



2.4 QUADRATIC FUNCTIONS

Def.:

- Let a , b , and c be real numbers $a \neq 0$. The function

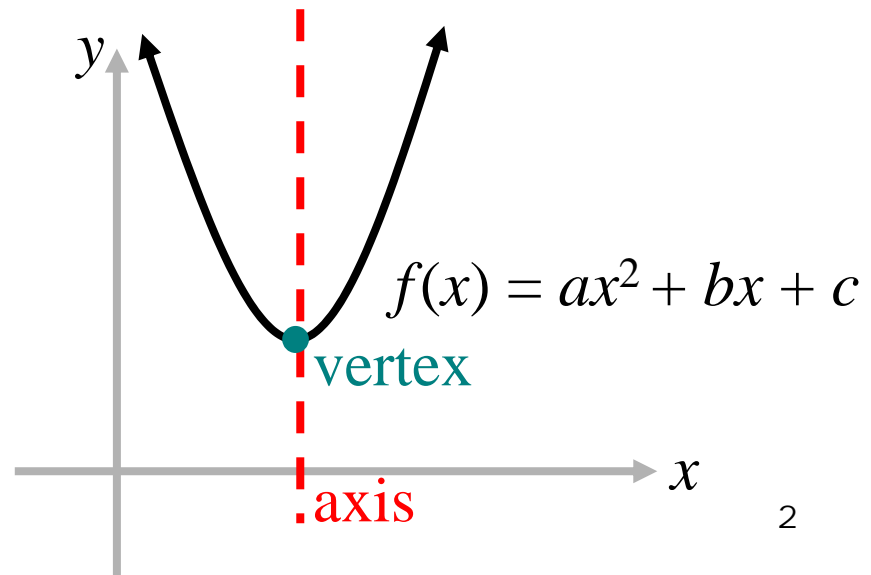
$$f(x) = ax^2 + bx + c$$

is called a **quadratic function**.

- The graph of a quadratic function is a **parabola** (قطع مكافئ).

- Every parabola is symmetrical about a line called the **axis of symmetry** (محور التناظر).

- The intersection point of the parabola and the axis of symmetry is called the **vertex** (رأس) of the parabola.



Def:

1) The **standard form** for the equation of a quadratic function is:

$$f(x) = a(x - h)^2 + k \quad (a \neq 0)$$

where

$$h = -b/2a$$

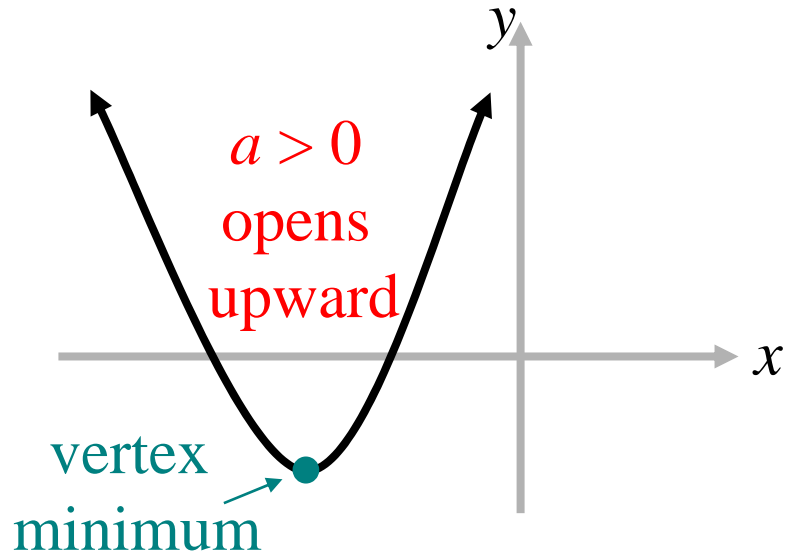
$$k = f(h) = f(-b/2a)$$

2) **Vertex** : (h, k)

3) **Axis of symmetry** : $x = h$

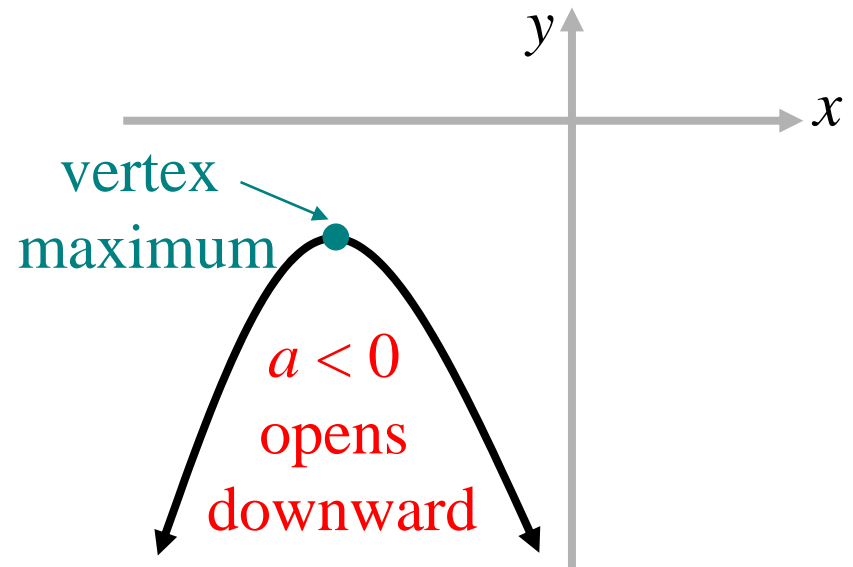
Case1: $a > 0$

- Minimum value: k
- Range: $[k, \infty)$
- Increasing: $[h, \infty)$
- Decreasing: $(-\infty, h]$



