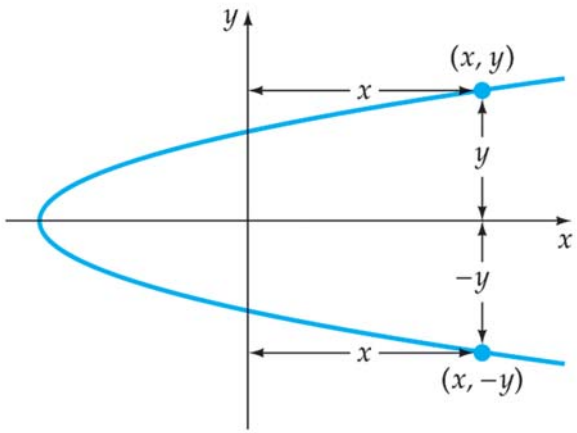
2.5 PROPERTIES OF GRAPHS

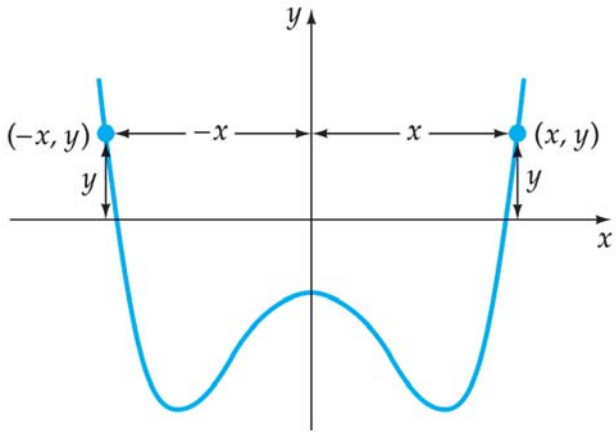
I) Symmetry (التماثل)

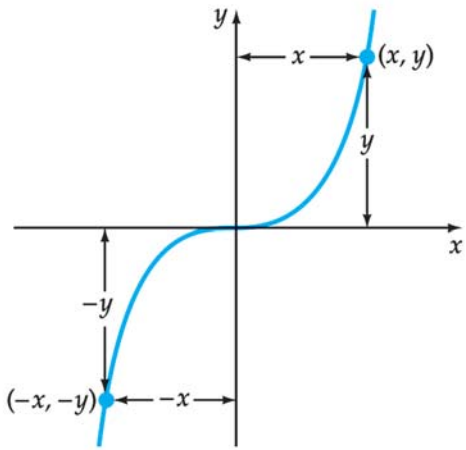
Symmetry with Respect to the *x*-axis: If (x,y) on the graph, then $(x, -y)$ is also in the graph

Symmetry with Respect to the *y*-axis: If (x,y) on the graph, then $(-x, y)$ is also in the graph

Symmetry with Respect to the origin: If (x,y) on the graph, then $(-x, -y)$ is also in the graph







Ex1: Determine whether the graph of each equation is symmetric with respect to the a. *x*-axis, b. *y*-axis, c. origin

1) $y = x^2 + 1$

2) $xy = 4$

3) $|x| - |y| = 6$

4) $x^2 = |x - y^2|$

5) $y = \dfrac{x}{|x|}$

Ex2: Find the image of the point *R*(-2, 3) with respect to the *x*-axis, *y*-axis, and the origin

II) Even and Odd Functions

A function f is **even** if $f(-x) = f(x)$ for every x in the domain of f.

A function f is **odd** if $f(-x) = -f(x)$ for every x in the domain of f.

Ex3: Determine each function is even, odd , or neither.

