NAME: ______KEY__________________________________ ID: ______________ Serial No: _________

Show all necessary steps for full marks.

Q1 (5 points): Find the vertex, focus, and directrix of the parabola given by $6x - 3y^2 - 12y + 4 = 0$. Sketch the graph.

Solution: $6x - 3y^2 - 12y + 4 = 0$

$$-3(y^2 + 4y) = -6x - 4$$

$$-3(y^2 + 4y + 2^2) = -6x - 4 - 12$$

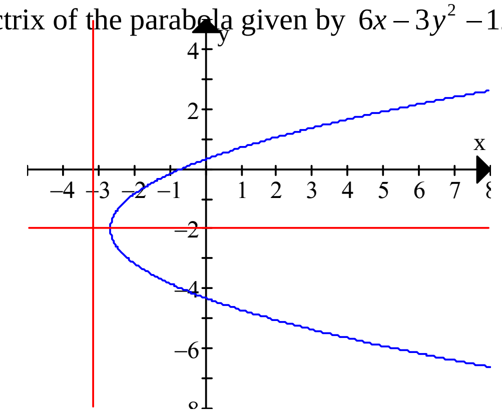
$$-3(y+2)^2 = -6x - 16$$

$$(y+2)^2 = 2x + \frac{16}{3} \quad \Rightarrow \quad (y+2)^2 = 2\left(x + \frac{8}{3}\right)$$

$$\text{vertex} = \left(-\frac{8}{3}, -2\right), \quad 4p = 2 \quad \Rightarrow \quad p = \frac{1}{2}$$

$$focus = (h+p, k) = \left(-\frac{8}{3} + \frac{1}{2}, -2\right) = \left(-\frac{13}{6}, -2\right)$$

$$Directrix: \quad x = h - p \quad \Rightarrow \quad x = -\frac{8}{3} - \frac{1}{2} \quad \Rightarrow \quad x = -\frac{19}{6}$$



Q2 (5 points): Find the vertex, focus, and directrix of the parabola given y $2x^2 - 8x - 4y + 3 = 0$. Sketch the graph.

Solution: $2(x^2 - 4x) = 4y - 3$

$$2(x^2 - 4x + 2^2) = 4y - 3 + 8$$

$$2(x-2)^2 = 4y + 5$$

$$(x-2)^2 = 2y + \frac{5}{2}$$

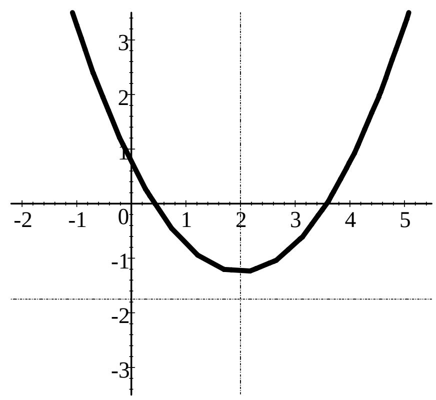
$$(x-2)^2 = 2\left(y + \frac{5}{4}\right)$$

$$vertex = \left(2, -\frac{5}{4}\right)$$

$$4p = 2 \Rightarrow p = \frac{1}{2}$$

$$\text{focus} = (h, p+k) = \left(2, \frac{1}{2} - \frac{5}{4}\right) = \left(2, \frac{-3}{4}\right)$$

$$\text{Directrix: } y = k - p \Rightarrow y = -\frac{5}{4} - \frac{1}{2} \Rightarrow y = -\frac{7}{4}$$



Q3 (5 points): Find the vertices and foci of the ellipse $9x^2 - 18x + 4y^2 + 8y - 23 = 0$. And ketch the graph of the equation.

Solution:

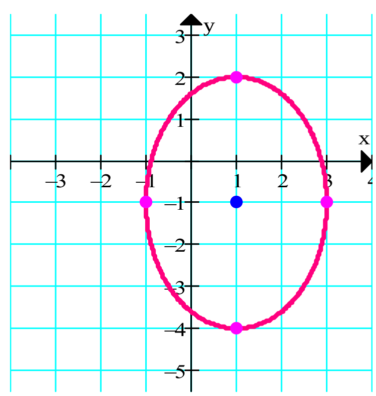
$$9x^2 - 18x + 4y^2 + 8y - 23 = 0$$

$$9(x^2 - 2x) + 4(y^2 + 2y) = 23$$

$$9(x^2 - 2x + 1) + 4(y^2 + 2y + 1) = 23 + 9 + 4$$

$$9(x-1)^2 + 4(y+1)^2 = 36$$

$$\frac{(x-1)^2}{4} + \frac{(y+1)^2}{9} = 1$$



$a=3, b=2$, center $=(1,-1)$

We locate the vertices by moving 3 units up or down from the center $(1,-1+3)=(1,2)$ and $(1,-1-3)$.

