Serial #: ________ ID ______________ NAME ____

Show all necessary steps for full marks.

Question 1 (5 points):

In the formula $P(t)=P_0e^{kt}$, if $P(25)=\frac{1}{2}P_0$, then $P(75)=?$

Solution:

Method I:

$P(t)=P_0e^{kt}$

$P(25)=P_0e^{25k}$

$\frac{1}{2}P_0=P_0e^{25k}$

$\frac{1}{2}=e^{25k}$

$\ln\frac{1}{2}=\ln e^{25k}$

$-\ln 2=25k$

$\boxed{k=-\frac{\ln 2}{25}}$

$P(t)=P_0e^{-\frac{\ln 2}{25}t}$

$P(75)=P_0e^{-\frac{\ln 2}{25}\cdot 75}=P_0e^{-3\ln 2}=P_0e^{\ln 2^{-3}}=\frac{P_0}{8}$

Method II:

$P(t)=P_0e^{kt}$

$P(25)=P_0e^{25k}$

$\frac{1}{2}P_0=P_0e^{25k}$

$\frac{1}{2}=e^{25k}$

$P(75)=P_0e^{75k}=P_0e^{3(25k)}=P_0\left(e^{25k}\right)^3=P_0\left(\frac{1}{2}\right)^3=\frac{P}{8}$

Question 2 (5 points) (Textbook Exercise 88):

Given $f(x)=\log x$ and $g(x)=x^2$. Find the functions $f\circ g$ and $g\circ f$ and their domains.

Solution:

$(f\circ g)(x)=f(g(x))$ where $x\in D_g=(-\infty,\infty)$ and $g(x)\in D_f=(0,\infty)$

$=f(x^2)$

$=\log_2(x^2)=2\log_2|x|$

Domain of $f\circ g=D_{f\circ g}=(-\infty,0)\cup(0,\infty)$ because $x\in D_g=(-\infty,\infty)$ and $x^2>0$

$(g\circ f)(x)=g(f(x))$ where $x\in D_f=(0,\infty)$ and $f(x)\in D_g=(-\infty,\infty)$

$=g(\log_2 x)$

$=(\log_2 x)^2$

Domain of $g\circ f=D_{g\circ f}=(0,\infty)$ because $x\in D_f=(0,\infty)$ and $f(x)\in D_g=(-\infty,\infty)$

Question 3 (5 points): If $\log_a 2 = x$ and $\log_a 3 = y$ then write the following expression in terms of x and y. $\log_a 65 - \log_a \frac{104}{3} + \frac{1}{2}\log_a \frac{1}{100} = ?$

Solution:

$$\log_a 65 - \log_a \frac{104}{3} + \frac{1}{2}\log_a \frac{1}{100} = \log_a 65 + \log_a \left(\frac{104}{3}\right)^{-1} + \log_a \left(\frac{1}{100}\right)^{1/2}$$

$$= \log_a \left[65\left(\frac{104}{3}\right)^{-1}\left(\frac{1}{10}\right)\right]$$

$$= \log_a \left[13(5)\left(\frac{3}{8(13)}\right)\left(\frac{1}{2(5)}\right)\right]$$

$$= \log_a \left(\frac{3}{16}\right) = \log_a 3 - \log_a 2^4$$

$$= \log_a 3 - 4\log_a 2$$

$$= y - 4x$$

2	104
2	52
2	26
13	13

5	65
13	13

$104 = 8(13)$, $65 = 2(13)$

Question 4 (5 points): If $\log_3 x + \log_3(x+3) + \log_3(x+2) - \log_3(x^2+5x+6) = 2$, then $x = ?$

Solution:

$$\log_3 x + \log_3(x+3) + \log_3(x+2) - \log_3(x^2+5x+6) = 2$$

$$\log_3\left[x(x+3)(x+2)\right] - \log_3\left[(x+3)(x+2)\right] = 2$$

$$\log_3 \frac{x(x+3)(x+2)}{(x+3)(x+2)} = 2$$

$$\log_3 x = 2$$

$$x = 3^2 = 9$$

$$SS = \{9\}$$