1) $f(x) = x^2$

Solution

$f(-x) = (-x)^2 = x^2 = f(x) \Rightarrow$ $f(x)$ is even

2) $g(x) = x^3$

Solution

$g(-x) = (-x)^3 = -x^3 = -\ g(x) \Rightarrow g(x)$ is odd

3) $h(x) = x^3 + x^2$

Solution

$h(-x) = (-x)^3 + (-x)^2 = -x^3 + x^2 \Rightarrow g(x)$ is neither even nor odd

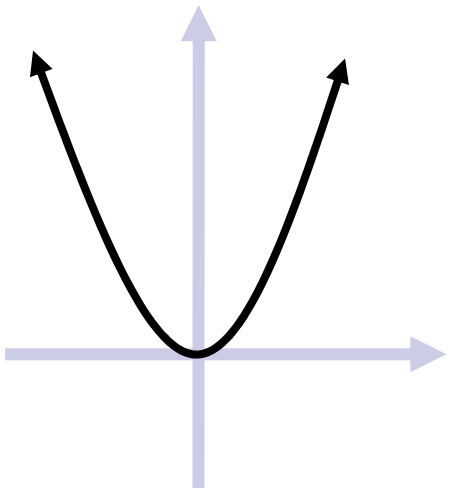
4) $k(x) = 5$ even

5) $f(x) = \dfrac{x^3}{x^2 + 1}$ odd

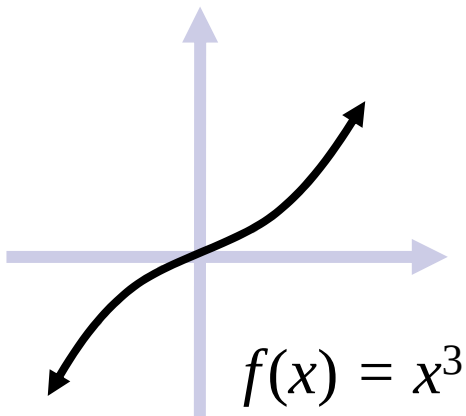
6) $f(x) = \dfrac{x^4}{\sqrt[5]{x^3 - x}}$ odd

Notes:

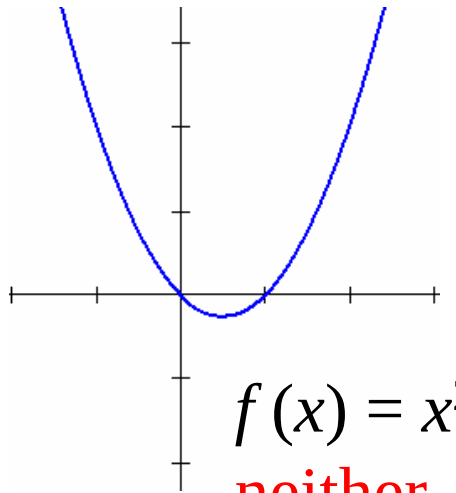
- If the graph of an even function is symmetric with respect to with respect to the y-axis.
- If the graph of an odd function is symmetric with respect to with respect to the origin.
- If the graph is not symmetric with respect to the y-axis or the origin, then function is neither even nor odd



$f(x) = x^2$ (even)



$f(x) = x^3$
(odd)



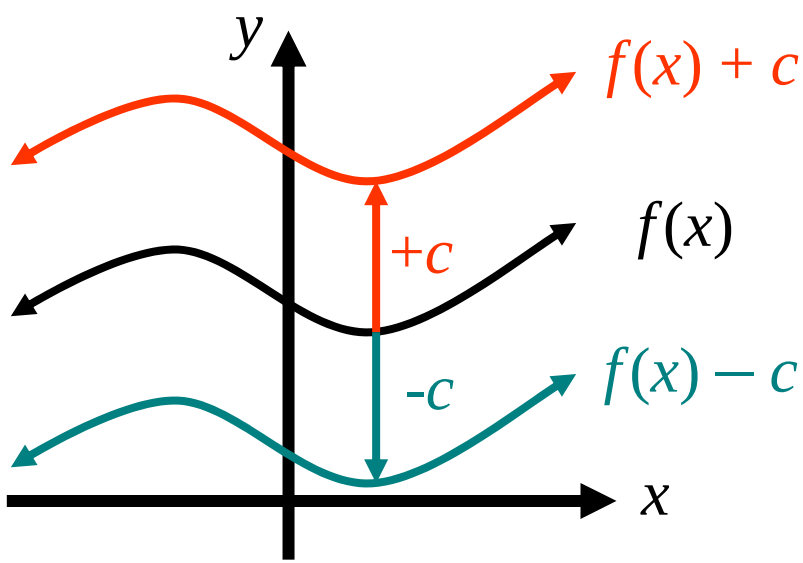
$f(x) = x^2 - x$
neither

III) Translations (الإزاحة) of Graphs

I) Vertical Translations

•If c is a positive real number, the graph of $f(x) + c$ is the graph of $y = f(x)$ **shifted upward** c units.

