King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 001 Class Test I
Textbook Sections: R.1 to R.7
Term 141
Time Allowed: 90 Minutes

Student's Name: …………… ………………………………………………
ID #: ………………………. **Section:** ……. **Serial Number:** …….

Provide neat and complete solutions.
Show all necessary steps for full credit and write the answer in simplest form.

No Calculators, Cameras, or Mobiles are allowed during this exam.

Question	Points	Student's Score
1	8	
2	6	
3	7	
4	8	
5	7	
6	7	
7	7	
8	8	
9	7	
10	7	
11	7	
12	7	
13	7	
14	7	
Total	**100**	___ / **100**

Q1. (8 points): Let U be the universal set, where

$U = \{\text{all whole numbers less than } 11\}$

$A = \{\text{all even natural numbers less than or equal to } 8\}$, and let

$B = \{y \mid y = x^2 + 2x \text{ where } x \text{ is an integer such that } 0 \le x < 3\}$,

Find each of the following.

(a): $U =$

(b): $A =$

(c): $A' =$

(d): $U' =$

(e): $\varnothing' =$

(f): $B =$

(g): $B' =$

(h): $A' \cap B' =$

Solution:

(a): $U = \{0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10\}$

(b): $A = \{2,\ 4,\ 6,\ 8\}$

(c): $A' = \{0,\ 1,\ 3,\ 5,\ 7,\ 9,\ 10\}$

(d): $U' = \varnothing$

(e): $\varnothing' = U$

(f): $B = \{0,\ 3,\ 8\}$

(g): $B' = \{1,2,4,5,6,7,9,10\}$

(h): $A' \cap B' = \{1,5,7,9,10\}$

Question 2: (6 points): Given the following numbers:

$$-1,\ \frac{0}{\pi},\ 1,\ 2,\ 5,\ 91,\ 2.12122123\ldots,\ \frac{22}{7},\ -41,\ 2.2\overline{3},\ \pi,\ \frac{\sqrt{2}}{3},\ \sqrt{\frac{27}{3}},\ 3.14,\ \frac{12.1}{1.1}$$

Complete the following:

Integers: ____

Rational Numbers: ____________________________________

Irrational Numbers: ___________________________________

Solution:

Integers: $-1,\ \frac{0}{\pi},\ 1,\ 2,\ 5,\ 91,\ -41,\ \sqrt{\frac{27}{3}},\ \frac{12.1}{1.1}$

Rational Numbers: $-1,\ \frac{0}{\pi},\ 1,\ 2,\ 5,\ 91,\ \frac{22}{7},\ -41,\ 2.2\overline{3},\ \sqrt{\frac{27}{3}},\ 3.14,\ \frac{12.1}{1.1}$

Irrational Numbers: $2.12122123...,\ \pi,\ \frac{\sqrt{2}}{3}$

Question 3: (7 points):

If $x=-\frac{3}{4}$ then find the value of the expression $-17+3\left[8x-4\left(3x-2\right)\right]=?$

Solution:

$$\begin{aligned}-17+3\left[8x-4\left(3x-2\right)\right]&=-17+3\left[8x-12x+8\right]\\&=-17+3\left[-4x+8\right]\\&=-17+3\left[(-4)\left(-\frac{3}{4}\right)+8\right]\\&=-17+3\left[3+8\right]\\&=-17+33\\&=16\end{aligned}$$

Another Method:

$$-17+3\left[8\left(-\frac{3}{4}\right)-4\left(3\left(-\frac{3}{4}\right)-2\right)\right]=-17+3\left[-6-4\left(-\frac{9}{4}-2\right)\right]$$

$$=-17+3\left[-6+9+8\right]=-17+3(11)=33-17=16$$

Question 4: (8 points): Write the following expression without absolute value symbols and in simplest form.

(a): $|x+3|-|x+6|,\quad -6\le x\le -3$

(b): $\left|\frac{x}{|x|+|x+3|}\right|,\quad -3<x<0$

(c): $|2x|+|-4x|+\left||6x|\right|,\ x\le -1$

(d): $\frac{x}{|x|}-\frac{|x|}{x}$, where $x\ne 0$.

Solution:

(a) $-(x+3)-(x+6)=-x-3-x-6=-2x-9$

(b) $\left|\frac{x}{|x|+|x+3|}\right|=\left|\frac{x}{-x+x+3}\right|=\left|\frac{x}{3}\right|=-\frac{x}{3}$

(c) $|2x|+|-4x|+\left||6x|\right|=|2x|+|4x|+|6x|=-2x-4x-6x=-12x$

(d) If $x>0\ \Rightarrow\ \frac{x}{|x|}-\frac{|x|}{x}=\frac{x}{x}-\frac{x}{x}=0$

If $x<0\ \Rightarrow\ \frac{x}{-x}-\frac{-x}{x}=-1-(-1)=0$

Question 5: (7 points): TRUE or FALSE

(A) The operation of division of real numbers is commutative. [ ]

