

Show all necessary steps for full marks.

Question 1: (7 points): (4.3 Textbook Exercise 54): Given $f(x) = \log_{1/3}(3-x)$. Find the following

- (a) Graph $f(x) = \log_{1/3}(3-x)$.
- (b) Find the domain and range of $f(x) = \log_{1/3}(3-x)$.
- (c) Graph the function $g(x) = |\log_{1/3}(3-x)|$.
- (d) Find the interval where the function $g(x) = |\log_{1/3}(3-x)|$ is increasing.
- (e) Find the interval where the function $g(x) = |\log_{1/3}(3-x)|$ is decreasing.
- (f) Find the inverse of $f(x) = \log_{1/3}(3-x)$.

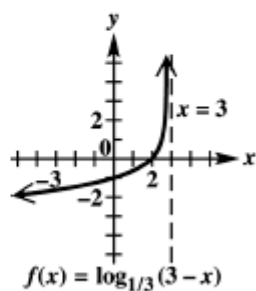
Solution: (a): $f(x) = \log_{1/3}(3-x)$

54. $f(x) = \log_{1/3}(3-x)$

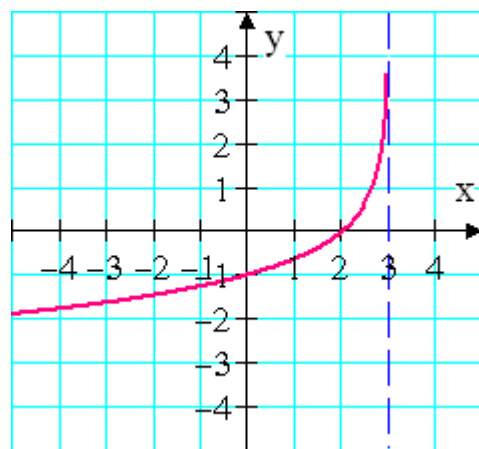
Since $f(x) = y = \log_{1/3}(3-x)$, we can write the exponential form as

$3-x = \left(\frac{1}{3}\right)^y \Rightarrow x = 3 - \left(\frac{1}{3}\right)^y$ to find ordered pairs that satisfy the equation. It is easier to choose values for y and find the corresponding values of x . Make a table of values.

x	$y = \log_{1/3}(3-x)$
-6	-2
0	-1
2	0
$\frac{8}{3} \approx 2.7$	1
$\frac{26}{9} \approx 2.9$	2

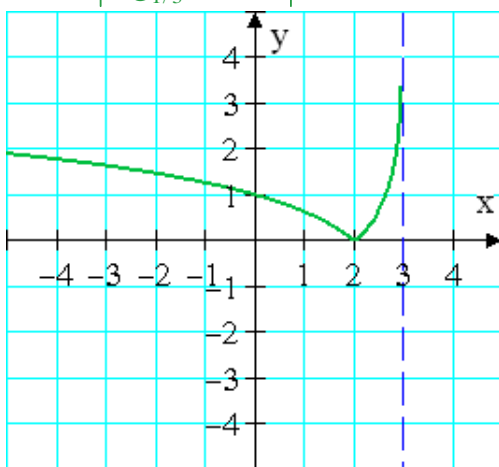


The graph has the line $x = 3$ as a vertical asymptote.



(b): $D_f = (-\infty, 3)$ $R_f = (-\infty, \infty)$

(c): $g(x) = |\log_{1/3}(3-x)|$



(d): The function $g(x) = |\log_{1/3}(3-x)|$ is increasing on $[2, 3)$

(e): The function $g(x) = |\log_{1/3}(3-x)|$ is decreasing on $(-\infty, 2]$.

(f): $y = \log_{1/3}(3-x)$

$$x = \log_{1/3}(3-y) \Rightarrow \left(\frac{1}{3}\right)^x = 3-y \Rightarrow y = f^{-1}(x) = 3 - \left(\frac{1}{3}\right)^x$$

Question 2: (6 points): (4.3 Textbook Summary Exercise 31 page 464):

Find the domain of $f(x) = \log \sqrt{\frac{x^2 - 2x - 63}{x^2 + x - 12}}$.

Solution: $f(x) = \log \sqrt{\frac{x^2 - 2x - 63}{x^2 + x - 12}}$

$$\frac{x^2 - 2x - 63}{x^2 + x - 12} > 0$$

$$\frac{(x - 9)(x + 7)}{(x + 4)(x - 3)} > 0$$

Critical values are: $-7, -4, 3, 9$

$$\begin{array}{cccccccc} & + & & - & & + & & - & & + \\ -\infty & & -7 & & -4 & & 3 & & 9 & & \infty \end{array}$$

Domain: $(-\infty, -7) \cup (-4, 3) \cup (9, \infty)$

Question 3: (7 points): (4.5 Textbook Exercise 32): Find the solution set of $3e^{2x} + 2e^x = 1$

Solution:

$$32. \quad 3e^{2x} + 2e^x = 1 \Rightarrow 3e^{2x} + 2e^x - 1 = 0$$

Let $u = e^x$.

$$3u^2 + 2u - 1 = 0$$

$$(3u - 1)(u + 1) = 0$$

$$3u - 1 = 0$$

$$3u = 1$$

$$u = \frac{1}{3}$$

$$e^x = \frac{1}{3}$$

$$\ln e^x = \ln \frac{1}{3}$$

$$x \ln e = \ln \frac{1}{3}$$

$$x = \ln \frac{1}{3}$$

$$u + 1 = 0$$

$$u = -1$$

$$e^x = -1$$

Disregard this value because e^x is always positive.

Solution set: $\left\{ \ln \frac{1}{3} \right\}$