

**King Fahd University of Petroleum and Minerals**  
**Prep-Year Math Program**  
**Math 002 - Term 151**  
**Recitation (7.2)**

**Question 1:** Simplify the following expression  $\frac{(\sec \theta - \tan \theta)^2 + 1}{\sec \theta \csc \theta - \tan \theta \csc \theta} = ?$

**Answer:**  $2 \tan \theta$

**Question 2:** Verify the following identities:

a)  $(\sin \theta + \csc \theta)^2 + (\cos \theta + \sec \theta)^2 - \tan^2 \theta - \cot^2 \theta = 7$

b)  $\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \csc x - \cot x, 0 < x < \frac{\pi}{2}$

**Solution:**

(a): 
$$\begin{aligned} & (\sin \theta + \csc \theta)^2 + (\cos \theta + \sec \theta)^2 - \tan^2 \theta - \cot^2 \theta \\ &= \sin^2 \theta + 2 \sin \theta \csc \theta + \csc^2 \theta + \cos^2 \theta + 2 \cos \theta \sec \theta + \sec^2 \theta - \sec^2 \theta + 1 - \csc^2 \theta + 1 \\ &= \sin^2 \theta + \cos^2 \theta + 1 + 1 + 2 \sin \theta \frac{1}{\sin \theta} + 2 \cos \theta \frac{1}{\cos \theta} \\ &= 3 + 2 + 2 \\ &= 7 \end{aligned}$$

(b): 
$$\begin{aligned} \sqrt{\frac{1 - \cos x}{1 + \cos x}} &= \sqrt{\frac{(1 - \cos x)(1 - \cos x)}{(1 + \cos x)(1 - \cos x)}} = \sqrt{\frac{(1 - \cos x)^2}{1 - \cos^2 x}} = \sqrt{\frac{(1 - \cos x)^2}{\sin^2 x}} = \frac{|1 - \cos x|}{|\sin x|} = \\ &= \frac{1 - \cos x}{\sin x} \quad \text{because } 1 - \cos x \geq 0 \text{ and for } 0 < x < \frac{\pi}{2}, \sin x > 0 \\ &= \frac{1}{\sin x} - \frac{\cos x}{\sin x} = \csc x - \cot x \end{aligned}$$

**Question 3:**  $\frac{\sin^2(-x) - \cos^2(-x)}{\sin(-x) - \cos(-x)} =$

- a)  $-\sin x - \cos x$
- b)  $-2 \sin x$
- c)  $\sin x - \cos x$
- d)  $\cos x - \sin x$
- e)  $\sin x + \cos x$

**Answer:** d)  $\cos x - \sin x$

**Question 4:**  $\frac{\sec x - 1}{\sec x + 1} - \frac{\sec x + 1}{\sec x - 1} =$

- a)  $-4 \csc x \cot x$
- b)  $-4 \sec x \tan x$
- c)  $4 \csc x \cot x$
- d)  $8 \sec x \tan x$
- e)  $-2 \sec x \tan x$