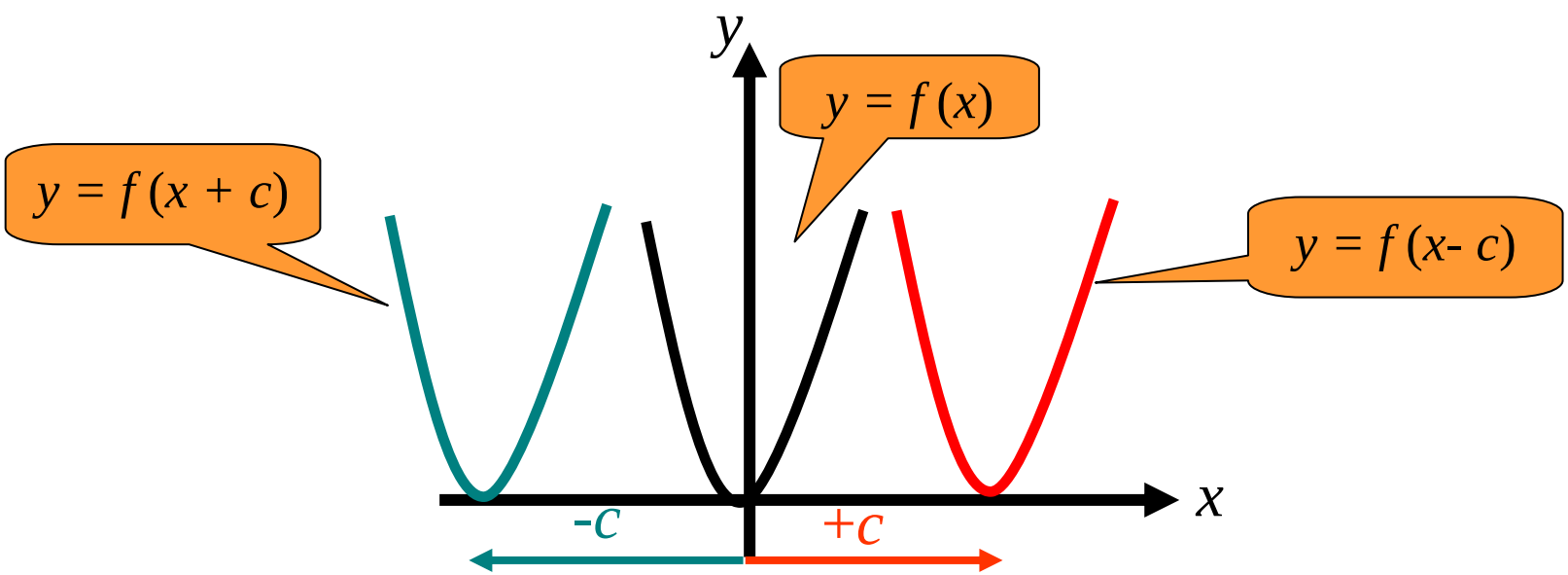
•If c is a positive real number, the graph of $f(x) - c$ is the graph of $y = f(x)$ **shifted downward** c units.



II) Horizontal Translations

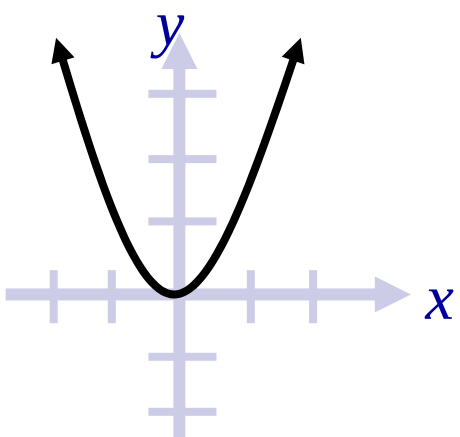
•If c is a positive real number, then the graph of $f(x - c)$ is the graph of $y = f(x)$ **shifted to the right** c units.

