Serial #: ________ ID ______________ NAME ____
Show all necessary steps for full marks.

Question 1: **(3 points) (7.1 Textbook Exercise 12):** Given $\cos(-\theta) = \frac{\sqrt{3}}{6}$ and $\cot\theta < 0$. Find $\sin\theta = ?$

Solution:

12. $\cos(-\theta) = \frac{\sqrt{3}}{6}$, $\cot\theta < 0$

Since $\cos(-\theta) = \frac{\sqrt{3}}{6}$, we have $\cos\theta = \frac{\sqrt{3}}{6}$ by a negative angle identity. An identity that relates sine and cosine is $\sin^2\theta + \cos^2\theta = 1$.

$$\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta + \left(\frac{\sqrt{3}}{6}\right)^2 = 1 \Rightarrow$$

$$\sin^2\theta + \frac{3}{36} = 1 \Rightarrow \sin^2\theta = 1 - \frac{3}{36} \Rightarrow$$

$$\sin^2\theta = 1 - \frac{1}{12} = \frac{11}{12} \Rightarrow$$

$$\sin\theta = \pm\frac{\sqrt{11}}{\sqrt{12}} = \pm\frac{\sqrt{11}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \pm\frac{\sqrt{33}}{6}$$

Since $\cot\theta < 0$ and $\cos(-\theta) > 0 \Rightarrow \cos\theta > 0$, θ is in quadrant IV, so $\sin\theta < 0$. Thus,

$$\sin\theta = -\frac{\sqrt{33}}{6}.$$

Question 2: **(5 points) (7.2 Textbook Exercise 65):**

Verify $\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta} = 4\cot\theta\csc\theta$

Solution:

65. Verify $\frac{1+\cos x}{1-\cos x} - \frac{1-\cos x}{1+\cos x} = 4\cot x\csc x$

$$\frac{1+\cos x}{1-\cos x} - \frac{1-\cos x}{1+\cos x}$$

$$= \frac{(1+\cos x)^2}{(1+\cos x)(1-\cos x)} - \frac{(1-\cos x)^2}{(1+\cos x)(1-\cos x)}$$

$$= \frac{1+2\cos x+\cos^2 x}{(1+\cos x)(1-\cos x)} - \frac{1-2\cos x+\cos^2 x}{(1+\cos x)(1-\cos x)}$$

$$= \frac{1+2\cos x+\cos^2 x-1+2\cos x-\cos^2 x}{(1+\cos x)(1-\cos x)}$$

$$= \frac{4\cos x}{1-\cos^2 x} = \frac{4\cos x}{\sin^2 x} = 4\cdot\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}$$

$$= 4\cot x\csc x$$

Question 3: (5 points) (7.4 Textbook Similar to Exercises 24 and 25): Find the exact value of the following:

(a): $\dfrac{\tan 15^\circ}{2(1-\tan^2 15^\circ)}$ **(b):** $\dfrac{1}{4}-\dfrac{1}{2}\sin^2\dfrac{\pi}{12}$

Solution

(a): $\dfrac{\tan 15^\circ}{2(1-\tan^2 15^\circ)}=\dfrac{1}{4}\cdot\dfrac{2\tan 15^\circ}{1-\tan^2 15^\circ}=\dfrac{1}{4}\tan 2(15^\circ)=\dfrac{1}{4}\tan(30^\circ)=\dfrac{1}{4}\cdot\dfrac{1}{\sqrt{3}}=\dfrac{\sqrt{3}}{12}$

(b): $\dfrac{1}{4}-\dfrac{1}{2}\sin^2\dfrac{\pi}{12}=\dfrac{1}{4}\left(1-2\sin^2\dfrac{\pi}{12}\right)=\dfrac{1}{4}\cos 2\left(\dfrac{\pi}{12}\right)=\dfrac{1}{4}\cos\left(\dfrac{\pi}{6}\right)=\dfrac{1}{4}\cdot\dfrac{\sqrt{3}}{2}=\dfrac{\sqrt{3}}{8}$

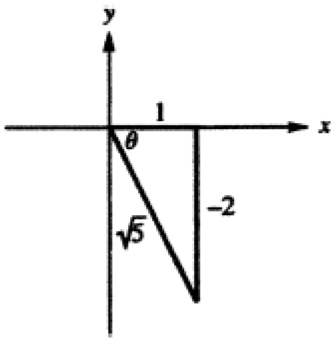
Question 4: (5 points) (7.5 Textbook Similar to Exercise 88): $\cos\left(2\tan^{-1}(-2)\right)=?$

Solution:

88. $\cos\left(2\tan^{-1}(-2)\right)$

Let θ = arctan (–2), so that tan θ = –2. Since arctan is defined only in quadrants I and IV, and –2 is negative, θ is in quadrant IV. Sketch θ and label a triangle with the hypotenuse equal to

$\sqrt{(-2)^2+1^2}=\sqrt{4+1}=\sqrt{5}.$



$\cos\theta=\dfrac{1}{\sqrt{5}}=\dfrac{\sqrt{5}}{5}$

$\cos\left(2\tan^{-1}(-2)\right)=\cos 2\theta=2\cos^2\theta-1$

$=2\left(\dfrac{1}{\sqrt{5}}\right)^2-1=\dfrac{2}{5}-1=-\dfrac{3}{5}$