King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 001 Class Test 2A
Textbook Sections: 1.1 to 2.5
Term 152
Time Allowed: 50 Minutes

Student's Name: …………… ………………………………………………
ID #:………………………. **Section:** ……. **Serial Number:** …….

Provide neat and complete solutions.
Show all necessary steps for full credit and write the answer in simplest form.

No Calculators, Cameras, or Mobiles are allowed during this exam.

Question	**Points**	Student's Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
8	5	
9	5	
10	5	
Total	**50**	____ 50
		____ 100

Q1. (5 points): Solve the equation for k.

$$\frac{k+1}{x}=\frac{x+1}{k}+\frac{k-1}{x}$$

Solution: $$\frac{k+1}{x}=\frac{x+1}{k}+\frac{k-1}{x}$$

Multiply both sides by kx :

$$kx\,\frac{k+1}{x}=kx\,\frac{x+1}{k}+kx\,\frac{k-1}{x}$$

$$k^2+k=x^2+x+k^2-k$$

$$2k=x^2+x$$

$$k=\frac{x^2+x}{2}$$

Another Method:

$$\frac{k+1}{x}=\frac{x+1}{k}+\frac{k-1}{x}$$

$$\frac{k+1}{x}-\frac{k-1}{x}=\frac{x+1}{k}$$

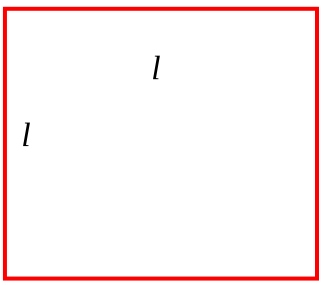
$$\frac{2}{x}=\frac{x+1}{k}$$

$$2k=x^2+x$$

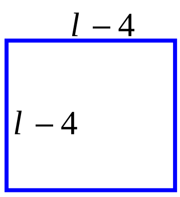
$$k=\frac{x^2+x}{2}$$

Q2. (5 points): If the length of each side of the original square is decreased by 4 inches, the perimeter of the new square is 10 inches more than half the perimeter of the original square. What are the dimensions of the original square?

Solution: $l =$ Length of the original rectangle in inches



Original square



Side is decreased 4

$$P_{new}=10+\frac{1}{2}P_{original}$$

$$4(l-4)=10+\frac{1}{2}(4l)$$

$$4l-16=10+2l$$

$$2l=26$$

$$l=13 \text{ inches}$$

The original square is 13 by 13 inches.

Q3. (5 points) Find the sum of the real part and the imaginary part of the complex number $\dfrac{\sqrt{-4}\left(\sqrt[3]{-27}-\sqrt{-16}\right)}{(1+i)^2}+(1-i)^3$.

Solution:

$$\frac{\sqrt{-4}\left(\sqrt[3]{-27}-\sqrt{-16}\right)}{(1+i)^2}+(1-i)^3=\frac{2i(-3-4i)}{1+2i+i^2}+1-3i+3i^2-i^3$$
$$=\frac{-6i-8i^2}{2i}+1-3i-3+i$$
$$=\frac{8-6i}{2i}-2-2i$$
$$=\frac{4-3i}{i}\cdot\frac{-i}{-i}-2-2i$$
$$=-4i+3i^2-2-2i$$
$$=-5-6i$$

The sum of the real part and the imaginary part $=-5+(-6)=-11$

Answer: -11

Q4. (5 points) (1.4 Textbook Exercise 69): Find the solution set of $x^3+27=0$

Solution:

69.
$$x^3+27=0$$
$$x^3+3^3=0$$
$$(x+3)\left(x^2-3x+9\right)=0$$
$x+3=0\Rightarrow x=-3$ or

$x^2-3x+9=0$

$a=1, b=-3,$ and $c=9$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(9)}}{2(1)}=\frac{3\pm\sqrt{9-36}}{2}$$
$$=\frac{3\pm\sqrt{-27}}{2}=\frac{3\pm3i\sqrt{3}}{2}=\frac{3}{2}\pm\frac{3\sqrt{3}}{2}i$$

Solution set: $\left\{-3,\frac{3}{2}\pm\frac{3\sqrt{3}}{2}i\right\}$

Q5. (5 points) (1.6 Textbook Exercise 21): Find the solution set $2=\dfrac{3}{2x-1}+\dfrac{-1}{(2x-1)^2}$

Solution:

21. $2=\dfrac{3}{2x-1}+\dfrac{-1}{(2x-1)^2}$

Multiply each term in the equation by the least common denominator, $(2x-1)^2$, assuming $x\neq\frac{1}{2}$.

$$(2x-1)^2(2)=(2x-1)^2\left[\frac{3}{2x-1}+\frac{-1}{(2x-1)^2}\right]$$
$$2\left(4x^2-4x+1\right)=3(2x-1)-1$$
$$8x^2-8x+2=6x-3-1$$
$$8x^2-8x+2=6x-4\Rightarrow 8x^2-14x+6=0$$
$$2\left(4x^2-7x+3\right)=0\Rightarrow 2(4x-3)(x-1)=0$$
$$4x-3=0\Rightarrow x=\tfrac{3}{4}\text{ or } x-1=0\Rightarrow x=1$$

The restriction $x\neq\frac{1}{2}$ does not affect the result.

Therefore the solution set is $\left\{\frac{3}{4},1\right\}$.