•If c is a positive real number, then the graph of $f(x + c)$ is the graph of $y = f(x)$ **shifted to the left** c units.

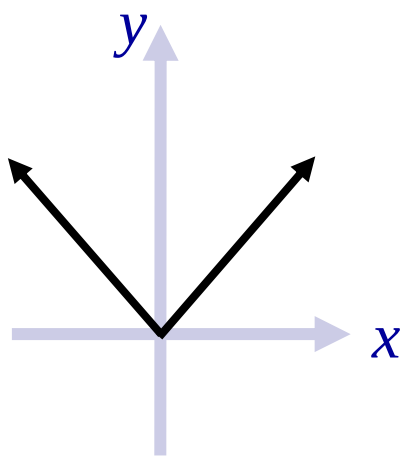


Basic Graphs:

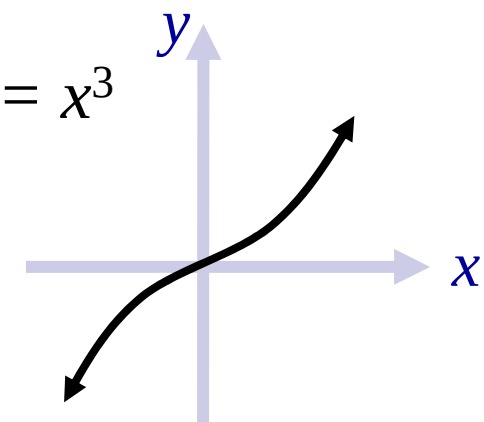
1) $y = x^2$



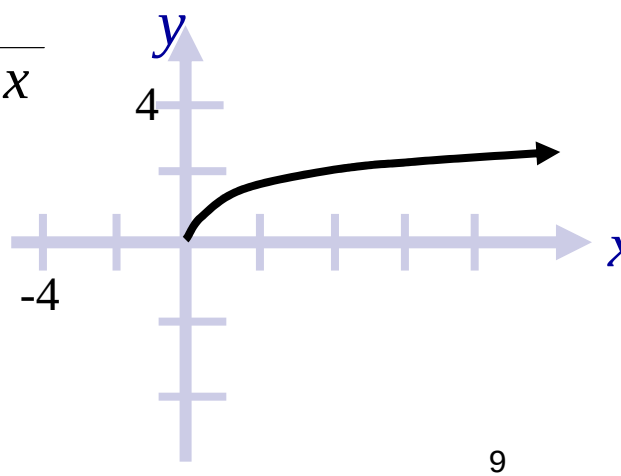
2) $f(x) = |x|$



3) $f(x) = x^3$



4) $y = \sqrt{x}$



Ex4: Sketch the graph of the following functions

1) $y = x^2$

2) $y = x^2 + 1$

3) $y = x^2 - 1$

4) $y = (x+1)^2$

5) $y = (x-1)^2$

6) $y = (x-1)^2 + 2$

7) $y = (x+1)^2 + 2$

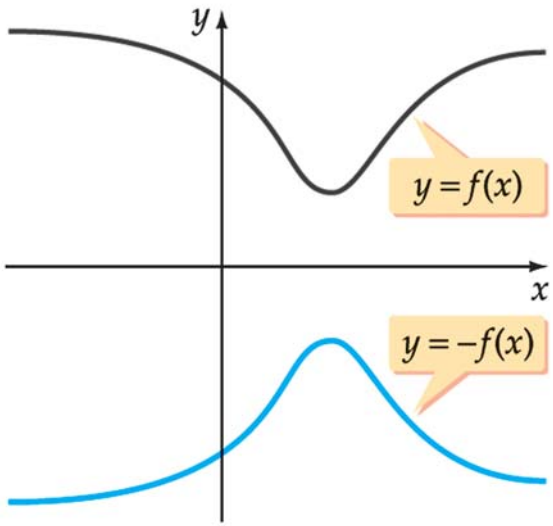
Ex5: Find the range of the function $f(x) = |x+1| - 3$

