King Fahd University of Petroleum and Minerals
DIPLOMA Math Prep-Year Program
Math 003Term 042

TEST 1

NAME: KEY **SEC**_____ **ID** _____

SHOW ALL NECESSARY WORKING

1. Which **ONE** of the following is **TRUE**:

a) The equation $x^2 = x$ has one real solution

b) 5 is the solution of the equation $\frac{x}{x-5} = 1 - \frac{5}{x-5}$

c) $\frac{x}{x-2} = \frac{2}{x-2}$ *and* $2x-3=1$ are equivalent equations

d) The equation $3|x-1|+12=9$ has two real solutions

(*e*) The equation $\frac{1}{5}x^2 - \frac{2}{3}x + \frac{1}{3}$ has two real distinct roots

$b^2 - 4ac$

$\frac{4}{9} - 4\left(\frac{1}{5}\right)\left(\frac{1}{3}\right) = \frac{4}{9} - \frac{4}{15} > 0$

2 Solve $(x+1)^{2/3} = 5(x+1)^{1/3} - 6$

Let $y = (x+1)^{1/3}$

$y^2 - 5y + 6 = 0$

$(y-3)(y-2) = 0$

$y = 3$ OR $y = 2$

$(x+1)^{1/3} = 3$ $\quad$ $(x+1)^{1/3} = 2$

$x+1 = 27$ $\quad$ $x+1 = 8$

$x = 26$ $\quad$ $x = 7$

S.S: $\{26, 7\}$

ANS: 26, 7

3 Solve $-2 < |x^2 - 1| < 6$

$|x^2-1| < 6$ and $|x^2-1| > -2$

$(-\infty, \infty)$

$x^2 - 1 < 6$ and $x^2 - 1 > -6$

$x^2 - 7 < 0$ $\quad x^2 + 5 > 0$

$(x-\sqrt{7})(x+\sqrt{7}) < 0$ $\quad (-\infty, \infty)$

$= | - | + | +$

$+ \quad -\sqrt{7} \quad - \quad \sqrt{7} \quad +$

$(-\sqrt{7}, \sqrt{7})$

ANS: $(-\sqrt{7}, \sqrt{7})$

4 Solve $4x^2 - 3x + 15 = x$ by completing the square

$4x^2 - 4x + 15 = 0$

$x^2 - x = \frac{-15}{4}$

$\left(x - \frac{1}{2}\right)^2 = \frac{-15}{4} + \frac{1}{4} = \frac{-14}{4}$

$\left(x - \frac{1}{2}\right)^2 = \frac{-14}{4}$

$x - \frac{1}{2} = \pm \frac{\sqrt{14}}{2} i$

$x = \frac{1}{2} \pm \frac{\sqrt{14}}{2} i$

ANS: $\frac{1}{2} \pm \frac{\sqrt{14}}{2} i$

5 Solve: $\frac{(x+1)^2 - 2x + 5}{x} \geq 5$

$\frac{x^2 + 2x + 1 - 2x + 5 - 5x}{x} \geq 0$

$\frac{x^2 - 5x + 6}{x} \geq 0$

$\frac{(x-2)(x-3)}{x} \geq 0$

$= | = | + | +$

$- \quad 0 \quad + \quad 2 \quad - \quad 3 \quad +$

ANS: $(0, 2] \cup [3, \infty)$

6. Given $Z = \dfrac{(1-i)^2}{\sqrt{-2}\sqrt{-8} - i^{203}}$. Find the conjugate

$$\frac{1-2i+i^2}{(\sqrt{2}i)(2\sqrt{2}i) - i^3} = \frac{-2i}{-4+i} \cdot \frac{-4-i}{-4-i}$$

$$= \frac{8i-2}{17} = -\frac{2}{17} + \frac{8}{17}i$$

The conjugate is $-\frac{2}{17} - \frac{8}{17}i$

ANS: $-\frac{2}{17} - \frac{8}{17}i$

7. Solve the equation $\sqrt{4x+1} - \sqrt{2x+4} = 1$

$$\left(\sqrt{4x+1}\right)^2 = \left(1 + \sqrt{2x+4}\right)^2$$

$$4x+1 = 1 + 2\sqrt{2x+4} + 2x + 4$$

$$2x - 4 = 2\sqrt{2x+4}$$

$$(x-2)^2 = \left(\sqrt{2x+4}\right)^2$$

$$x^2 - 4x + 4 = 2x + 4$$

$$x^2 - 6x = 0$$

$$x(x-6) = 0$$

$$x = 0 \quad \text{OR} \quad x = 6$$

Check:

$x = 0$ rejected.

$x = 6$ is a Solution.

ANS: 6