King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 Class Test I
Textbook Sections: 4.1 to 5.4
Term 142
Time Allowed: 90 Minutes

Student's Name: …………… ………………………………………………
ID #: ………………………. **Section:** ……. **Serial Number:** …….

Provide neat and complete solutions.
Show all necessary steps for full credit and write the answer in simplest form.

No Calculators, Cameras, or Mobiles are allowed during this exam.

Question	Points	Student's Score
1	7	
2	10	
3	7	
4	8	
5	7	
6	7	
7	6	
8	8	
9	6	
10	6	
11	8	
12	6	
13	7	
14	7	
Total	**100**	$\overline{\mathbf{100}}$

Q1. (7 points): If $f(x)=\dfrac{1-3x}{2+5x}$. If $f^{-1}(x)$ is written in the form $\dfrac{Ax+B}{x+C}$, then $A+B+C=?$

Solution: $y=\dfrac{1-3x}{2+5x}$

$$x=\frac{1-3y}{2+5y}$$

$$1-3y=2x+5xy$$

$$-5xy-3y=-1+2x$$

$$y(-5x-3)=-1+2x$$

$$y=\frac{-1+2x}{-5x-3}$$

$$f^{-1}(x)=\frac{-1+2x}{-5x-3}$$

We have to write the function $f^{-1}(x)=\dfrac{2x-1}{-5x-3}$ in the required form $\dfrac{Ax+B}{x+C}$

Therefore, we have to divide numerator and denominator by -5

$$f^{-1}(x)=\frac{2x-1}{-5x-3}=\frac{\frac{2x-1}{-5}}{\frac{-5x-3}{-5}}=\frac{-\frac{2}{5}x+\frac{1}{5}}{x+\frac{3}{5}}\Rightarrow A=-\frac{2}{5},\ B=\frac{1}{5}\text{ and }C=\frac{3}{5}$$

$$A+B+C=-\frac{2}{5}+\frac{1}{5}+\frac{3}{5}=\frac{-2+1+3}{5}=\frac{2}{5}$$

Q2. (10 points) (4.2 Exercise 52, page 409): Given the function $f(x)=\left(\dfrac{1}{3}\right)^{x+3}-2$

(a): Find the y-intercept
(b): Find the x-intercept
(c): Find the domain
(d): Sketch the graph
(e): Find the range

Solution: **(a):** To find y-intercept, put $x=0$:

$$y=f(0)=\left(\frac{1}{3}\right)^{0+3}-2=\frac{1}{27}-2=\frac{1-54}{27}=-\frac{53}{27}\Rightarrow \boxed{y=-\frac{53}{27}}$$

(b): To find x-intercept, put $y=0$ and solve for x:

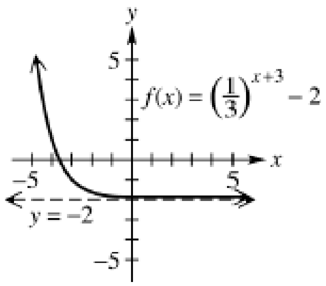
$$0=\left(\frac{1}{3}\right)^{x+3}-2\Rightarrow\left(\frac{1}{3}\right)^{x+3}=2\Rightarrow\ln\left(\frac{1}{3}\right)^{x+3}=\ln 2\Rightarrow(x+3)\ln\left(\frac{1}{3}\right)=\ln 2$$

