King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 001 Class Test II
Textbook Sections: 1.1 to 2.3
Term 141
Time Allowed: 80 Minutes

Student's Name: …………… ………………………………………………
ID #: ………………………. **Section:** ……. **Serial Number:** …….

Provide neat and complete solutions.
Show all necessary steps for full credit and write the answer in simplest form.

No Calculators, Cameras, or Mobiles are allowed during this exam.

Question	Points	Student's Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
8	5	
9	5	
10	5	
Total	**50**	**/50**

/100

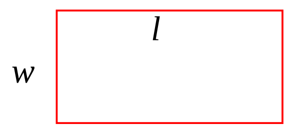
Q1. (5 points): (1.1 Exercise 58): Solve $-x = (5x+3)(3k+1)$ for $x = ?$

Solution:

58.
$$-x = (5x+3)(3k+1)$$
$$-x = 15xk + 5x + 9k + 3$$
$$-6x - 15xk = 9k + 3$$
$$(-6-15k)x = 9k + 3$$
$$x = \frac{9k+3}{-6-15k} = \frac{3(3k+1)}{-3(2+5k)}$$
$$= -\frac{3k+1}{5k+2}$$

Q2. (5 points): The length of a room is 9 feet less than twice the width of the room. The perimeter of the room is 48 feet. Find the width and the length of the room.

Solution:



Label the rectangle. We have used w for its width and l for its length. The problem states that the length is 9 feet less than twice the width. Thus w and l are related by the equation
$$l = 2w - 9$$
The perimeter of a rectangle is given by the formula $P = 2l + 2w$. To produce an equation that involves only constants and a single variable (say w), substitute 48 for P and $2w - 9$ for l .
$$P = 2l + 2w$$
$$48 = 2(2w - 9) + 2w$$
Solve for w .
$$48 = 4w - 18 + 2w$$
$$48 = 6w - 18$$
$$8 = w - 3$$
$$w = 11 \text{ feet}$$
The length is 9 feet less than the twice of the width. Thus $l = 2(11) - 9 = 22 - 9 = 13$ feet.
A check verifies that 13 is 9 less than the twice of width (22).

Q3. (5 points): (1.4 Exercise 77): Solve $h = -16t^2 + v_0 t + s_0$ for $t = ?$

Solution: $h = -16t^2 + v_0 t + s_0$

$16t^2 - v_0 t + h - s_0 = 0 \Rightarrow a = 16$, $b = -v_0$, $c = h - s_0$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-v_0) \pm \sqrt{(-v_0)^2 - 4(16)(h - s_0)}}{2(16)}$$
$$= \frac{v_0 \pm \sqrt{v_0^2 - 64(h - s_0)}}{32}$$
$$= \frac{v_0 \pm \sqrt{v_0^2 - 64h + 64s_0}}{32}$$

Q4. (5 points): (1.6 Exercise 78): Find the solution set of $\sqrt{5x-15}-\sqrt{x+1}=2$.

Solution:

78. $\sqrt{5x-15}-\sqrt{x+1}=2$

$$\sqrt{5x-15}=\sqrt{x+1}+2$$
$$\left(\sqrt{5x-15}\right)^2=\left(\sqrt{x+1}+2\right)^2$$
$$5x-15=(x+1)+4\sqrt{x+1}+4$$
$$5x-15=x+5+4\sqrt{x+1}$$
$$4x-20=4\sqrt{x+1}$$
$$x-5=\sqrt{x+1}$$
$$(x-5)^2=\left(\sqrt{x+1}\right)^2$$
$$x^2-10x+25=x+1$$
$$x^2-11x+24=0\Rightarrow(x-3)(x-8)=0$$
$$x-3=0\Rightarrow x=3 \quad\text{or}\quad x-8=0\Rightarrow x=8$$

Check $x=3$.

$$\sqrt{5x-15}-\sqrt{x+1}=2$$
$$\sqrt{5(3)-15}-\sqrt{3+1}=2$$
$$\sqrt{15-15}-\sqrt{4}=2$$
$$\sqrt{0}-2=2$$
$$0-2=2$$
$$-2=2$$

$x=3$ is not a solution

Check $x=8$.

$$\sqrt{5x-15}-\sqrt{x+1}=2$$
$$\sqrt{5(8)-15}-\sqrt{8+1}=2$$
$$\sqrt{40-15}-\sqrt{9}=2$$
$$\sqrt{25}-3=2$$
$$5-3=2$$
$$2=2$$

$x=8$ is a solution

$\boxed{\text{Solution set}=\{8\}}$

Q5. (5 points): (1.6 Review Exercise 71, page 164): Find the solution set of $(2x+3)^{2/3}+(2x+3)^{1/3}-6=0$

Solution:

71. $(2x+3)^{2/3}+(2x+3)^{1/3}-6=0$

Let $u=(2x+3)^{1/3}$. Then

$u^2=[(2x+3)^{1/3}]^2=(2x+3)^{2/3}$.

With this substitution, the equation becomes

$u^2+u-6=0$. Solve by factoring.

$(u+3)(u-2)=0$

$u+3=0\Rightarrow u=-3 \quad\text{or}\quad u-2=0\Rightarrow u=2$

To find x, replace u with $(2x+3)^{1/3}$.

