Serial #: ________ ID ______________ NAME ____

Show all necessary steps for full marks.

Question1: (4 points) (7.1 Textbook Exercise 85):

Let $\cos x = \frac{1}{5}$. Find all possible values of $\frac{\sec x + \tan x}{\sin x}$

Solution: Since $\cos x = \frac{1}{5}$, which is positive, x is in **Quadrant I** or quadrant IV.

$$\sin x = \pm\sqrt{1-\cos^2 x} = \pm\sqrt{1-\left(\frac{1}{5}\right)^2} = \pm\sqrt{\frac{24}{25}} = \pm\frac{\sqrt{24}}{5} = \pm\frac{2\sqrt{6}}{5}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{\pm\frac{2\sqrt{6}}{5}}{\frac{1}{5}} = \pm 2\sqrt{6} \qquad \sec x = \frac{1}{\cos x} = 5$$

Quadrant I: $$\frac{\sec x + \tan x}{\sin x} = \frac{5+2\sqrt{6}}{\frac{2\sqrt{6}}{5}} = \frac{25+10\sqrt{6}}{2\sqrt{6}} = \frac{25+10\sqrt{6}}{2\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}} = \frac{25\sqrt{6}+60}{12}$$

Quadrant IV: $$\frac{\sec x + \tan x}{\sin x} = \frac{5-2\sqrt{6}}{-\frac{2\sqrt{6}}{5}} = \frac{25-10\sqrt{6}}{-2\sqrt{6}} = \frac{25-10\sqrt{6}}{-2\sqrt{6}}\cdot\frac{-\sqrt{6}}{-\sqrt{6}} = \frac{-25\sqrt{6}+60}{12}$$

Question 2: (4 points) (7.2 Textbook Exercise 16): $(\tan x + \cot x)^2 - (\tan x - \cot x)^2 = ?$

Solution:

$$\begin{aligned}(\tan x + \cot x)^2 - (\tan x - \cot x)^2 &= \tan^2 x + 2\tan x \cot x + \cot^2 x - (\tan^2 x - 2\tan x \cot x + \cot^2 x)\\ &= +2\tan x \cot x + +2\tan x \cot x\\ &= 2+2\\ &= 4\end{aligned}$$

Another Method:

16. $(\tan x + \cot x)^2 - (\tan x - \cot x)^2$

$$\begin{aligned}&= \left[(\tan x + \cot x) + (\tan x - \cot x)\right]\\ &\quad\cdot\left[(\tan x + \cot x) - (\tan x - \cot x)\right]\\ &= (\tan x + \cot x + \tan x - \cot x)\\ &\quad\cdot(\tan x + \cot x - \tan x + \cot x)\\ &= (2\tan x)(2\cot x) = 4\cdot\frac{\sin x}{\cos x}\cdot\frac{\cos x}{\sin x} = 4\end{aligned}$$

Question 3: (4 points) (7.3 Textbook Exercise 51): Find the exact value of $\tan\frac{\pi}{12} = ?$

Solution:

$$\tan\frac{\pi}{12} = \tan\left(\frac{4\pi}{12}-\frac{3\pi}{12}\right) = \tan\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{3}-\tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}} = \frac{\sqrt{3}-1}{1+\sqrt{3}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}\cdot\frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3-2\sqrt{3}+1}{3-1} = \frac{4-2\sqrt{3}}{2}$$

$$= 2-\sqrt{3}$$

Another Method:

51. $\tan\frac{\pi}{12}=\tan\left(\frac{\pi}{4}-\frac{\pi}{6}\right)=\frac{\tan\frac{\pi}{4}-\tan\frac{\pi}{6}}{1+\tan\frac{\pi}{4}\tan\frac{\pi}{6}}$

$$=\frac{1-\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}}=\frac{1-\frac{\sqrt{3}}{3}}{1+\frac{\sqrt{3}}{3}}\cdot\frac{3}{3}=\frac{3-\sqrt{3}}{3+\sqrt{3}}$$

$$=\frac{3-\sqrt{3}}{3+\sqrt{3}}\cdot\frac{3-\sqrt{3}}{3-\sqrt{3}}=\frac{\left(3-\sqrt{3}\right)^2}{3^2-\left(\sqrt{3}\right)^2}$$

$$=\frac{9-6\sqrt{3}+3}{9-3}=\frac{12-6\sqrt{3}}{6}=2-\sqrt{3}$$

Question 4: **(4 points) (7.4 Textbook Exercise 22):** Find the exact value of $\cos^2\frac{\pi}{8}-\frac{1}{2}=?$

Solution:

22. $\cos^2\frac{\pi}{8}-\frac{1}{2}=\frac{1}{2}\left(2\cos^2\frac{\pi}{8}-1\right)$

$$=\frac{1}{2}\left[\cos\left(2\cdot\frac{\pi}{8}\right)\right]=\frac{1}{2}\cos\frac{\pi}{4}$$

$$=\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)=\frac{\sqrt{2}}{4}$$

Question 5: **(4 points) (Reduction ID Ex 3):** Given the function $f(x)=2\sin\frac{x}{3}-2\sqrt{3}\cos\frac{x}{3}$

(a): Rewrite $f(x)$ in the form $f(x)=k\sin(bx+\alpha)$

(b): Find the amplitude, the phase shift, the period, and the range for the graph of $f(x)$.

Solution: (a): $f(x)=a\sin\frac{x}{3}+b\cos\frac{x}{3}=k\sin\left(\frac{x}{3}+\alpha\right)$

$a=2,\ b=-2\sqrt{3}\ \Rightarrow\ (2,-2\sqrt{3})$ is in Quadrant IV.

$k=\sqrt{a^2+b^2}=\sqrt{2^2+(-2\sqrt{3})^2}=\sqrt{4+12}=4$

$\left.\begin{array}{l}\sin\alpha=\frac{b}{k}=\frac{-2\sqrt{3}}{4}=-\frac{\sqrt{3}}{2}\\ \cos\alpha=\frac{a}{k}=\frac{2}{4}=\frac{1}{2}\end{array}\right\}\Rightarrow\ \alpha$ is in Quadrant IV and $\alpha=-\frac{\pi}{3}$ OR $\alpha=\frac{5\pi}{3}$

$f(x)=4\sin\left(\frac{x}{3}-\frac{\pi}{3}\right)$ OR $f(x)=4\sin\left(\frac{x}{3}+\frac{5\pi}{3}\right)$

(b): Amplitude $=4$ Phase shift $=-\frac{-\pi/3}{1/3}=\pi$ units to the right.

OR Phase shift $=-\frac{5\pi/3}{1/3}=-5\pi$ $|-5\pi|$ units to the left.

Period $=\frac{2\pi}{1/3}=6\pi$ Range $=[-4,4]$

Question 6: **(2 points) (7.4 Textbook Exercise 22):** Find the exact value of $2\cos^2 67\frac{1}{2}^\circ - 1 = ?$

Solution: $2\cos^2\theta - 1 = \cos 2\theta$

$$2\cos^2 67\frac{1}{2}^\circ - 1 = 2\cos^2\left(67+\frac{1}{2}\right)^\circ - 1 = 2\cos^2\left(\frac{135}{2}\right)^\circ - 1 = \cos 2\left(\frac{135}{2}\right)^\circ = \cos 135^\circ = -\frac{\sqrt{2}}{2}$$

Another Method:

21. $2\cos^2 67\frac{1}{2}^\circ - 1 = \cos^2 67\frac{1}{2}^\circ - \sin^2 67\frac{1}{2}^\circ$

$$= \cos 2\left(67\frac{1}{2}^\circ\right) = \cos 135^\circ$$

$$= -\frac{\sqrt{2}}{2}$$