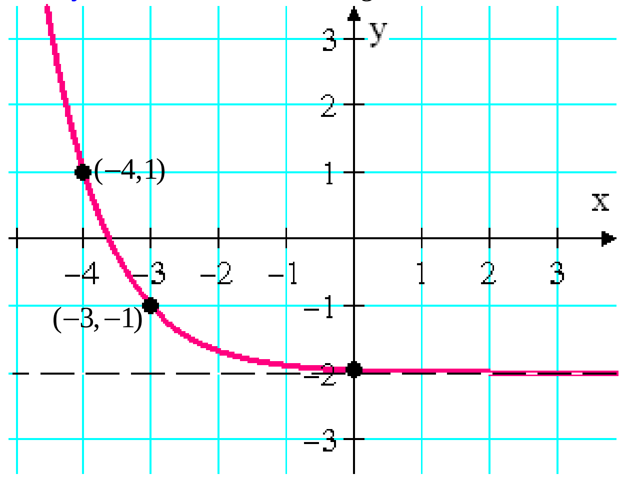
$$\Rightarrow(x+3)\ln\left(\frac{1}{3}\right)=\ln 2\Rightarrow x+3=\frac{\ln 2}{\ln\frac{1}{3}}\Rightarrow x=\frac{\ln 2}{-\ln 3}-3\Rightarrow\boxed{x=-\log_3 2-3}$$

(c): $D_f=(-\infty,\infty)$

(d): $f(x)=\left(\dfrac{1}{3}\right)^{x+3}-2$

52. The graph of $f(x)=\left(\frac{1}{3}\right)^{x+3}-2$ is obtained by translating the graph of $f(x)=\left(\frac{1}{3}\right)^{x}$ three units to the left and two units down.





(e): $R_f=(-2,\infty)$

Q3. (7 points): (4.3 Additional Exercise 14): Given $f(x)=\log_4\left(\dfrac{3-x}{x^2+x-2}\right)$. Find domain of $f(x)$.

Solution:

$$\frac{3-x}{x^2+x-2}>0$$

$$\frac{3-x}{(x+2)(x-1)}>0$$

+	−	+	−
	−2	1	3

$D_f=(-\infty,-2)\cup(1,3)$

Q4. (8 points): (4.3 Textbook Exercises 32 and 35): Solve the following equations

(a): $\log_{1/3}(x+6)=-2$

(b): $3x-15=\log_x 1$

Solution:

32. $\log_{1/3}(x+6)=-2\Rightarrow x+6=\left(\dfrac{1}{3}\right)^{-2}\Rightarrow$

$x+6=3^2\Rightarrow x+6=9\Rightarrow x=3$

Solution set: $\{3\}$

35. $3x-15=\log_x 1 \quad (x>0,\ x\neq 1)$

Note that $\log_x 1=0$ since $x^0=1$ for any number x. Thus,

$3x-15=\log_x 1\Rightarrow 3x-15=0\Rightarrow 3x=15\Rightarrow$

$x=5$

Solution set: $\{5\}$

Q5. (7 points): (4.4 Recitation #2): Write the logarithmic expression:

$2-\log_3 x^2 - 8\log_9 y + \log_{\sqrt{3}} xy$ as a single logarithm with a base of 3

Solution:

$$\begin{aligned}2-\log_3 x^2 - 8\log_9 y + \log_{\sqrt{3}} xy &= \log_3 3^2 - \log_3 x^2 - 8\frac{\log_3 y}{\log_3 9} + \frac{\log_3 xy}{\log_3 \sqrt{3}}\\ &= \log_3 9 - \log_3 x^2 - 8\frac{\log_3 y}{2} + \frac{\log_3 xy}{\frac{1}{2}}\\ &= \log_3 9 - \log_3 x^2 - 4\log_3 y + 2\log_3 xy\\ &= \log_3 9 - \log_3 x^2 - \log_3 y^4 + \log_3 (xy)^2\\ &= \log_3 9 + \log_3 (xy)^2 - \left(\log_3 x^2 + \log_3 y^4\right)\\ &= \log_3 9(xy)^2 - \log_3 x^2 y^4\\ &= \log_3 \frac{9(xy)^2}{x^2 y^4} = \log_3 \frac{9}{y^2}\end{aligned}$$

Q6. (7 points): (4.5 Textbook Exercises 58): Solve $\ln(10-x)+\ln(-6-x)=\ln(-34-15x)$

Solution:

58. $\ln(10-x)+\ln(-6-x)=\ln(-34-15x)$

$$\begin{aligned}\ln\left[(10-x)(-6-x)\right] &= \ln(-34-15x)\\ -60-4x+x^2 &= -34-15x\\ x^2+11x-26 &= 0\\ (x+13)(x-2) = 0 &\Rightarrow x=-13,\ 2\end{aligned}$$

If 2 is substituted for x in $\ln(-6-x)$, the argument becomes -8. Since this is not allowed, we reject this proposed solution.

