

King Fahd University of Petroleum and Minerals
Prep-Year Math Program
Math 002 - Term 142
Recitation (7.2)

Question 1: Simplify the following expression $\frac{(\sec \theta - \tan \theta)^2 + 1}{\sec \theta \csc \theta - \tan \theta \cot \theta} = ?$

Answer: $2 \tan \theta$

Question 2: Verify the following identities:

a) $(\sin \theta + \csc \theta)^2 + (\cos \theta + \sec \theta)^2 - \tan^2 \theta - \cot^2 \theta = 7$

b) $\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \csc x - \cot x, 0 < x < \frac{\pi}{2}$

Solution: (a):

$$\begin{aligned} & (\sin \theta + \csc \theta)^2 + (\cos \theta + \sec \theta)^2 - \tan^2 \theta - \cot^2 \theta \\ &= \sin^2 \theta + 2 \sin \theta \csc \theta + \csc^2 \theta + \cos^2 \theta + 2 \cos \theta \sec \theta + \sec^2 \theta - \tan^2 \theta - \cot^2 \theta \\ &= \sin^2 \theta + \cos^2 \theta + 2 \frac{\sin \theta}{\sin \theta} + 2 \frac{\cos \theta}{\cos \theta} + 1 + \cot^2 \theta + 1 + \tan^2 \theta - \tan^2 \theta - \cot^2 \theta \\ &= 1 + 2 + 2 + 1 + 1 = 7 \end{aligned}$$

(b):
$$\begin{aligned} \sqrt{\frac{1 - \cos x}{1 + \cos x}} &= \sqrt{\frac{(1 - \cos x)(1 - \cos x)}{(1 + \cos x)(1 - \cos x)}} = \sqrt{\frac{(1 - \cos x)^2}{1 - \cos^2 x}} = \sqrt{\frac{(1 - \cos x)^2}{\sin^2 x}} = \frac{|1 - \cos x|}{|\sin x|} = \\ &= \frac{1 - \cos x}{\sin x} \quad \text{because } 1 - \cos x \geq 0 \text{ and for } 0 < x < \frac{\pi}{2}, \sin x > 0 \\ &= \frac{1}{\sin x} - \frac{\cos x}{\sin x} = \csc x - \cot x \end{aligned}$$

Question 3:

$$\frac{\sin^2(-x) - \cos^2(-x)}{\sin(-x) - \cos(-x)} =$$

- a) $-\sin x - \cos x$
- b) $-2 \sin x$
- c) $\sin x - \cos x$
- d) $\cos x - \sin x$
- e) $\sin x + \cos x$

Question 4: $\frac{\sec x - 1}{\sec x + 1} - \frac{\sec x + 1}{\sec x - 1} =$

- a) $-4 \csc x \cot x$
- b) $-4 \sec x \tan x$
- c) $4 \csc x \cot x$
- d) $8 \sec x \tan x$
- e) $-2 \sec x \tan x$