

King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 - Term 151

Recitation (7.4 Reduction Identity)

$$a \sin x + b \cos x = k \sin(x + \alpha)$$

where $k = \sqrt{a^2 + b^2}$ and α is determined by: $\cos \alpha = \frac{a}{\sqrt{a^2 + b^2}}$ and $\sin \alpha = \frac{b}{\sqrt{a^2 + b^2}}$

Or $\tan \alpha = \frac{b}{a}$ where α can be determined from the quadrant that contains the point (a, b)

Question 1:

Given the function $f(x) = 2 \sin \frac{x}{3} - 2\sqrt{3} \cos \frac{x}{3}$

- a) Rewrite $f(x)$ in the form $f(x) = k \sin(bx + \alpha)$
- b) Find the amplitude, the phase shift, the period, and the range for the graph of $f(x)$.

Answer:

(a): $f(x) = 4 \sin\left(\frac{x}{3} - \frac{\pi}{3}\right)$ OR $f(x) = 4 \sin\left(\frac{x}{3} + \frac{5\pi}{3}\right)$

(b):

Amplitude = 4

Phase shift = π units to the right. OR Phase shift = -5π $| -5\pi |$ units to the left.

Period = 6π

Range = $[-4, 4]$

Question 2: If $\sin 20^\circ - \sqrt{3} \cos 20^\circ = k \sin \theta$, $0^\circ < \theta < 90^\circ$. Then k and θ are equal to

- a) $-2, 40^\circ$
- b) $2, 20^\circ$
- c) $1 - \sqrt{3}, 20^\circ$
- d) $-2, 20^\circ$
- e) $-2, 30^\circ$

Answer: $-2, 40^\circ$