Solution set: $\{-13\}$

Q7. (6 points): (5.1 Textbook Example 5): Find the angles of least positive measure that are coterminal with each angle.

(a): $908°$ **(b):** $-75°$ **(c):** $-800°$

Solution: (a):

▶ EXAMPLE 5 FINDING MEASURES OF COTERMINAL ANGLES

Find the angles of least possible positive measure coterminal with each angle.

(a) $908°$ (b) $-75°$ (c) $-800°$

Solution

(a) Add or subtract $360°$ as many times as needed to obtain an angle with measure greater than $0°$ but less than $360°$. Since

$$908° - 2 \cdot 360° = 188°,$$

an angle of $188°$ is coterminal with an angle of $908°$. See Figure 11.

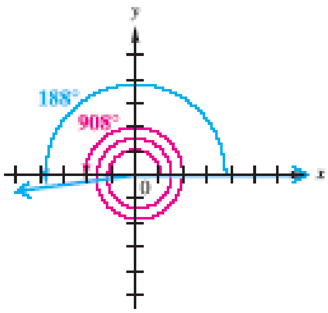


Figure 11

(b) Use a rotation of $360° + (-75°) = 285°$. See Figure 12.

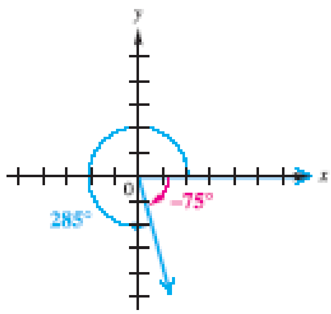


Figure 12

(c) The least integer multiple of $360°$ greater than $800°$ is

$$360° \cdot 3 = 1080°.$$

Add $1080°$ to $-800°$ to obtain

$$1080° + (-800°) = 280°.$$

NOW TRY EXERCISES 73, 83, AND 87. ◀

Q8. (8 points): (5.1 Textbook Exercise 12): Given $50°\ 40'\ 50''$.

(a): Find the complement of the angle.

(b): Find the supplement of the angle.

Solution:

12. $50°40'50''$

(a) $90° - 50°40'50'' = 89°59'60'' - 50°40'50''$
$= 39°19'10''$

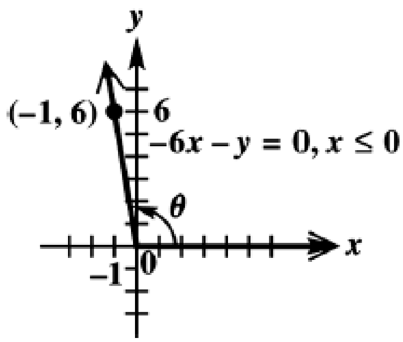
(b) $180° - 50°40'50''$
$= 179°59'60'' - 50°40'50''$
$= 129°19'10''$

Q9. (6 points): (5.2 Textbook Exercise 37): Find the six trigonometric function valued of the angle θ in standard position, if the terminal side of θ is defined by $-6x - y = 0,\ x \le 0$.

