

King Fahd University of Petroleum and Minerals

Prep-Year Math Program

Math 002 - Term 132

Recitation (7.2)

Question 1: Simplify the following expression $\frac{(\sec \theta - \tan \theta)^2 + 1}{\sec \theta \csc \theta - \tan \theta \csc \theta} = ?$

Answer: $2 \tan \theta$

Question 2: Verify the following identities

(a): $\frac{\cot \theta - \csc \theta}{\cot \theta + \csc \theta} = \frac{\cos \theta - 1}{\cos \theta + 1}$

(b): $\frac{1}{1 - \cos x} - \frac{\cos x}{1 + \cos x} = 2 \csc^2 x - 1$

Solution:

(a):
$$LHS = \frac{\cot \theta - \csc \theta}{\cot \theta + \csc \theta} = \frac{\frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}}{\frac{\cos \theta}{\sin \theta} + \frac{1}{\sin \theta}} = \frac{\sin \theta}{\sin \theta} \cdot \frac{\frac{\cos \theta}{\sin \theta} - \frac{1}{\sin \theta}}{\frac{\cos \theta}{\sin \theta} + \frac{1}{\sin \theta}} = \frac{\cos \theta - 1}{\cos \theta + 1}$$

(b):

$$\begin{aligned} LHS &= \frac{1}{1 - \cos x} - \frac{\cos x}{1 + \cos x} = \frac{1 + \cos x - \cos x(1 - \cos x)}{(1 - \cos x)(1 + \cos x)} \\ &= \frac{\cos^2 x + 1}{1 - \cos^2 x} \\ &= \frac{\cos^2 x + 1}{\sin^2 x} \\ &= \frac{\cos^2 x}{\sin^2 x} + \frac{1}{\sin^2 x} \\ &= \cot^2 x + \csc^2 x \\ &= \csc^2 x - 1 + \csc^2 x \\ &= 2 \csc^2 x - 1 \end{aligned}$$

Question 3:

$$\frac{\sin^2(-x) - \cos^2(-x)}{\sin(-x) - \cos(-x)} =$$

- a) $-\sin x - \cos x$
- b) $-2 \sin x$
- c) $\sin x - \cos x$
- d) $\cos x - \sin x$
- e) $\sin x + \cos x$

Answer: d)

Question 4: $\frac{\sec x - 1}{\sec x + 1} - \frac{\sec x + 1}{\sec x - 1} =$

- a) $-4 \csc x \cot x$
- b) $-4 \sec x \tan x$
- c) $4 \csc x \cot x$
- d) $8 \sec x \tan x$
- e) $-2 \sec x \tan x$

Answer: a) $-4 \csc x \cot x$