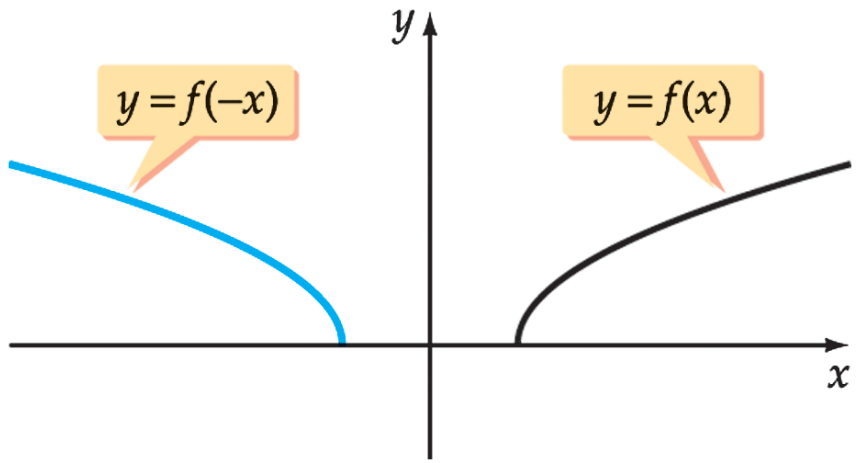
Ex6: If the graph of the function $y = \dfrac{x-1}{x+3}$ is shifted horizontally two units to the left and vertically three units up, hen find the equation of the new graph.

IV) Reflections (الإنعاس) of Graphs

•The graph of the function $y = -f(x)$ is the graph of $y = f(x)$ **reflected across the *x*-axis**.

• The graph of the function $y = f(-x)$ is the graph of $y=f\,x)$ **reflected across the *y*-axis**.





Ex7: Sketch the graph of the following functions:

1) $y = -\sqrt{x}$

2) $y = \sqrt{-x}$

3) $y = -\sqrt{x} + 1$

4) $y = -\sqrt{x+1}$

5) $y = \sqrt{-x} + 1$

6) $y = \sqrt{-x+1}$

7) $y = -\sqrt{x-1} + 2$

8) $y = \sqrt{-x+1} - 2$

9) $y = -\sqrt{-x+1} + 2$

Note: I) If $(x, y) \in f(x)$ then $\left(\frac{1}{a}x, y\right) \in f(ax)$ and $(x, ay) \in af(x)$

Ex: If the point (-1, 2) lies on the graph of $y = f(x)$, then find the image of this point on the graph of the following functions:

1) $y = 2f(x)$

2) $y = f(2x)$

3) $y = 2f\left(\frac{1}{3}x\right)$

4) $y = 3f(x) + 1$

5) $y = -3f(2x) + 2$

V) Stretching (التمدد) and Compressing (التقلص) of Graphs

1) Vertical Stretching and Compressing

If $c > 1$ then the graph of $y = c\,f(x)$ is the graph of $y = f(x)$ **stretched vertically by *c***.

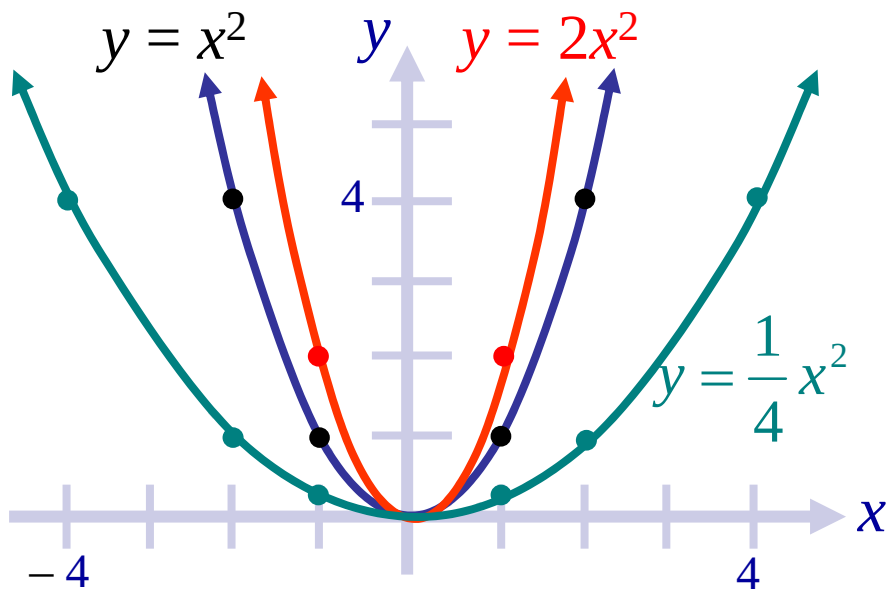
If $0 < c < 1$ then the graph of $y = c\,f(x)$ is the graph of $y = f(x)$ **shrunk vertically by *c***.

