SN ________ ID ______________ NAME ______________________________________

Show all necessary steps for full marks.

Question 1: (5 points)(7.1 Example 2): Write $\cos x$ in terms of $\tan x$

Solution:

$$\cos x = \frac{1}{\sec x} = \frac{1}{\pm\sqrt{1+\tan^2 x}} = \pm\frac{1}{\sqrt{1+\tan^2 x}}\cdot\frac{\sqrt{1+\tan^2 x}}{\sqrt{1+\tan^2 x}} = \pm\frac{\sqrt{1+\tan^2 x}}{1+\tan^2 x}$$

Another Method:

▶ EXAMPLE 2 EXPRESSING ONE FUNCTION IN TERMS OF ANOTHER

Express $\cos x$ in terms of $\tan x$.

Solution Since $\sec x$ is related to both $\cos x$ and $\tan x$ by identities, start with $1 + \tan^2 x = \sec^2 x$.

$$\frac{1}{1+\tan^2 x} = \frac{1}{\sec^2 x}$$ Take reciprocals.

$$\frac{1}{1+\tan^2 x} = \cos^2 x$$ Reciprocal identity

$$\pm\sqrt{\frac{1}{1+\tan^2 x}} = \cos x$$ Take the square root of each side.

Remember both the positive and negative roots.

$$\cos x = \frac{\pm 1}{\sqrt{1+\tan^2 x}}$$ Quotient rule for radicals: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ (Section R.7); rewrite.

$$\cos x = \frac{\pm\sqrt{1+\tan^2 x}}{1+\tan^2 x}$$ Rationalize the denominator. (Section R.7)

Choose the + sign or the − sign, depending on the quadrant of x.

NOW TRY EXERCISE 47. ◀

Question 2: (5 points) (7.2 Exercise 59): Verify that the following equation is an identity.

$$\frac{\tan^2 t - 1}{\sec^2 t} = \frac{\tan t - \cot t}{\tan t + \cot t}$$

Solution:

59. Verify $\frac{\tan^2 t - 1}{\sec^2 t} = \frac{\tan t - \cot t}{\tan t + \cot t}$.

Simplify the right side

$$\frac{\tan t - \cot t}{\tan t + \cot t} = \frac{\tan t - \frac{1}{\tan t}}{\tan t + \frac{1}{\tan t}}$$

$$= \frac{\tan t - \frac{1}{\tan t}}{\tan t + \frac{1}{\tan t}}\cdot\frac{\tan t}{\tan t}$$

$$= \frac{\tan^2 t - 1}{\tan^2 t + 1} = \frac{\tan^2 t - 1}{\sec^2 t}$$

Question 3: (5 points) (7.3 Example 2): Find one value of θ that satisfies each of the following.

(a): $\cot\theta = \tan 25°$ (b): $\sin\theta = \cos(-30°)$ (c): $\csc\frac{3\pi}{4} = \sec x$

Solution:

(a) Since tangent and cotangent are cofunctions, $\tan(90° - \theta) = \cot\theta$.

$\cot\theta = \tan 25°$

$\tan(90° - \theta) = \tan 25°$ Cofunction identity

$90° - \theta = 25°$ Set angle measures equal.

$\theta = 65°$ Solve for θ.

(b)

$$\sin\theta = \cos(-30^\circ)$$
$$\cos(90^\circ - \theta) = \cos(-30^\circ) \quad \text{Cofunction identity}$$
$$90^\circ - \theta = -30^\circ$$
$$\theta = 120^\circ$$

(c)

$$\csc\frac{3\pi}{4} = \sec\theta$$
$$\sec\left(\frac{\pi}{2} - \frac{3\pi}{4}\right) = \sec\theta \quad \text{Cofunction identity}$$
$$\sec\left(-\frac{\pi}{4}\right) = \sec\theta \quad \text{Combine terms.}$$
$$-\frac{\pi}{4} = \theta$$

Question 4: **(5 points) (7.4 Recitation Q2):** If $\sin\frac{\alpha}{2} = \frac{4}{5}$ and α terminates in quadrant III, then find $\sin\alpha + \cos\alpha$

Solution:

$$\pi < \alpha < \frac{3\pi}{2} \quad \Rightarrow \quad \frac{\pi}{2} < \frac{\alpha}{2} < \frac{3\pi}{4} \quad \Rightarrow \quad \frac{\alpha}{2} \text{ is in Quadrant II}$$

$$\sin\frac{\alpha}{2} = +\sqrt{\frac{1-\cos\alpha}{2}} \quad \Rightarrow \quad \frac{4}{5} = \sqrt{\frac{1-\cos\alpha}{2}} \quad \Rightarrow \quad \frac{16}{25} = \frac{1-\cos\alpha}{2}$$

$$\Rightarrow \quad \frac{16}{25} = \frac{1-\cos\alpha}{2} \quad \Rightarrow \quad 32 = 25 - 25\cos\alpha \quad \Rightarrow \quad 25\cos\alpha = -7$$

$$\Rightarrow \quad \cos\alpha = -\frac{7}{25} \Rightarrow \sin\alpha = -\sqrt{1-\left(\frac{7}{25}\right)^2} = -\sqrt{\frac{25^2-7^2}{25^2}} = -\frac{\sqrt{625-49}}{25} = -\frac{\sqrt{576}}{25} = -\frac{24}{25}$$

$$\sin\alpha + \cos\alpha = -\frac{24}{25} - \frac{7}{25} = -\frac{31}{25}$$

Answer: $-\frac{31}{25}$

Question 5: **(5 points):** Graph one cycle of the equation $y = -\sin x - \sqrt{3}\cos x$.

Solution:

$$a = -1,\ b = -\sqrt{3} \quad \Rightarrow \quad k = \sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = 2$$

$$\left.\begin{aligned} \sin\alpha &= \frac{b}{k} = \frac{-\sqrt{3}}{2} \\ \cos\alpha &= \frac{a}{k} = \frac{-1}{2} \end{aligned}\right\} \quad \Rightarrow \quad \alpha \in QIII,\ \alpha = -\frac{2\pi}{3} \quad \text{or} \quad \alpha = \frac{4\pi}{3}$$

$$y = -\sin x - \sqrt{3}\cos x = 2\sin\left(x + \frac{4\pi}{3}\right) \qquad \text{Or} \qquad y = -\sin x - \sqrt{3}\cos x = 2\sin\left(x - \frac{2\pi}{3}\right)$$