



2.6 ALGEBRA OF FUNCTIONS

Def.:

Let f and g be two functions.

$$\star (f + g)(x) = f(x) + g(x)$$

$$D_{f+g} = D_f \cap D_g$$

$$\star (f - g)(x) = f(x) - g(x)$$

$$D_{f-g} = D_f \cap D_g$$

$$\star (f \cdot g)(x) = f(x) \cdot g(x)$$

$$D_{fg} = D_f \cap D_g$$

$$\star \frac{f}{g}(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$$

$$D_{\frac{f}{g}} = D_f \cap D_g \quad \text{except} \\ \{x \mid g(x) = 0\}$$

Ex1:

If $f(x) = 2x - 1$ and $g(x) = x^2 + x - 2$, find:

a) $(f + g)(-2)$

b) $(f - g)(x)$

c) $(f \cdot g)(x)$

d) $\left(\frac{f}{g}\right)(x)$

e) Domain of $f \cdot g$, $f + g$ and f / g

Ex2:

Find the domain of $(f + g)$ and $\left(\frac{f}{g}\right)$ where

$$(i) f(x) = \frac{1}{x-4} \quad \text{and} \quad g(x) = \sqrt{x-3}$$

$$(ii) f(x) = \sqrt{16-x^2} \quad \text{and} \quad g(x) = x^2 + 4x$$

Difference Quotient

$$\frac{f(x+h) - f(x)}{h}$$

Ex3: Find the difference quotient of $f(x) = 2x - x^2$

Solution

$$\frac{f(x+h) - f(x)}{h} = \frac{[2(x+h) - (x+h)^2] - [2x - x^2]}{h}$$

substitute

$$= \frac{2x + 2h - x^2 - 2hx - h^2 - 2x + x^2}{h}$$

Expand and Simplify

$$= \frac{2h - 2hx - h^2}{h} = \frac{h(2 - 2x - h)}{h}$$

Takeout common factor

$$= 2 - 2x - h$$

Ex4: Find the difference quotient of the following functions

$$1) f(x) = \frac{x}{2x-1}$$

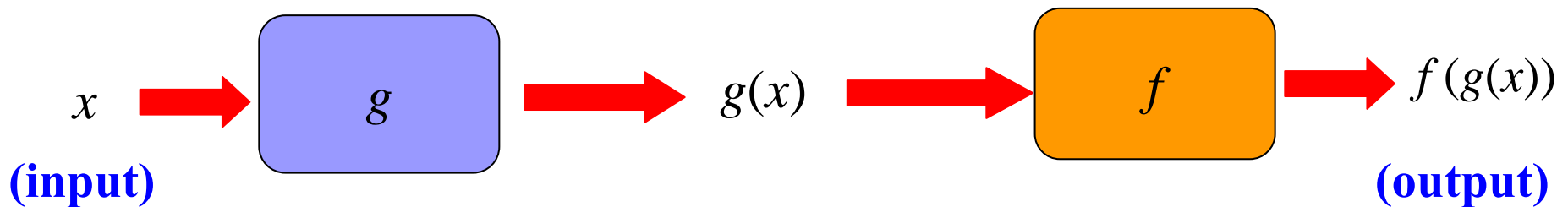
$$2) f(x) = \sqrt{x-1}$$

Composition of Functions (تركيب الدوال)

Let f and g be two functions.

Then we define the composition of the two functions by

$$(f \circ g)(x) = f[g(x)], \quad \text{Where } g(x) \in D_f \text{ for all } x \in D_g$$