Example:

$y=2x^2$ is the graph of $y = x^2$

stretched vertically by 2.

$y = \frac{1}{4}x^2$ is the graph of $y = x^2$ *compressed vertically* by $\frac{1}{4}$.

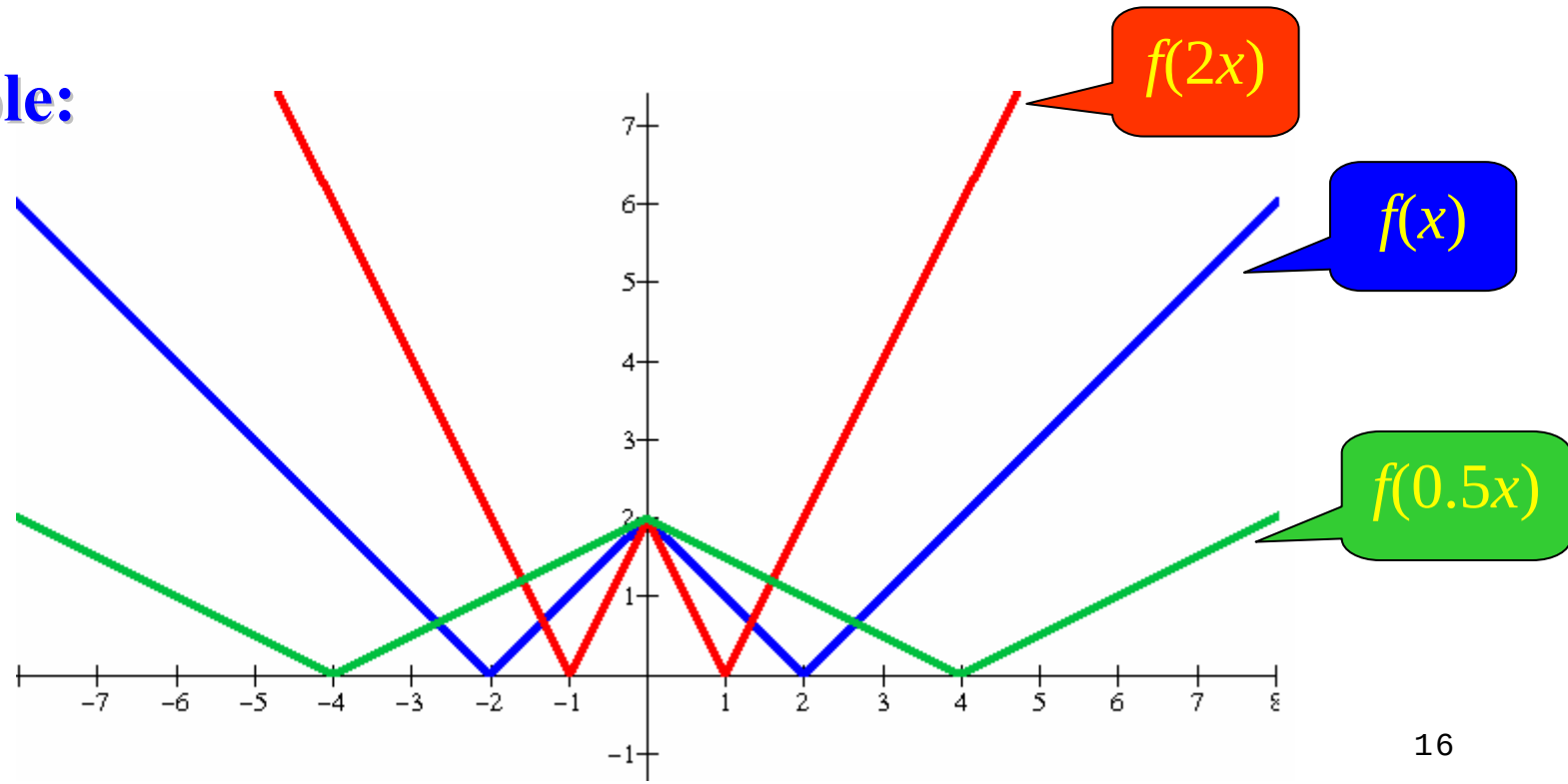