Case2: $a < 0$

- Maximum value: k
- Range: $(-\infty, k]$
- Increasing: $(-\infty, h]$
- Decreasing: $[h, \infty)$



Ex1:

For the following functions

1. Write the function in the standard form
2. Find the vertex
3. Find the axis of symmetry
4. Find , if any, the maximum value of the function
5. Find , if any, the minimum value of the function
6. Find the range of the function
7. Find the interval(s) of increasing and decreasing
8. Sketch the graph of the function and show on the graph the intercept(s), the vertex, and the axis of symmetry

$$a) f(x) = 2x^2 + 4x + 3$$

Solution

$$1. \quad h = -\frac{b}{2a} = \frac{-4}{2(2)} = -1$$

$$k = f(h) = f(-1) = 1$$

$$\begin{aligned} \text{The standard form is: } f(x) &= a(x - h)^2 + k \\ &= 2(x + 1)^2 + 1 \end{aligned}$$

2. The vertex is $(h, k) = (-1, 1)$

3. The axis of symmetry: $x = h$

$$x = -1$$

4. $f(x)$ has no maximum value.

5. The minimum value of is $k = 1$.

6. The range = $[k, \infty) = [1, \infty)$

7. $f(x)$ is increasing on $[h, \infty) = [-1, \infty)$

$f(x)$ is decreasing on $[-\infty, h) = [-\infty, -1)$

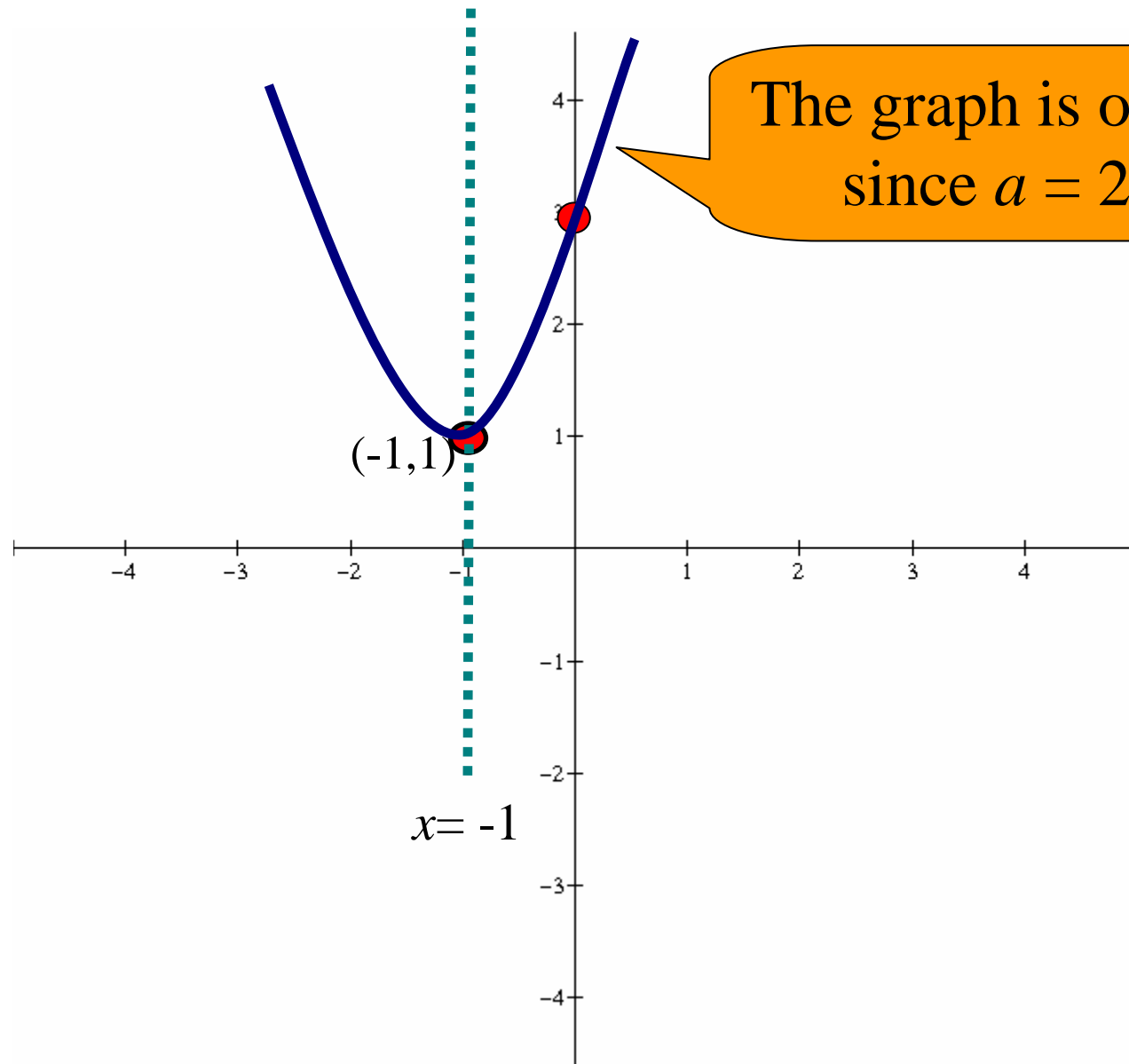
8. x -intercept:

$$y = 0 \Rightarrow 2x^2 + 4x + 3 = 0 \Rightarrow \text{no solution} \\ \Rightarrow \text{no } x - \text{intercept}$$

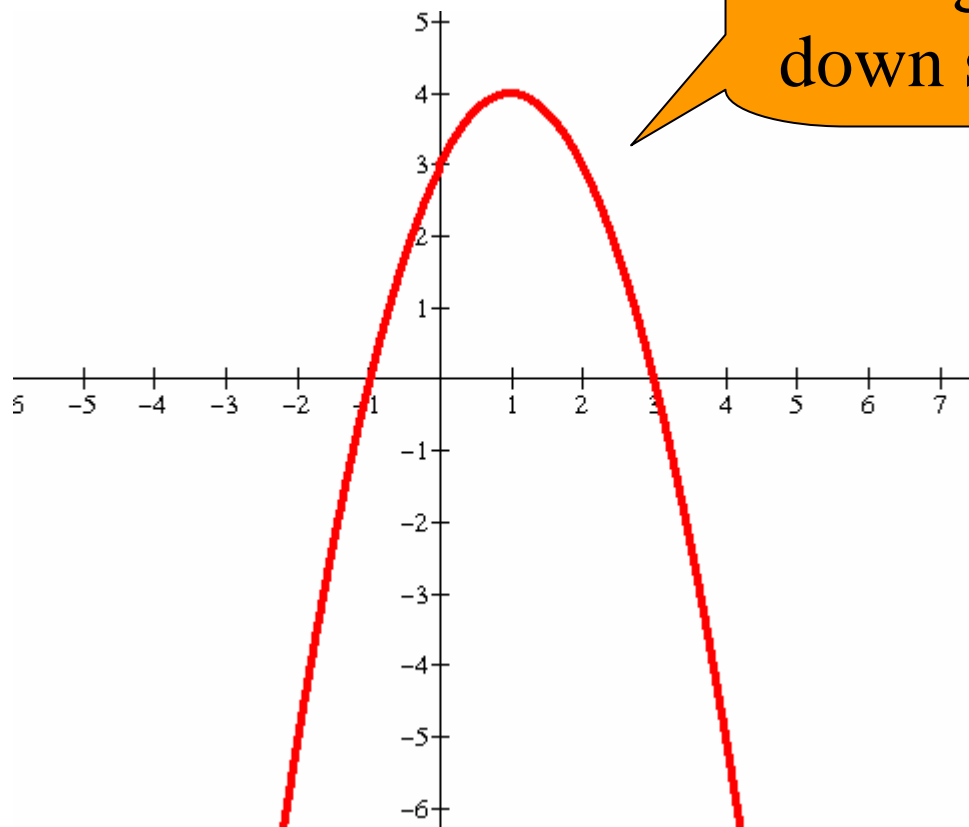
y -intercept:

$$x = 0 \Rightarrow y = 3 \Rightarrow \text{the } y\text{-intercept is } (0, 3)$$

Hence the graph of $f(x)$ is



b) $g(x) = -x^2 + 2x + 3$



The graph is open down since $a = -1 < 0$

Note:

We can find the standard form for any quadratic function by using the idea of completing the square

Ex2:

Use the idea of completing the square to write the function $f(x) = 2x^2 + 4x - 1$ in the standard form.

Solution

$$\begin{aligned} f(x) &= (2x^2 + 4x) - 1 \\ &= 2(x^2 + 2x) - 1 \\ &= 2(x^2 + 2x + 1 - 1) - 1 \\ &= 2(x^2 + 2x + 1) - 2 - 1 \end{aligned}$$

The standard form of $f(x)$ is

$$f(x) = 2(x + 1)^2 - 3$$

Ex3: A basketball is thrown from the free throw line from a height of six feet. What is the maximum height of the ball if the path of the ball is: $-3x^2 + 4x + 2$

Ex4: Find the quadratic function of x whose graph has a minimum at $(2, 1)$ and passes through the point $(0, 4)$

Ex5: Find two numbers whose sum is 8 and whose product is a maximum.

Ex6: If x is a real number, then find the maximum area of a rectangle of length $3+2x$ and width $1-2x$.

Ex7: If $x=3$ is the axis of symmetry of the parabola $y=-2x^2+cx+2$, then find c