Serial #: ________ ID ______________ NAME ____

Show all necessary steps for full marks.

Question 1: (5 points) (1.4 Textbook Exercise 48):

(a): After completing the square in the $3x^2 - 9x + 7 = 0$, we get $(x-a)^2 = b$. Find $a+b=?$

(b): Use completing the square to find the solution set of $3x^2 - 9x + 7 = 0$.

Solution: (a):

48. $3x^2 - 9x = -7$

$$x^2 - 3x = -\frac{7}{3}$$

$$x^2 - 3x + \frac{9}{4} = -\frac{7}{3} + \frac{9}{4} = \frac{-28}{12} + \frac{27}{12}$$

$$\text{Note:}\left[\frac{1}{2}\cdot(-3)\right]^2 = \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{-1}{12}$$

$$(x-a)^2 = b$$

$$a = \frac{3}{2} \qquad b = -\frac{1}{12} \qquad \Rightarrow \qquad a+b = \frac{3}{2} - \frac{1}{12} = \frac{17}{12}$$

(b):

$$\left(x - \frac{3}{2}\right)^2 = \frac{-1}{12}$$

$$x - \frac{3}{2} = \pm\sqrt{\frac{-1}{12}} = \pm\frac{i\sqrt{12}}{12} = \pm\frac{2\sqrt{3}}{12}i = \pm\frac{\sqrt{3}}{6}i$$

$$x = \frac{3}{2} \pm \frac{\sqrt{3}}{6}i$$

$$\text{Solution set: } \left\{\frac{3}{2} \pm \frac{\sqrt{3}}{6}i\right\}$$

Question 2: (5 points) (1.4 Textbook Exercise 60): Find the solution set of $\frac{2}{3}x^2 + \frac{1}{4}x = 3$

Solution:

60. $\frac{2}{3}x^2 + \frac{1}{4}x = 3$

$$12\left(\frac{2}{3}x^2 + \frac{1}{4}x\right) = 12\cdot 3$$

$$8x^2 + 3x = 36$$

$$8x^2 + 3x - 36 = 0$$

Let $a = 8, b = 3,$ and $c = -36$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{3^2 - 4(8)(-36)}}{2(8)} = \frac{-3 \pm \sqrt{9 + 1152}}{16}$$

$$= \frac{-3 \pm \sqrt{1161}}{16} = \frac{-3 \pm 3\sqrt{129}}{16}$$

$$\text{Solution set: } \left\{\frac{-3 \pm 3\sqrt{129}}{16}\right\}$$

Another Method by completing the square: Multiply both sides by 12:

$8x^2 + 3x = 36$

$x^2 + \frac{3}{8}x = \frac{9}{2}$

$x^2 + \frac{3}{8}x + \left(\frac{3}{16}\right)^2 = \frac{9}{2} + \frac{9}{16^2}$

$\left(x + \frac{3}{16}\right)^2 = \frac{9(8)(16) + 9}{16^2}$

$\left(x + \frac{3}{16}\right)^2 = \frac{9[(8)16 + 1]}{16^2}$

$\left(x + \frac{3}{16}\right)^2 = \frac{9[128 + 1]}{16^2}$

$\left(x + \frac{3}{16}\right)^2 = \frac{9(129)}{16^2}$

$x + \frac{3}{16} = \pm\sqrt{\frac{9(129)}{16^2}}$

$x = -\frac{3}{16} \pm \frac{3\sqrt{129}}{16} = \frac{-3 \pm 3\sqrt{129}}{16}$

Question 3: **(5 points) (1.6Textbook exercise 22):** Find the solution set of $6 = \frac{7}{2x - 3} + \frac{3}{(2x - 3)^2}$

Solution: Multiply both sides by LCD $(2x - 3)^2$:

$6(2x - 3)^2 = 7(2x - 3) + 3$

$6(2x - 3)^2 - 7(2x - 3) - 3 = 0$

$6u^2 - 7u - 3 = 0$ where $u = 2x - 3$

$2u \quad -3$

$3u \quad +1$

$(2u - 3)(3u + 1) = 0$

$u = \frac{3}{2}$ or $u = -\frac{1}{3}$

$2x - 3 = \frac{3}{2}$ or $2x - 3 = -\frac{1}{3}$

$2x = 3 + \frac{3}{2}$ or $2x = 3 - \frac{1}{3}$

$2x = \frac{9}{2}$ or $2x = \frac{8}{3}$

$x = \dfrac{9}{4}$ or $x = \dfrac{4}{3}$

$SS = \left\{\dfrac{9}{4}, \dfrac{4}{3}\right\}$

Another Method:

22. $6 = \dfrac{7}{2x-3} + \dfrac{3}{(2x-3)^2}$

Multiply each term in the equation by the least common denominator, $(2x-3)^2$, assuming $x \neq \frac{3}{2}$.

$$(2x-3)^2(6) = (2x-3)^2\left[\frac{7}{2x-3} + \frac{3}{(2x-3)^2}\right]$$

$$6\left(4x^2 - 12x + 9\right) = 7(2x-3) + 3$$
$$24x^2 - 72x + 54 = 14x - 21 + 3$$
$$24x^2 - 72x + 54 = 14x - 18$$
$$24x^2 - 86x + 72 = 0$$
$$2\left(12x^2 - 43x + 36\right) = 0 \Rightarrow 2(4x-9)(3x-4) = 0$$

$4x - 9 = 0 \Rightarrow x = \frac{9}{4}$ or $3x - 4 = 0 \Rightarrow x = \frac{4}{3}$

The restriction $x \neq \frac{3}{2}$ does not affect the result.

Therefore, the solution set is $\left\{\frac{9}{4}, \frac{4}{3}\right\}$.

Question 4: **(5 points) (1.6Textbook exercise 84):** Solve $(x+5)^{2/3} + (x+5)^{1/3} - 20 = 0$.

Solution:

84. $(x+5)^{2/3} + (x+5)^{1/3} - 20 = 0$

Let $u = (x+5)^{1/3}$ then,

$u^2 = \left[(x+5)^{1/3}\right]^2 = (x+5)^{2/3}$.

$u^2 + u - 20 = 0 \Rightarrow (u+5)(u-4) = 0 \Rightarrow$

$u = -5$ or $u = 4$

To find x, replace u with $(x+5)^{1/3}$.

$(x+5)^{1/3} = -5 \Rightarrow \left[(x+5)^{1/3}\right]^3 = (-5)^3$

$x + 5 = -125 \Rightarrow x = -130$ or

$(x+5)^{1/3} = 4 \Rightarrow \left[(x+5)^{1/3}\right]^3 = 4^3 \Rightarrow$

$x + 5 = 64 \Rightarrow x = 59$

Solution set: $\{-130, 59\}$