

Show all necessary steps for full marks.

Question 1: (4 points): (R.3 Exercise 72): $[(2m + 7) - n]^2$

Solution:

$$\begin{aligned} 72. \quad & [(2m + 7) - n]^2 \\ & = (2m + 7)^2 - 2(2m + 7)(n) + n^2 \\ & = (4m^2 + 28m + 49) - 2(2m + 7)(n) + n^2 \\ & = 4m^2 + 28m + 49 - 4mn - 14n + n^2 \end{aligned}$$

Question 2: (4 points): (R.3 Example 12, page 28): Divide $3x^3 - 2x^2 - 150$ by $x^2 - 4$.

Write your answer as $\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$

Solution:

$$\begin{array}{r} \overline{3x - 2} \\ 3x^3 - 2x^2 - 150 \\ \underline{3x^3 - 12x} \\ -2x^2 + 12x - 150 \\ \underline{-2x^2 + 8} \\ + - 158 \\ \underline{ - 158} \end{array}$$

$$\frac{3x^3 - 2x^2 - 150}{x^2 - 4} = 3x - 2 + \frac{12x - 158}{x^2 - 4}$$

Another Method:

$$\begin{array}{r} \overline{3x - 2} \\ x^2 + 0x - 4 \overline{) 3x^3 - 2x^2 + 0x - 150} \\ \underline{3x^3 + 0x^2 - 12x} \\ -2x^2 + 12x - 150 \\ \underline{-2x^2 + 0x + 8} \\ 12x - 158 \end{array}$$

Missing term \rightarrow

Missing term \rightarrow

Insert placeholders for missing terms.

$12x - 158 \leftarrow$ Remainder

Question 3: (4 points): (R.4 Exercise 32 and 40): Factor

(a): $36x^3 + 18x^2 - 4x$

(b): $18x^5 + 15x^4z - 75x^3z^2$

Solution:

(a):

32. Factor out the greatest common factor, $2x$:

$$36x^3 + 18x^2 - 4x = 2x(18x^2 + 9x - 2).$$

Now factor the trinomial by trial and error:

$$18x^2 + 9x - 2 = (6x - 1)(3x + 2).$$

Thus,

$$36x^3 + 18x^2 - 4x = 2x(6x - 1)(3x + 2).$$

(b):

40. First, factor out the greatest common factor,

$$3x^3 :$$

$$18x^5 + 15x^4z - 75x^3z^2 = 3x^3(6x^2 + 5xz - 25z^2)$$

Now factor the trinomial by trial and

$$\text{error: } 6x^2 + 5xz - 25z^2 = (3x - 5z)(2x + 5z)$$

Thus,

$$\begin{aligned} 18x^5 + 15x^4z - 75x^3z^2 &= 3x^3(6x^2 + 5xz - 25z^2) \\ &= 3x^3(3x - 5z)(2x + 5z) \end{aligned}$$

Question 4: (4 points): (R.4 Exercise 96 and 97): Factor

(a): $q^2 + 6q + 9 - p^2$

(b): $64 + (3x + 2)^3$

Solution: (a):

$$\begin{aligned} 96. \quad q^2 + 6q + 9 - p^2 &= (q^2 + 6q + 9) - p^2 \\ &= [q^2 + 2(q)(3) + 3^2] - p^2 \\ &= (q + 3)^2 - p^2 \\ &= [(q + 3) + p][(q + 3) - p] \\ &= (q + 3 + p)(q + 3 - p) \end{aligned}$$

(b):

$$\begin{aligned} 97. \quad 64 + (3x + 2)^3 &= 4^3 + (3x + 2)^3 \\ &= [4 + (3x + 2)][4^2 - (4)(3x + 2) + (3x + 2)^2] \\ &= [4 + (3x + 2)] \left[\begin{array}{l} 4^2 - (4)(3x + 2) \\ + 9x^2 + 12x + 4 \end{array} \right] \\ &= (4 + 3x + 2)(16 - 12x - 8 + 9x^2 + 12x + 4) \\ &= (3x + 6)(9x^2 + 12) \\ &= 3(x + 2)(3)(3x^2 + 4) = 9(x + 2)(3x^2 + 4) \end{aligned}$$