Q6. (5 points) (1.6 Textbook Exercise 55): Find the solution set of $3-\sqrt{x}=\sqrt{2\sqrt{x}-3}$

Solution:

55.
$$3-\sqrt{x}=\sqrt{2\sqrt{x}-3}$$
$$\left(3-\sqrt{x}\right)^2=\left(\sqrt{2\sqrt{x}-3}\right)^2$$
$$9-6\sqrt{x}+x=2\sqrt{x}-3$$
$$12+x=8\sqrt{x}$$
$$(12+x)^2=\left(8\sqrt{x}\right)^2$$
$$144+24x+x^2=64x$$
$$x^2-40x+144=0$$
$$(x-36)(x-4)=0\Rightarrow x=36\text{ or }x=4$$

Check $x=36$.
$$3-\sqrt{x}=\sqrt{2\sqrt{x}-3}$$
$$3-\sqrt{36}\stackrel{?}{=}\sqrt{2\sqrt{36}-3}$$
$$3-6=\sqrt{2(6)-3}$$
$$-3=\sqrt{12-3}\Rightarrow -3=\sqrt{9}\Rightarrow -3=3$$
This is a false statement. 36 is not a solution.

Check $x=4$.
$$3-\sqrt{x}=\sqrt{2\sqrt{x}-3}$$
$$3-\sqrt{4}\stackrel{?}{=}\sqrt{2\sqrt{4}-3}$$
$$3-2=\sqrt{2(2)-3}$$
$$1=\sqrt{4-3}\Rightarrow 1=\sqrt{1}\Rightarrow 1=1$$
This is a true statement. 4 is a solution.

Solution set: $\{4\}$

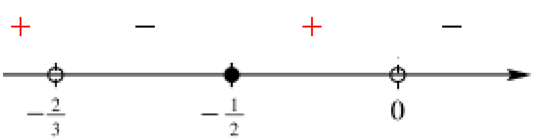
Q7. (5 points) (1.7 Textbook Exercise 82): Find the solution set of $\dfrac{-5}{3x+2}\geq\dfrac{5}{x}$

Solution:

82. $\dfrac{-5}{3x+2}\geq\dfrac{5}{x}$

Step 1: Rewrite the inequality so that 0 is on one side and there is a single fraction on the other side.

$$\frac{-5}{3x+2}-\frac{5}{x}\geq 0$$
$$\frac{-5x}{x(3x+2)}-\frac{5(3x+2)}{x(3x+2)}\geq 0$$
$$\frac{-5x-5(3x+2)}{x(3x+2)}\geq 0$$
$$\frac{-5x-15x-10}{x(3x+2)}\geq 0\Rightarrow\frac{-20x-10}{x(3x+2)}\geq 0$$



Solution set: $\left(-\infty,-\frac{2}{3}\right)\cup\left[-\frac{1}{2},0\right)$

Q8. **(5 points)** If A is the solution set of $\frac{x^2+14x+49}{x^2+x-12}\le 0$ and B is the solution set of $\left|\frac{x-4}{3x+1}\right|\ge 0$, then $A\cap B=?$

Solution:

$$A:\quad \frac{x^2+14x+49}{x^2+x-12}\le 0 \Rightarrow \frac{(x+7)^2}{(x-3)(x+4)}\le 0$$

$+ \quad -4 \quad - \quad 3 \quad +$

$A=\{-7\}\cup(-4,3)$

$$\text{B}:\quad \left|\frac{x-4}{3x+1}\right|\ge 0 \Rightarrow x=4 \text{ and } x\ne -\frac{1}{3} \text{ and } x\in R \Rightarrow B=(-\infty,-\frac{1}{3})\cup(-\frac{1}{3},\infty)$$

$$A\cap B=\{-7\}\cup\left(-4,-\frac{1}{3}\right)\cup\left(-\frac{1}{3},3\right)$$

Q9. **(5 points)** If $M\left(5,\frac{3}{2}\right)$ is the midpoint of the line segment joining the points $P_1(x,8)$ and $P_2(3,y)$, then find the distance between P_1 and P_2 is

Solution:

$$\left(\frac{3+x}{2},\frac{y+8}{2}\right)=\left(5,\frac{3}{2}\right) \Rightarrow \frac{3+x}{2}=5 \quad\text{and}\quad \frac{y+8}{2}=\frac{3}{2}$$

$$\Rightarrow 3+x=10 \quad\text{and}\quad 2y+16=6$$

$$\Rightarrow x=7 \quad\text{and}\quad y=-5$$

$$P_1(x,8)=(7,8)\ ,\ P_2(3,y)=(3,-5)$$

$$d(P_1,P_2)=\sqrt{(7-3)^2+(8+5)^2}=\sqrt{16+169}=\sqrt{185}$$

Q10. **(5 points)** Find the value of k so that the line through the points $(4,-1)$ and $(k,2)$ is perpendicular to the line $2y-5x=1$.

Solution:

$$m_1=\frac{2-(-1)}{k-4}=\frac{3}{k-4}$$

$$2y-5x=1\Rightarrow m_2=\frac{5}{2}$$

$$m_1m_2=-1$$

$$\frac{3}{k-4}\cdot\frac{5}{2}=-1$$

$$15=(-1)2(k-4)$$

$$15=-2k+8$$

$$2k=-7$$

$$\Rightarrow k=-\frac{7}{2}$$