Solution:

37. Since $x \le 0$, the graph of the line $-6x - y = 0$ is shown to the left of the y-axis. A point on this graph is $(-1, 6)$ since $-6(-1) - 6 = 0$. The corresponding value of r is

$$r = \sqrt{(-1)^2 + 6^2} = \sqrt{1+36} = \sqrt{37}.$$



$$\sin\theta = \frac{y}{r} = \frac{6}{\sqrt{37}} = \frac{6}{\sqrt{37}} \cdot \frac{\sqrt{37}}{\sqrt{37}} = \frac{6\sqrt{37}}{37}$$

$$\cos\theta = \frac{x}{r} = \frac{-1}{\sqrt{37}} = -\frac{1}{\sqrt{37}} \cdot \frac{\sqrt{37}}{\sqrt{37}} = -\frac{\sqrt{37}}{37}$$

$$\tan\theta = \frac{y}{x} = \frac{6}{-1} = -6$$

$$\cot\theta = \frac{x}{y} = \frac{-1}{6} = -\frac{1}{6}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{37}}{-1} = -\sqrt{37}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{37}}{6}$$

Q10. (6 points): (5.2 Textbook Exercise 122): Find $\sec\theta$, given that $\tan\theta = \frac{\sqrt{7}}{3}$ and θ is in quadrant III.

Solution:

122. If $\tan\theta = \frac{\sqrt{7}}{3}$ and θ is in quadrant III, then

$x = -3$ and $y = -\sqrt{7}$. So $r^2 = x^2 + y^2 \Rightarrow$

$r^2 = (-3)^2 + \left(-\sqrt{7}\right)^2 \Rightarrow r^2 = 9 + 7 \Rightarrow$

$r^2 = 16 \Rightarrow r = 4$. Therefore, $\sec\theta = \frac{r}{x} = -\frac{4}{3}$.

Another Method: Using the Identity:

$\tan^2\theta + 1 = \sec^2\theta$:

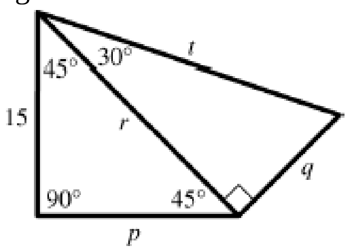
$$\left(\frac{\sqrt{7}}{3}\right)^2 + 1 = \sec^2\theta \Rightarrow \frac{7}{9} + 1 = \sec^2\theta \Rightarrow$$

$$\frac{16}{9} = \sec^2\theta \Rightarrow \pm\frac{4}{3} = \sec\theta$$

θ is in quadrant III, so $\sec\theta$ is negative.

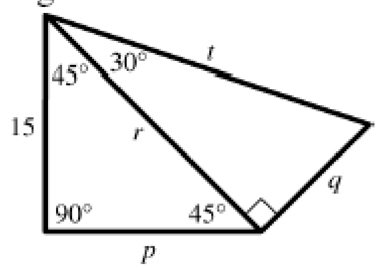
Thus $\sec\theta = -\dfrac{4}{3}$

Q11. (8 points): (5.3 Textbook Exercise 55): Find the exact value of each part labeled with a variable in the figure.



Solution:

55. Apply the relationships between the lengths of the sides of a $45° - 45°$ right triangle to the triangle on the left to find the values of *p* and *r*. In the $45° - 45°$ right triangle, the sides opposite the $45°$ angles measure the same. The hypotenuse is $\sqrt{2}$ times the measure of a leg.



Thus, we have $p = 15$ and $r = p\sqrt{2} = 15\sqrt{2}$

Apply the relationships between the lengths of the sides of a $30° - 60°$ right triangle next to the triangle on the right to find the values of *q* and *t*. In the $30° - 60°$ right triangle, the side opposite the $60°$ angle is $\sqrt{3}$ times as long as the side opposite to the $30°$ angle. The length of the hypotenuse is 2 times as long as the shorter leg (opposite the $30°$ angle). Thus, we have $r = q\sqrt{3} \Rightarrow$

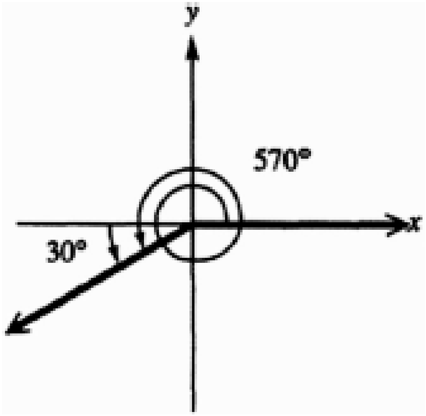
$$q = \frac{r}{\sqrt{3}} = \frac{15\sqrt{2}}{\sqrt{3}} = \frac{15\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = 5\sqrt{6} \text{ and}$$

$$t = 2q = 2\left(5\sqrt{6}\right) = 10\sqrt{6}$$

Q12. (6 points): (5.3 Textbook Exercise 79): Find exact values of the six trigonometric functions of $570°$.