Ex4

If $f(x) = x^2 - 3$ and $g(x) = \sqrt{x+1}$, find

a) $(f \circ g)(x)$

b) $(g \circ f)(x)$

c) $(f \circ f)(x)$

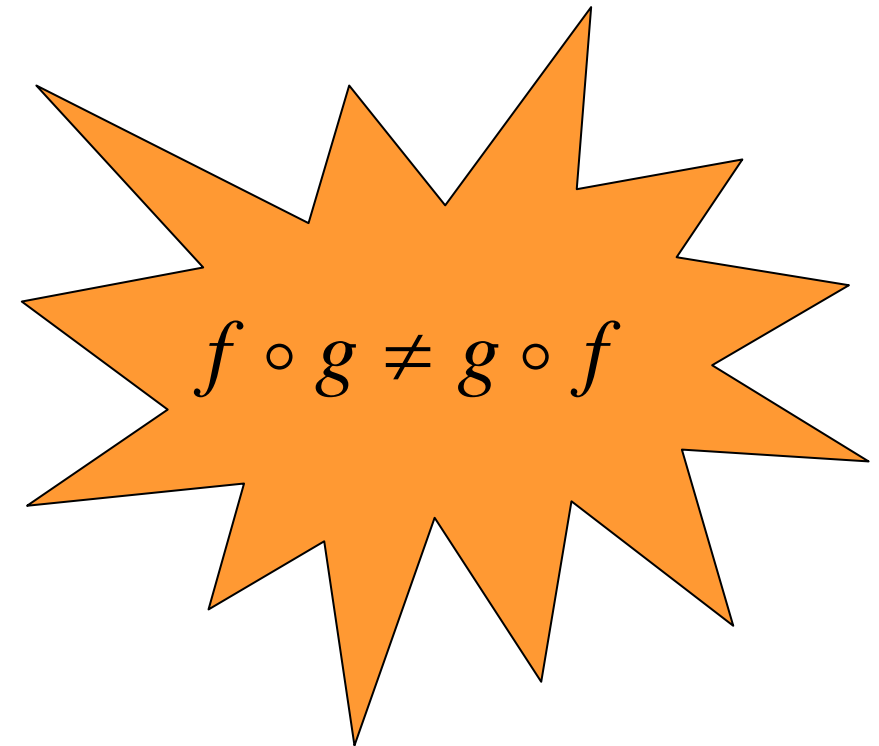
d) $(g \circ g)(x)$

e) $(f \circ g \circ f)(x)$

Solution

$$\begin{aligned} a) \quad (f \circ g)(x) &= f[g(x)] \\ &= f(\sqrt{x+1}) \\ &= (\sqrt{x+1})^2 - 3 \\ &= x - 2 \end{aligned}$$

$$\begin{aligned} b) \quad (g \circ f)(x) &= g[f(x)] \\ &= g(x^2 - 3) \\ &= \sqrt{x^2 - 3} + 1 \\ &= \sqrt{x^2 - 2} \end{aligned}$$



$$\begin{aligned}c) \quad (f \circ f)(x) &= f[f(x)] \\ &= f(x^2 - 3) \\ &= (x^2 - 3)^2 - 3 \\ &= x^4 - 6x^2 + 6\end{aligned}$$

$$\begin{aligned}d) \quad (g \circ g)(x) &= g[g(x)] \\ &= g(\sqrt{x+1}) \\ &= \sqrt{\sqrt{x+1} + 1}\end{aligned}$$

$$\begin{aligned}e) \quad (f \circ g \circ f)(x) &= (f \circ g)[f(x)] \\ &= (f \circ g)(x^2 - 3) \\ &= f[g(x^2 - 3)] \\ &= f(\sqrt{x^2 - 2}) \\ &= x^2 - 3 - 2 \\ &= x^2 - 5\end{aligned}$$

Ex 5

If $f(x) = 2 + 3x - x^2$ and $g(x) = 2x - 1$, find

a) $(fg)(x)$

b) $(f \circ g)(x)$

c) $(g \circ f)(x)$

Ex 6

If $f(x) = 2x + 5$, $g(x) = 5x + 5k$ and $(f \circ g)(2) = 10$, then find k .

Ex 7

If $f(x) = 3x + 2$ and $(f \circ h)(x) = 5x - 1$, then find $h(x)$.

Ex8

Given $f(x) = x^2 + x$ and $g(x) = x + k$, find all values of k such that $(f \circ g)(3) = (g \circ f)(3)$