y is irrational.

(d): The distributive law states that: $(a + b) + c = a + (b + c)$.

(e): Any irrational number has a terminating or repeating decimal expansion.

Solution: (a): **False**, since $-\sqrt{2} + \sqrt{2} = 0 \notin H$

(b): **TRUE:** The set $\{-1, 0, 1\}$ is closed with respect to multiplication.

(c): **False**, since $x = 0 \in I$ and $y = \sqrt{3} \in H$ but $\frac{0}{\sqrt{3}} = 0 \notin H$

(d): **False**, It is NOT distributive law.

(e): **False**, Any irrational number does not have a terminating and repeating decimal expansion.