

King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 - Term 142

Recitation (7.5)

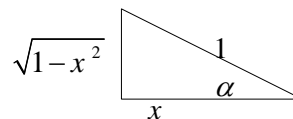
**Question 1:** Find the exact value of  $\tan\left[\sin^{-1}\left(-\frac{4}{5}\right) - \cos^{-1}\left(\frac{12}{13}\right)\right]$ . **Answer:**  $-\frac{63}{16}$

**Question 2** Find the exact value of  $\sec^{-1}(-2) + \tan^{-1}\left(\tan\frac{3\pi}{5}\right)$  **Answer:**  $\frac{4\pi}{15}$

**Question 3:** Verify the identity  $\tan(2\cos^{-1}x) = \frac{2x\sqrt{1-x^2}}{2x^2-1}$ .

**Solution:**

Let  $\alpha = \cos^{-1}x \Rightarrow \cos\alpha = x \Rightarrow \cos\alpha = \frac{x}{1}$



$\tan(2\cos^{-1}x) = \tan 2\alpha$

$$= \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{2\sin\alpha\cos\alpha}{2\cos^2\alpha - 1} = \frac{2(\sqrt{1-x^2})x}{2x^2 - 1} = \frac{2x\sqrt{1-x^2}}{2x^2 - 1}$$

**Another Method:**

$\tan(2\cos^{-1}x) = \tan 2\alpha$

$$= \frac{2\tan\alpha}{1 - \tan^2\alpha} = \frac{2\frac{\sqrt{1-x^2}}{x}}{1 - \frac{1-x^2}{x^2}} = \frac{2x\sqrt{1-x^2}}{x^2 - (1-x^2)} = \frac{2x\sqrt{1-x^2}}{2x^2 - 1}$$

**Question 4:** Which one of the following statements is **FALSE** ?

- A)  $\sin^{-1}(\sin x) = x, \quad 0 \leq x < 2\pi$
- B)  $\sec^{-1}x = \cos^{-1}\frac{1}{x}, \quad x \leq -1 \text{ or } x \geq 1$
- C)  $\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x, \quad -\infty < x < \infty$
- D)  $\csc^{-1}x = \sin^{-1}\frac{1}{x}, \quad x \leq -1 \text{ or } x \geq 1$
- E)  $\sin^{-1}(-x) = -\sin^{-1}x, \quad -1 \leq x \leq 1$

**Question 5**  $\csc^{-1}\left(\frac{-2\sqrt{3}}{3}\right) + \cos^{-1}\left(\sin\frac{\pi}{5}\right) =$

- A)  $\frac{\pi}{20}$
- B)  $-\frac{2\pi}{15}$
- C)  $\frac{\pi}{30}$
- D)  $\frac{2\pi}{15}$
- E)  $\frac{3\pi}{20}$