2) Horizontal Compressing and Stretching

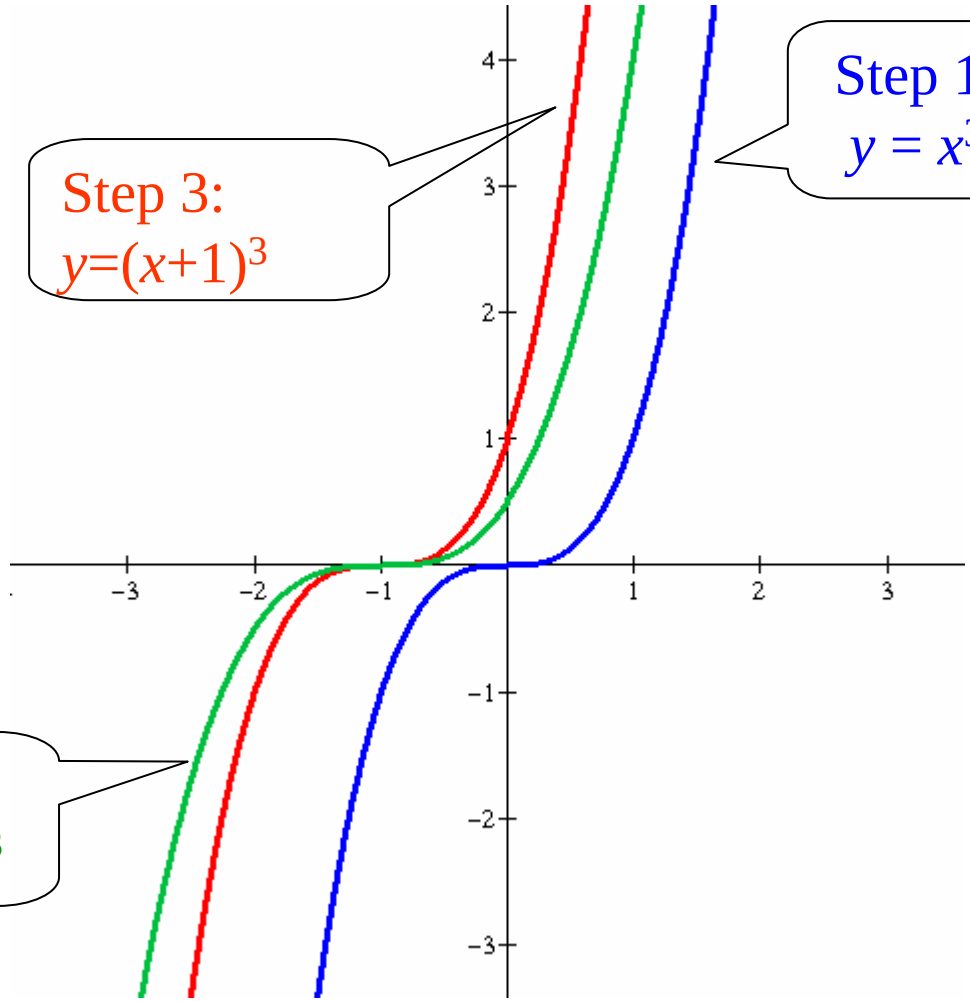
If $c > 1$, the graph of $y = f(cx)$ is the graph of $y = f(x)$ **compressed horizontally by *c*.**