(B) The multiplicative inverse of $-2\frac{2}{3}$ is $-\frac{3}{4}$. [ ]

(C) If $x < 0$, then $|-x| = -x$. [ ]

(D) If x is any real number, then $|-x| = x$ [ ]

(E) If $x < 0$, then $|x-1| = x+1$. [ ]

(F) Any positive integer is either prime or composite. [ ]

(G) The set $\{0,1\}$ is closed with respect to addition. [ ]

Solution:

(A) The operation of division of real numbers is commutative. **F**

(B) The multiplicative inverse of $-2\frac{2}{3}$ is $-\frac{3}{4}$. **F**

Because $-2\frac{2}{3} = -\left(2+\frac{2}{3}\right) = -\frac{8}{3}$ so the multiplicative inverse is $-\frac{3}{8}$

(C) If $x < 0$, then $|-x| = -x$. **T**

(D) If x is any real number, then $|-x| = x$ **F**

Because if x is a negative number then $|-x| = -x$.

(E) If $x < 0$, then $|x-1| = x+1$. **F**

Because $|x-1| = -(x-1) = -x+1$.

(F) Any positive integer is either prime or composite. **F**

Because 1 is neither a prime nor a composite number.

(G) The set $\{0,1\}$ is closed with respect to addition. **F**

Because $1+1=2 \notin \{0,1\}$

Question 6: (7 points): (R.3 Exercise 94, page 30): Divide $k^4 - 4k^2 + 2k + 5$ by $k^2 + 1$.

Write your answer as $\frac{Dividend}{Divisor} = Quotient + \frac{Remainder}{Divisor}$

Solution:

$$
\begin{array}{r|rrrr}
 & k^2 & -5 & & \quad \text{Quotient} \\
\hline
\text{Divisor} \quad k^2+1 & k^4 & -4k^2 & +2k & +5 \quad \text{Dividend} \\
 & k^4 & +k^2 & & \\
 & - & - & & \\
\hline
 & & -5k^2 & +2k & +5 \\
 & & -5k^2 & & -5 \\
 & & + & & + \\
\hline
 & & & 2k & +\ 10 \\
\end{array}
$$

Remainder

$$\frac{k^4 - 4k^2 + 2k + 5}{k^2+1} = k - 5 + \frac{2k+10}{k^2+1}$$

Question 7: (7 points): (R.3 Example 9 (c), page 27): $(2a+b)^4=?$

$$\begin{aligned}(2a+b)^4 &= (2a+b)^2(2a+b)^2\\ &= (4a^2+4ab+b^2)(4a^2+4ab+b^2) \qquad \text{Square each } 2a+b.\\ &= 16a^4+16a^3b+4a^2b^2+16a^3b+16a^2b^2\\ &\quad +4ab^3+4a^2b^2+4ab^3+b^4\\ &= 16a^4+32a^3b+24a^2b^2+8ab^3+b^4\end{aligned}$$

Question 8: (8 points): (R.4 Exercise 23 and 31): Factor each polynomial.

(a): $p^2q^2-10-2q^2+5p^2$

(b): $12a^3+10a^2-42a$

Solution:

(a):

23. $p^2q^2-10-2q^2+5p^2$

$$\begin{aligned}&= p^2q^2-2q^2+5p^2-10\\ &= q^2(p^2-2)+5(p^2-2)\\ &= (p^2-2)(q^2+5)\end{aligned}$$

(b):

31. Factor out the greatest common factor, $2a$: $12a^3+10a^2-42a=2a(6a^2+5a-21)$. Now factor the trinomial by trial and error: $6a^2+5a-21=(3a+7)(2a-3)$. Thus, $12a^3+10a^2-42a=2a(3a+7)(2a-3)$.

Question 9: (7 points): (R.5 Exercise 36): Find the following

$$\frac{x^2-y^2}{(x-y)^2}\cdot\frac{x^2-xy+y^2}{x^2-2xy+y^2}\div\frac{x^3+y^3}{(x-y)^4}=?$$

Solution:

$$\begin{aligned}\frac{x^2-y^2}{(x-y)^2}\cdot\frac{x^2-xy+y^2}{x^2-2xy+y^2}\div\frac{x^3+y^3}{(x-y)^4} &= \frac{(x-y)(x+y)}{(x-y)^2}\cdot\frac{x^2-xy+y^2}{(x-y)^2}\cdot\frac{(x-y)^4}{x^3+y^3}\\ &= \frac{(x-y)(x+y)(x^2-xy+y^2)}{x^3+y^3}\\ &= \frac{(x-y)(x+y)(x^2-xy+y^2)}{(x+y)(x^2-xy+y^2)}\\ &= x-y\end{aligned}$$

Question 10: (7 points): (R.6 Recitation Q#4): Simplify $\left(y^{-2}-x^{-2}\right)^{-3n}\left(x^{2}-y^{2}\right)^{2n}\left(x^{2}y^{2}\right)^{-3n}$

