

KING FAHD UNIVERSITY OF PETROLUUM AND MINERALS
 Faculty of Science, Prep-Year Math Program
 Math 001 - Term 041

CLASS TEST 2

Code 2

Name: _____ ID#: _____ Sr. #: _____ Section: _____

[Provide neat and complete solution. Show all necessary steps for full credit.]

Question 1

If $f(x) = 2x^2 + 5$ and $g(x) = 3x + a$, find a so that the graph of $f \circ g$ crosses the y -axis at 23.

$$(f \circ g)(x) = f(g(x)) = f(3x + a) = 2(3x + a)^2 + 5$$

$$(f \circ g)(x) \text{ cross the } y\text{-axis when } x = 0 \Rightarrow 2a^2 + 5 = 23$$

$$\Rightarrow a^2 = 9$$

$$\Rightarrow a = \pm 3$$

(4 pts)

Question 2 Given the equation of a circle $2x^2 + 2y^2 + 8x - 4y + 2 = 0$.

a) Write this equation in the standard form.

$$x^2 + y^2 + 4x - 2y = -1$$

$$(x^2 + 4x + 4)(y^2 - 2y + 1) = 4 + 1 - 1$$

$$(x+2)^2 + (y-1)^2 = 4$$

(3 pts)

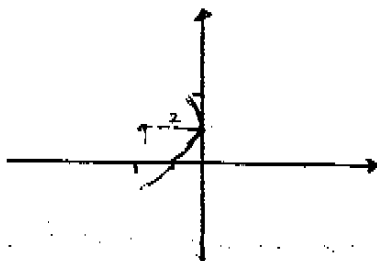
b) Find the center and the radius of this circle.

$$C: (-2, 1)$$

$$r = 2$$

(1 pt)

c) Determine whether the circle is tangent to the x -axis, y -axis or both.



only y -axis

(1 pt)

(9)

Question 3

2

Given the function $f(x) = 4x - x^2, x \leq 2$.d) Write $f^{-1}(x)$.Let $f(y) = x \Rightarrow 4y - y^2 = x$, Complete the square for y

$$\Rightarrow y^2 - 4y + 4 = -x + 4$$

$$\Rightarrow (y-2)^2 = -x + 4$$

$$\Rightarrow y-2 = \pm \sqrt{-x+4}, y \leq 2$$

$$\Rightarrow f^{-1}(x) = 2 - \sqrt{-x+4}$$

(4 pts)

e) State the domain and range of f^{-1}

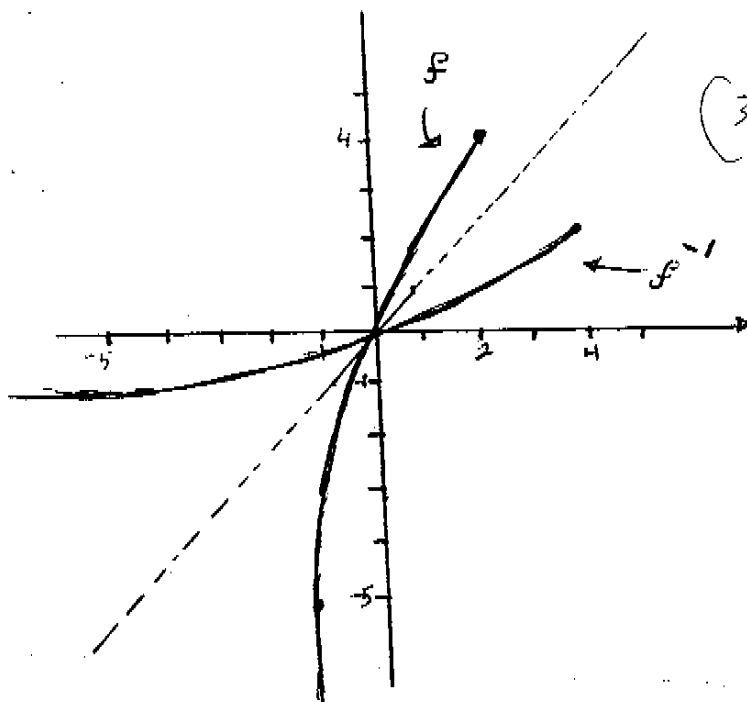
$$D_{f^{-1}}: -x+4 \geq 0 \Rightarrow x \leq 4 \Rightarrow D_{f^{-1}} = (-\infty, 4]$$

$$R_{f^{-1}} = D_f = (-\infty, 2)$$

(2 pts)

f) Sketch the graphs of f and f^{-1} in the same coordinate system.

$$h = -\frac{b}{2a} = 2, k = f(2) = 4 \Rightarrow \text{Vertex} = (2, 4)$$



(9)

Question 4 If the line L passes through the vertex of the quadratic function $f(x) = -2x^2 + 4x + 1$ and is perpendicular to the line $3y - 2x + 1 = 0$. Write the equation of the line L .

