SN _________ ID ______________ NAME ____

Show all necessary steps for full marks.

Q1. (7 points): Given the line L having the equation $5x+4y-20=0$, find the equation of the line perpendicular to the line L and passing through $(2,-3)$.

Solution: $5x+4y-20=0$

$$4y=-5x+20$$

$$y=-\frac{5}{4}x+5$$

The slope of L is $-\frac{5}{4}$.

The slope of the line perpendicular to L is $\frac{4}{5}$.

The equation of the line required is

$$y-(-3)=\frac{4}{5}(x-2) \Rightarrow y=\frac{4}{5}(x-2)-3=\frac{4}{5}x-\frac{8}{5}-3 \Rightarrow \boxed{y=\frac{4}{5}x-\frac{23}{5}}$$

Q2. (8 points) (Textbook 2.6 Exercise 46): Given $g(x)=[\![2x-1]\!]$

(a): Graph $g(x)=[\![2x-1]\!]$.

(b): Find the set of all x-intercepts.

(c): Find domain,

(d): Find range

(e): Find y-intercept

Solution:

$g(x)=[\![2x-1]\!]$

To get $y=0$, we need

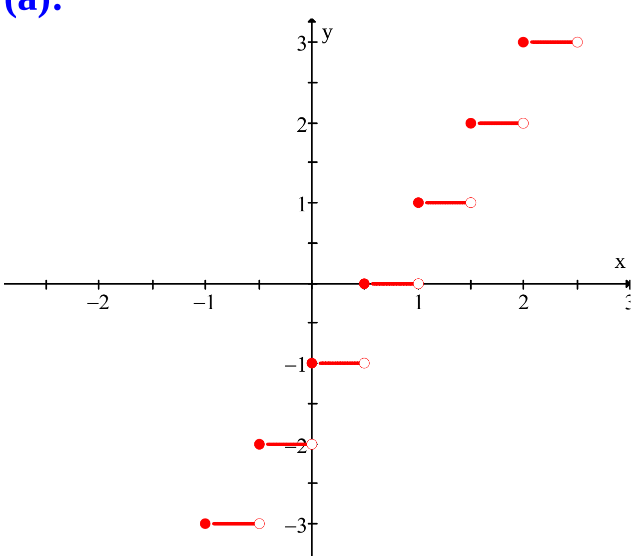
$0\le 2x-1<1 \Rightarrow 1\le 2x<2 \Rightarrow \frac{1}{2}\le x<1.$

To get $y=1$, we need

$1\le 2x-1<2 \Rightarrow 2\le 2x<3 \Rightarrow 1\le x<\frac{3}{2}.$

Follow this pattern to graph the step function.

(a):



(b): $domain = (-\infty, \infty)$

(c): $\{0, \pm 1, \pm 2, \pm 3, \cdots\}$

(d): x-intercepts: $\frac{1}{2} \le x < 1$

(e): y-intercepts: $y = -1$

Q3. (4 points) (Recitation 2.7)

If the graph of $y = \frac{2x}{x+1}$ is translated one unit to the right and three units downward, what is the new equation of the new graph?

Solution:

Shift the graph of $y = \frac{2x}{x+1}$ 1 unit to the right $=$Replace x with $x-1$:

$$y = \frac{2(x-1)}{(x-1)+1} = \frac{2x-2}{x}$$

Now, shift 3 units downward $=$Replace y with $y+3$:

$$y + 3 = \frac{2x-2}{x}$$

$$y = \frac{2x-2}{x} - 3 = \frac{2x-2-3x}{x} = \frac{-x-2}{x}$$

Answer: $y = \frac{-x-2}{x}$

Q4. (6 points) (2.7 Exercise 25 - 27, page 255): Suppose the point $(8,12)$ is on the graph of $y = f(x)$. Find a point on the graph of each function.

25. (a): $y = f(x+4)$ $\boxed{}$

25 (b): $y = f(x)+4$ $\boxed{}$

26. (a): $y = \frac{1}{4}f(x)$ $\boxed{}$

26 (b): $y = 4f(x)$ $\boxed{}$

27. (a): $y = f(4x)$ $\boxed{}$

27 (b): $y = f\left(\frac{1}{4}x\right)$ $\boxed{}$

Solution:

25. **(a)** $y = f(x+4)$ is a horizontal translation of f, 4 units to the left. The point that corresponds to (8, 12) on this translated function would be $(8-4, 12) = (4, 12)$.

(b) $y = f(x)+4$ is a vertical translation of f, 4 units up. The point that corresponds to (8, 12) on this translated function would be $(8, 12+4) = (8, 16)$.

$\boxed{(-4,12) = (4,12)}$ $\boxed{(8,12+4) = (8,16)}$

26. **(a)** $y = \frac{1}{4} f(x)$ is a vertical shrinking of f, by a factor of $\frac{1}{4}$. The point that corresponds to (8, 12) on this translated function would be $\left(8, \frac{1}{4} \cdot 12\right) = (8, 3)$.

(b) $y = 4f(x)$ is a vertical stretching of f, by a factor of 4. The point that corresponds to (8, 12) on this translated function would be $(8, 4 \cdot 12) = (8, 48)$.

$\boxed{\left(8, \frac{12}{4}\right) = (8,3)}$ $\boxed{(8, 4 \cdot 12) = (8, 48)}$

27. **(a)** $y = f(4x)$ is a horizontal shrinking of f, by a factor of 4. The point that corresponds to (8, 12) on this translated function is $\left(8 \cdot \frac{1}{4}, 12\right) = (2, 12)$.

(b) $y = f\left(\frac{1}{4}x\right)$ is a horizontal stretching of f, by a factor of 4. The point that corresponds to (8, 12) on this translated function is $(8 \cdot 4, 12) = (32, 12)$.

$\boxed{\left(\frac{8}{4}, 12\right) = (2,12)}$ $\boxed{(8 \cdot 4, 12) = (32, 12)}$