Solution:

$$\left(y^{-2}-x^{-2}\right)^{-3n}\left(x^{2}-y^{2}\right)^{2n}\left(x^{2}y^{2}\right)^{-3n}=\left(x^{2}y^{2}\right)^{-3n}\left(y^{-2}-x^{-2}\right)^{-3n}\left(x^{2}-y^{2}\right)^{2n}$$

$$=\left[\left(x^{2}y^{2}\right)\left(y^{-2}-x^{-2}\right)\right]^{-3n}\left(x^{2}-y^{2}\right)^{2n}$$

$$=\left(x^{2}-y^{2}\right)^{-3n}\left(x^{2}-y^{2}\right)^{2n}$$

$$=\left(x^{2}-y^{2}\right)^{-n}$$

$$=\frac{1}{\left(x^{2}-y^{2}\right)^{n}}$$

Question 11: (7 points): Simplify the following expression: $2+\dfrac{1}{2+\dfrac{1}{1+\dfrac{1}{x}}}=?$

Solution:

$$2+\frac{1}{2+\frac{1}{1+\frac{1}{x}}}=2+\frac{1}{2+\frac{1}{\frac{x+1}{x}}}=2+\frac{1}{2+\frac{x}{x+1}}=2+\frac{1}{\frac{2x+2+x}{x+1}}$$

$$=2+\frac{1}{\frac{2x+2+x}{x+1}}=2+\frac{x+1}{3x+2}=\frac{6x+4+x+1}{3x+2}=\frac{7x+5}{3x+2}$$

Question 12: (7 points): (R.7 Recitation Q#2b): Simplify $\dfrac{2}{\sqrt[3]{54}}+\dfrac{4}{\sqrt[3]{16}}-\dfrac{1}{\sqrt[3]{2}}$

Solution:

$$\frac{2}{\sqrt[3]{54}}+\frac{4}{\sqrt[3]{16}}-\frac{1}{\sqrt[3]{2}}=\frac{2}{\sqrt[3]{2(27)}}+\frac{4}{\sqrt[3]{2(8)}}-\frac{1}{\sqrt[3]{2}}$$

$$=\frac{2}{3\sqrt[3]{2}}+\frac{4}{2\sqrt[3]{2}}-\frac{1}{\sqrt[3]{2}}$$

$$=\frac{1}{\sqrt[3]{2}}\left(\frac{2}{3}+2-1\right)$$

$$=\frac{\sqrt[3]{4}}{\sqrt[3]{2}\sqrt[3]{4}}\left(\frac{2}{3}+1\right)=\frac{\sqrt[3]{4}}{2}\left(\frac{5}{3}\right)=\frac{5\sqrt[3]{4}}{6}$$

Question 13: (7 points): (R.7 Recitation Q#2d): Simplify $\dfrac{\sqrt[13]{(-2)^{13}}-\sqrt[10]{(-2)^{10}}}{\sqrt{2}-1}$

Solution:

$$\frac{\sqrt[13]{(-2)^{13}}-\sqrt[10]{(-2)^{10}}}{\sqrt{2}-1}=\frac{-2-2}{\sqrt{2}-1}=\frac{-4}{\sqrt{2}-1}=\frac{-4\left(\sqrt{2}+1\right)}{\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)}=-4\left(\sqrt{2}+1\right)$$

Question 14: (7 points) (R.7 Exercise 102, page 68): Rationalize the denominator of $\dfrac{1}{\sqrt[3]{3}-\sqrt[3]{5}}$.

Solution:

102. The denominator is in the form of the difference of cubes, so multiply the numerator and denominator by $\sqrt[3]{3^2}+\sqrt[3]{3\cdot 5}+\sqrt[3]{5^2}$ or $\sqrt[3]{9}+\sqrt[3]{15}+\sqrt[3]{25}$ in order to rationalize the denominator.

$$\frac{1}{\sqrt[3]{3}-\sqrt[3]{5}}\cdot\frac{\sqrt[3]{9}+\sqrt[3]{15}+\sqrt[3]{25}}{\sqrt[3]{9}+\sqrt[3]{15}+\sqrt[3]{25}}$$

$$=\frac{\sqrt[3]{9}+\sqrt[3]{15}+\sqrt[3]{25}}{\sqrt[3]{3}\left(\sqrt[3]{9}+\sqrt[3]{15}+\sqrt[3]{25}\right)-\sqrt[3]{5}\left(\sqrt[3]{9}+\sqrt[3]{15}+\sqrt[3]{25}\right)}$$

$$=\frac{\sqrt[3]{9}+\sqrt[3]{15}+\sqrt[3]{25}}{3+\sqrt[3]{45}+\sqrt[3]{75}-\sqrt[3]{45}-\sqrt[3]{75}-5}$$

$$=\frac{\sqrt[3]{9}+\sqrt[3]{15}+\sqrt[3]{25}}{-2}$$