* $k = \frac{-b}{2a} = 1$, $k = f(-h) = 3 \Rightarrow \text{Vertex} : (1, 3)$

* $y = \frac{2}{3}x - \frac{1}{3} \Rightarrow m = 2/3 \Rightarrow \text{the slope of the required line is } -\frac{3}{2}$.

* $\therefore y - 3 = -\frac{3}{2}(x - 1)$ (5 pts)
 $\Rightarrow y = \frac{3}{2}x + \frac{9}{2}$

Question 5 The graph of the equation $y = |x - 2| + 1$ is reflected across the y -axis, then shifted 2 units left, then shifted 1 unit down. The equation of the new graph is $y = |ax + b| + c$. Find the value of $a + b + c$

* $y_1 = |-x - 2| + 1$

* $y_2 = |-(x + 2) - 2| + 1$
 $= |-x - 4| + 1$

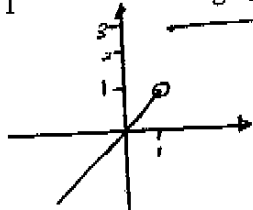
* $y_{\text{new}} = |-x - 4| + 1 - 2$
 $= |-x - 4| - 1$

$\therefore a = -1$
 $b = -4$
 $c = -1$ (4 pts)

so, $a + b + c = -6$
 or 5

Question 6

a) If $g(x) = \begin{cases} x & \text{if } x < 1 \\ 3 & \text{if } x \geq 1 \end{cases}$, then find the range of $g(x)$.



$R = (-\infty, 1) \cup \{3\}$ (3 pts)

b) If $h(x) = [x]$, where $[x]$ is the greatest integer of less than or equal to x , then find the value of $h(-0.5) + h^2(\sqrt{8})$.

sol. $h(-0.5) + [h(\sqrt{8})]^2 = [-0.5] + [2.828]^2$ (3 pts)
 $= -1 + 4 = 3$

Question 8 If $f(x) = x^2 + 1$ and $g(x) = |x| - 4$, find:

a) $\frac{f(1+h) - f(1)}{h}$

Sol: $\frac{(1+h)^2 + 1 - 2}{h} = \frac{1 + 2h + h^2 + 1 - 2}{h} = \frac{h(2+h)}{h} = 2+h$.
3 pts

b) $(f \circ g)(2) + (g \circ f)(-1)$

Sol: $f(g(2)) + g(f(-1)) = f(-2) + g(-3) = 5 - 1 = 4$

c) The Domain of $\left(\frac{f}{g}\right)(x)$

$\left(\frac{f}{g}\right)(x) = \frac{x^2 + 1}{\sqrt{|x| - 4}}$

$|x| - 4 > 0 \Rightarrow x < -4 \text{ or } x > 4$

$\therefore D_{\frac{f}{g}} = (-\infty, -4) \cup (4, \infty)$

(3 pts)

Question 9 If the quadratic function $f(x) = 2x^2 + bx + c$ decreases on $(-\infty, 2]$ and has x-intercept $(3, 0)$ find the value of $b + c$

Sol.

* $h = \frac{-b}{2a} = 2 \Rightarrow \frac{-b}{4} = 2 \Rightarrow \boxed{b = -8}$

(5 pts)

* $(3, 0)$ on the graph

$2(3)^2 - 8(3) + c = 0$

$\Rightarrow 18 - 24 + c = 0$

$\Rightarrow \boxed{c = 6}$

$-8 + 6 = -2$

(2 pts)

Question 3

Identify the equation or the set of ordered pair that define y as a function of x .

a) $\{(2,5), (-4,3), (-2,5)\}$

function (1 pt)

b) $|x| + y^3 = 1$

Sol.:

$$y^3 = 1 - |x|$$

(2 pts)

$$y = \sqrt[3]{1 - |x|}$$

\Rightarrow function