

Show all necessary steps for full marks.

Question 1: (5 points)(1.1 Textbook Example): Solve $\frac{2x + 4}{3} + \frac{1}{2}x = \frac{1}{4}x - \frac{7}{3}$

Solution:

SOLUTION

$$\frac{2x + 4}{3} + \frac{1}{2}x = \frac{1}{4}x - \frac{7}{3}$$

Distribute to all terms within the parentheses.

$$12\left(\frac{2x + 4}{3} + \frac{1}{2}x\right) = 12\left(\frac{1}{4}x - \frac{7}{3}\right)$$

Multiply by 12, the LCD of the fractions. (Section R.5)

$$12\left(\frac{2x + 4}{3}\right) + 12\left(\frac{1}{2}x\right) = 12\left(\frac{1}{4}x\right) - 12\left(\frac{7}{3}\right)$$

Distributive property

$$4(2x + 4) + 6x = 3x - 28$$

Multiply.

$$8x + 16 + 6x = 3x - 28$$

Distributive property

$$14x + 16 = 3x - 28$$

Combine like terms.

$$11x = -44$$

Subtract 3x. Subtract 16.

$$x = -4$$

Divide each side by 11.

Question 2: (5 points): Solve for R: $\frac{AR - B}{BR - A} = \frac{A}{B} + 1$

Solution:

$$\frac{AR - B}{BR - A} = \frac{A}{B} + 1$$

$$\frac{AR - B}{BR - A} = \frac{A + B}{B}$$

$$ABR - B^2 = ABR + B^2R - A^2 - AB$$

$$A^2 + AB - B^2 = ABR - ABR + B^2R$$

$$A^2 + AB - B^2 = B^2R$$

$$R = \frac{A^2 + AB - B^2}{B^2}$$

Question 3: (5 points): $A + iB = \frac{\sqrt[3]{-125} + i^{103} - \sqrt{-4}\sqrt{-1}}{(2i - 1) - (i + 5)}$, $A = ?$, $B = ?$

Solution:

$$A + iB = \frac{\sqrt[3]{-125} + i^{103} - \sqrt{-4}\sqrt{-1}}{(2i - 1) - (i + 5)} = \frac{\sqrt[3]{(-5)^3} + i(i^2)^{51} - (\sqrt{4})i(i)}{2i - 1 - i - 5}$$

$$= \frac{-5 - i + 2}{i - 6} = \frac{-3 - i}{-6 + i} = \frac{(-3 - i)(-6 - i)}{(-6 + i)(-6 - i)} = \frac{18 + 3i + 6i - 1}{36 + 1} = \frac{17 + 9i}{37}$$

$$= \frac{17}{35} + \frac{9}{35}i \quad \text{Answer: } A = \frac{17}{37}, B = \frac{9}{37}$$

Question 4: (5 points): If $z = (1 - i)^3 + i^{15}$, where $i = \sqrt{-1}$, then find the conjugate of z .

Solution:

$$z = (1 - i)^3 + i^{15} = z = (1 - i)(1 - i)^2 + i \cdot i^{14} = (1 - i)(1 - 2i - 1) + i \cdot (i^2)^7$$

$$= -2i(1 - i) - i = -2i + 2i^2 - i = -2 - 3i \Rightarrow \bar{z} = -2 + 3i$$

Another Method:

Q16. If $z = (1-i)^3 + i^{15}$, where $i = \sqrt{-1}$, then the conjugate of z is:

Sec.(1.3) Complex numbers

A) $-2+3i$

B) $1+6i$

C) $2-3i$

D) $3i$

E) $-3i$

$$z = 1 - 3i + 3i^2 - i^3 + i^{12} \cdot i^3$$

$$= 1 - 3i - 3 + i + i^3$$

$$= 1 - 3i - 3 + i - i$$

$$\underline{z} = -2 - 3i$$

$$\overline{z} = -2 + 3i$$