We locate the endpoints of the minor axis by moving 2 units' right or left from center:

$(1-2,-1)=(-1,-1)$ and $(1+2,-1)$

To find the coordinates of foci, we find c. $c^2=a^2-b^2=9-4=5 \Rightarrow c=\sqrt{5}$

foci $=(1,-1-\sqrt{5}),(1,-1+\sqrt{5})$

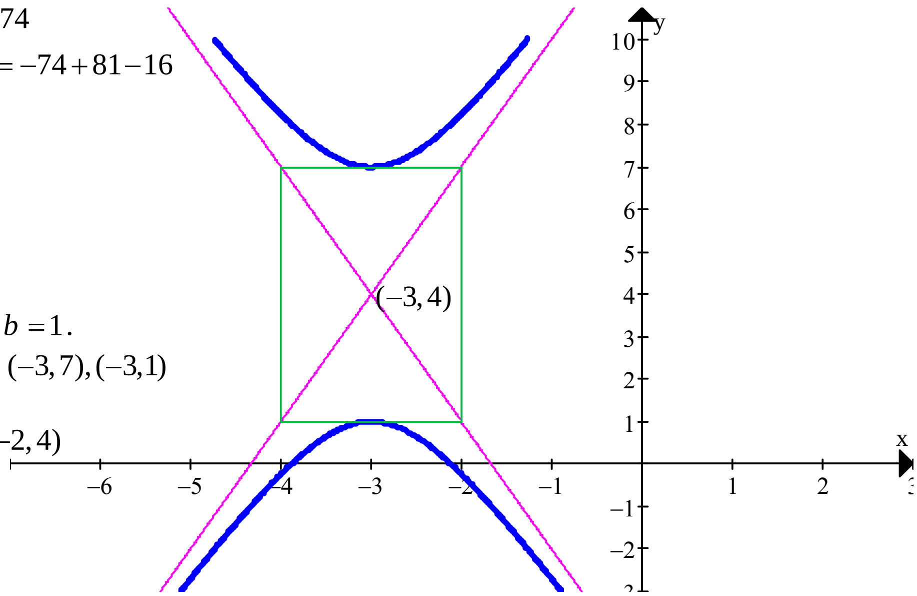
Q4 (5 points): The equation of hyperbola is given $9x^2-y^2+54x+8y+74=0$

(a): Sketch the graph of the hyperbola.

(b): Find the range, foci, eccentricity and the equations of asymptotes of the hyperbola

$$\frac{(y-4)^2}{9}-(x+3)^2=1$$

Solution:



(a): $9(x^2+6x)-(y^2-8y)=-74$

$9(x^2+6x+3^2)-(y^2-8y+4^2)=-74+81-16$

$9(x+3)^2-(y-4)^2=-9$

$$\frac{9(x+3)^2}{-9}-\frac{(y-4)^2}{-9}=\frac{-9}{-9}$$

$$\frac{(y-4)^2}{9}-(x+3)^2=1$$

The center is $(-3,4)$, $a=3$ and $b=1$.

$vertices:(h,k\pm a)=(-3,4\pm 3)=(-3,7),(-3,1)$

Endpoints of conjugate axis are:

$(h\pm b,k)=(-3\pm 1,4)=(-4,4),(-2,4)$

(b): $\frac{(y-4)^2}{9}-(x+3)^2=1$

$Range=(-\infty,1]\cup[7,\infty)$

$c^2=a^2+b^2=9+1=10 \Rightarrow c=\pm\sqrt{10}$

$$eccentricity=\frac{c}{a}=\frac{\sqrt{10}}{3}$$

$Foci:(h,k\pm c)=(-3,4\pm\sqrt{10})=\left(-3,4+\sqrt{10}\right),\left(-3,4-\sqrt{10}\right)$

Equations of asymptotes are: $y-4=\pm 3(x+3)$

$y=-3x-5$, $y=3x+13$

Q5 (5 points): Sketch the graph of $x=-\sqrt{1+4y^2}$ and find the domain and range of the equation.

Solution: Note that x is a negative number.

Squaring both sides gives $x^2=1+4y^2$

$x^2-4y^2=1$

$$x^2-\frac{y^2}{1/4}=1$$

$a=1$ and $b=1/2$

$Domain=(-\infty,-1]$ $Range=(-\infty,\infty)$