SN ________ ID ______________ NAME ____

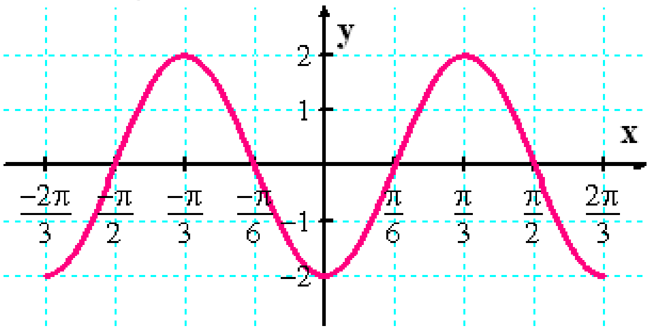
Show all necessary steps for full marks.

Question 1: **(6 points)(6.3 Exercise 31):** Graph $y = -2\cos 3x$ over two periods.

Solution:

$0 \le 3x \le 2\pi$

$0 \le x \le \dfrac{2\pi}{3}$

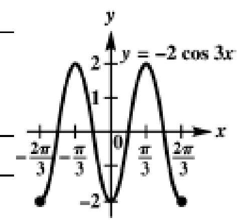


31. $y = -2\cos 3x$

Period: $\frac{2\pi}{3}$ and amplitude: $|-2| = 2$

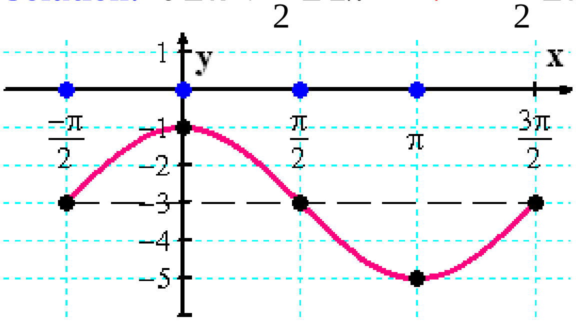
Divide the interval $\left[0, \frac{2\pi}{3}\right]$ into four equal parts to get the x-values that will yield minimum and maximum points and x-intercepts. Then make a table. Repeat this cycle for the interval $\left[-\frac{2\pi}{3}, 0\right]$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$
$3x$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\cos 3x$	1	0	-1	0	1
$-2\cos 3x$	-2	0	2	0	-2



Question 2: **(6 points) (6.4 Exercise 53):** Graph $y = -3 + 2\sin\left(x + \dfrac{\pi}{2}\right)$ over one period.

Solution: $0 \le x + \dfrac{\pi}{2} \le 2\pi \Rightarrow -\dfrac{\pi}{2} \le x \le \dfrac{3\pi}{2}$



53. $y = -3 + 2\sin\left(x + \frac{\pi}{2}\right)$

Step 1: Find the interval whose length is $\frac{2\pi}{b}$.

$0 \le x + \frac{\pi}{2} \le 2\pi \Rightarrow 0 - \frac{\pi}{2} \le x \le 2\pi - \frac{\pi}{2} \Rightarrow$
$-\frac{\pi}{2} \le x \le \frac{3\pi}{2}$

Step 2: Divide the period into four equal parts to get the following x-values: $-\frac{\pi}{2}$, 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$

Step 3: Evaluate the function for each of the five x-values

x	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$x + \frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin\left(x + \frac{\pi}{2}\right)$	0	1	0	-1	0
$2\sin\left(x + \frac{\pi}{2}\right)$	0	2	0	-2	0
$-3 + 2\sin\left(x + \frac{\pi}{2}\right)$	-3	-1	-3	-5	-3

Question 3: **(6 points):** Find all vertical asymptotes of $y = 2\tan\left(3\pi x + \frac{\pi}{4}\right)$ over the interval $\left[0, \frac{5}{12}\right]$.

Solution:

All vertical asymptotes are given by: $3\pi x + \frac{\pi}{4} = (2n+1)\frac{\pi}{2}$, $n = 0, \pm 1, , \pm 2, , \pm 3, \cdots$

$$3x = (2n+1)\frac{1}{2} - \frac{1}{4} = n + \frac{1}{2} - \frac{1}{4} = n + \frac{1}{4}$$

$$x = \frac{n}{3} + \frac{1}{12}$$

$$x = \frac{4n+1}{12}$$

If $n = 0$ then $x = \frac{1}{12} \in \left[0, \frac{5}{12}\right]$

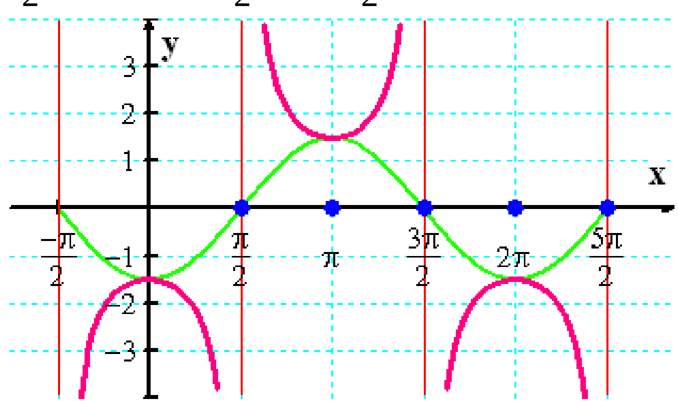
If $n = 1$ then $x = \frac{5}{12} \in \left[0, \frac{5}{12}\right]$

If $n = 2$ then $x = \frac{9}{12} \notin \left[0, \frac{5}{12}\right]$

The vertical asymptotes are: $\boxed{x = \frac{1}{12}}$ and $\boxed{x = \frac{5}{12}}$

Question 4: **(7 points) (6.6 Example 2:** Graph $y = \frac{3}{2}\csc\left(x - \frac{\pi}{2}\right)$.

Solution: $y = \frac{3}{2}\sin\left(x - \frac{\pi}{2}\right) \Rightarrow 0 \le x - \frac{\pi}{2} \le 2\pi \Rightarrow \frac{\pi}{2} \le x \le \frac{5\pi}{2}$



Step 1 Use the guidelines of **Section 6.4** to graph the corresponding reciprocal function defined by

$$y = \frac{3}{2}\sin\left(x - \frac{\pi}{2}\right),$$

shown as a red dashed curve in Figure 63.

Step 2 Sketch the vertical asymptotes through the x-intercepts of the graph of $y = \frac{3}{2}\sin\left(x - \frac{\pi}{2}\right)$. These have the form $x = (2n+1)\frac{\pi}{2}$, where n is any integer. See the black dashed lines in Figure 63.

Step 3 Sketch the graph of $y = \frac{3}{2}\csc\left(x - \frac{\pi}{2}\right)$ by drawing the typical U-shaped branches between adjacent asymptotes. See the solid blue graph in Figure 63.

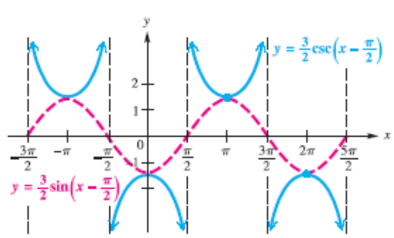


Figure 63