$(2x+3)^{1/3}=-3\Rightarrow\left[(2x+3)^{1/3}\right]^3=(-3)^3\Rightarrow$

$2x+3=-27\Rightarrow 2x=-30\Rightarrow x=-15$

or

$(2x+3)^{1/3}=2\Rightarrow\left[(2x+3)^{1/3}\right]^3=2^3\Rightarrow$

$2x+3=8\Rightarrow 2x=5\Rightarrow x=\frac{5}{2}$

$\boxed{\text{Solution set}=\left\{-15,\frac{5}{2}\right\}}$

Q6. (5 points): (1.7 Exercise 92): Find the solution set of $\dfrac{(9x-11)(2x+7)}{(3x-8)^3} > 0$.

Solution:

92. $\dfrac{(9x-11)(2x+7)}{(3x-8)^3} > 0$

The values $-\frac{7}{2}$, $\frac{11}{9}$, and $\frac{8}{3}$ divide the number line into four intervals.

$9x-11$	- - -	- - -	+ + +	+ +
$2x+7$	- - -	+ + +	- - -	+ +
$(3x-8)^3$	- - -	- - -	- - -	+ +
	- - -	+ + +	- - -	+ +

$-\infty \quad -\frac{7}{2} \quad \frac{11}{9} \quad \frac{8}{3} \quad \infty$

Solution set $=\left(-\frac{7}{2},\frac{11}{9}\right)\cup\left(\frac{8}{3},\infty\right)$

Q7. (5 points) (1.8 Recitation Question3): Find the solution set, in interval notation, of

A) $\left|\dfrac{2x+5}{3}\right|-\dfrac{3}{4}<\dfrac{1}{2}$ B) $-3\left|2x-\dfrac{1}{3}\right|>\dfrac{3}{2}$ C) $|3x+1|>0$

Solution:

(A): $\left|\dfrac{2x+5}{3}\right|<\dfrac{5}{4}$

$-\dfrac{5}{4}<\dfrac{2x+5}{3}<\dfrac{5}{4}$ Multiply by LCD $=(3)(4)$:

$-15<8x+20<15$

$-35<8x<-5$

$-\dfrac{35}{8}<x<-\dfrac{5}{8}$ $\quad SS=\left(-\dfrac{35}{8},-\dfrac{5}{8}\right)$

(B): $-3\left|2x-\dfrac{1}{3}\right|>\dfrac{3}{2}$

Multiplying both sides of the inequality by $-\dfrac{1}{3}$:

$\Rightarrow \left|2x-\dfrac{1}{3}\right|<-\dfrac{1}{2}$ which is impossible. $\Rightarrow$ $SS=\varnothing$

(C): $|3x+1|>0$ is always when $x\neq-\dfrac{1}{3}$

$\Rightarrow SS=\left\{x\in R \,\middle|\, x\neq-\dfrac{1}{3}\right\}$ $\Rightarrow$ $SS=\left(-\infty,-\dfrac{1}{3}\right)\cup\left(-\dfrac{1}{3},\infty\right)$

Q8. (5 points) (2.1 Recitation Question4): Find the distance between the point $(-1,3)$ and the midpoint of the line segment with endpoint $\left(\frac{7}{2},-\frac{16}{3}\right)$ and $\left(\frac{5}{2},-\frac{14}{3}\right)$.

Solution: The midpoint is

$$M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)=\left(\frac{1}{2}\left(\frac{7}{2}+\frac{5}{2}\right),\frac{1}{2}\left(-\frac{16}{3}-\frac{14}{3}\right)\right)=\left(\frac{6}{2},\frac{-10}{2}\right)=(3,-5)$$

The distance between the pints $(-1,3)$ and $(3,-5)$ is

$$d=\sqrt{(3+1)^2+(-5-3)^2}=\sqrt{16+64}=\sqrt{80}=\sqrt{4(4)(5)}=4\sqrt{5}$$

Q9. (5 points): (2.2 Recitation Question4): If the point $(0,-5)$ and (a,b) are the endpoints of a diameter of the circle $x^2+y^2-2x+4y-5=0$. Then find a and b.

Solution:

$x^2-2x+y^2+4y=5$

$x^2-2x+1+y^2+4y+4=5+1+4$

$(x-1)^2+(y+2)^2=10 \Rightarrow$ center $=(1,-2)$

The point $(1,-2)$ is the midpoint of the diameter from $(0,-5)$ to (a,b).

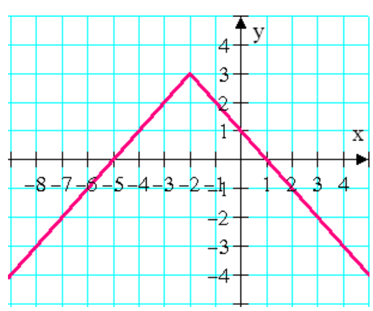
$1=\frac{0+a}{2}$ and $-2=\frac{-5+b}{2}$

$\boxed{a=2}$ and $\boxed{b=1}$

Q10. (5 points): (2.3 Similar to Recitation Question 2): If $f(x)=-|x+2|+3$

a) Find $f(3)$.
b) Give the domain and the range of *f*.
c) Give the largest interval for which *f* is (a) increasing. (b) decreasing.
d) Find the x-intercepts.

Solution:



a) $f(3)=-|3+2|+3=-5+3=-2$

b) $Domain=(-\infty,\infty)$, $Range=(-\infty,3]$

c) f is increasing on $(-\infty,-2]$

f is decreasing on $[-2,\infty)$

d) To find x-intercept, let $y=0$:

$0=-|x+2|+3$

$|x+2|=3$

$x+2=\pm 3 \Rightarrow \boxed{x=-5}, \boxed{x=1}$