Solution:

79. To find the reference angle for 570° sketch this angle in standard position.



570° is coterminal with 570° − 360° = 210°. The reference angle is 210° − 180° = 30°. Since 570° lies in quadrant III, the sine, cosine, secant, and cosecant are negative.

$$\sin 570° = -\sin 30° = -\frac{1}{2}$$

$$\cos 570° = -\cos 30° = -\frac{\sqrt{3}}{2}$$

$$\tan 570° = \tan 30° = \frac{\sqrt{3}}{3}$$

$$\cot 570° = \cot 30° = \sqrt{3}$$

$$\sec 570° = -\sec 30° = -\frac{2\sqrt{3}}{3}$$

$$\csc 570° = -\csc 30° = -2$$

Q13. (7 points): (5.3 Recitation Q#2): If $\tan 40° = 0.84$, then $3\tan 140° + 5\cot 410° = ?$

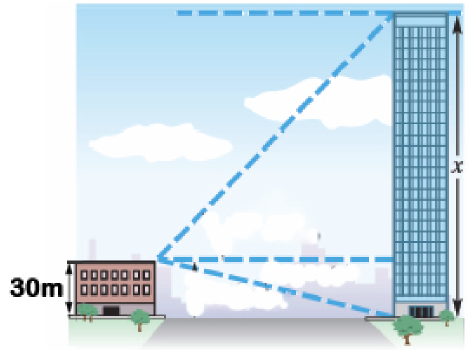
Solution:

$$\begin{aligned} 3\tan 140° + 5\cot 410° &= 3[-\tan(180° - 140°)] + 5[\cot(410° - 360°] \\ &= -3\tan 40° + 5\cot 50° \\ &= -3\tan 40° + 5[\tan(90° - 50°] \\ &= -3\tan 40° + 5\tan 40° \\ &= 2\tan 40° \\ &= 2(0.84) \\ &= 1.68 \end{aligned}$$

Answer: (A): 1.68

Q14. (7 points): (5.4 Recitation Q#4):

The angle of elevation from the top of a small building to the top of a taller building is 60°, while the angle of depression to the bottom is 30°. If the shorter building is 30 m high, then the height of the taller building is



A) $\left(30+60\sqrt{3}\right)\text{m}$

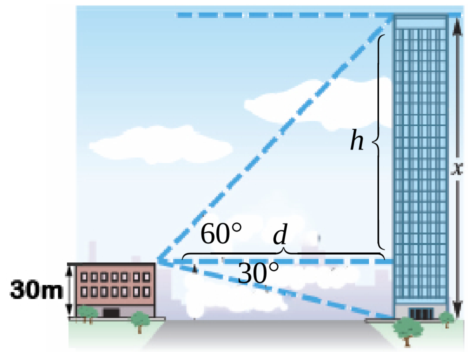
B) $150\,\text{m}$

C) $100\sqrt{3}\,\text{m}$

D) $120\,\text{m}$

E) $90\sqrt{3}\,\text{m}$

Solution:



$$\tan 30^\circ = \frac{30}{d} \Rightarrow \frac{1}{\sqrt{3}} = \frac{30}{d} \Rightarrow d = 30\sqrt{3}\ m$$

$$\tan 60^\circ = \frac{h}{d} \Rightarrow \sqrt{3} = \frac{h}{30\sqrt{3}} \Rightarrow h = 90\ m$$

$x = h + 30 = 90 + 30 = 120\ m$

The height of the taller building is $120\ m$.