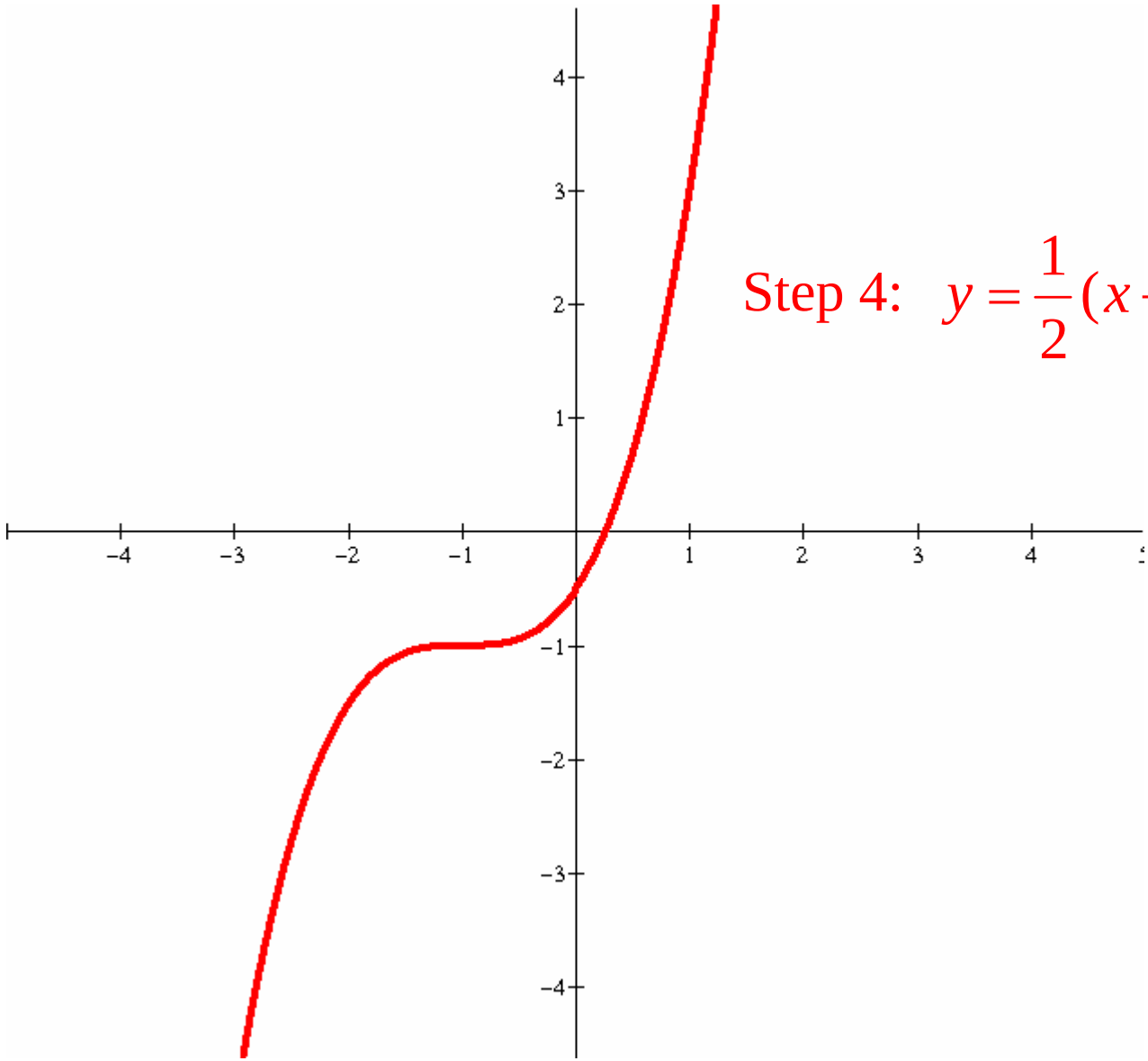
If $0 < c < 1$, the graph of $y = f(cx)$ is the graph of $y = f(x)$ **stretched horizontally by *c*.**

Example:



Ex8: Graph $y = \frac{1}{2}(x+1)^3 - 1$ using the graph of $y = x^3$.





Step 4: $y = \frac{1}{2}(x+1)^3 - 1$