



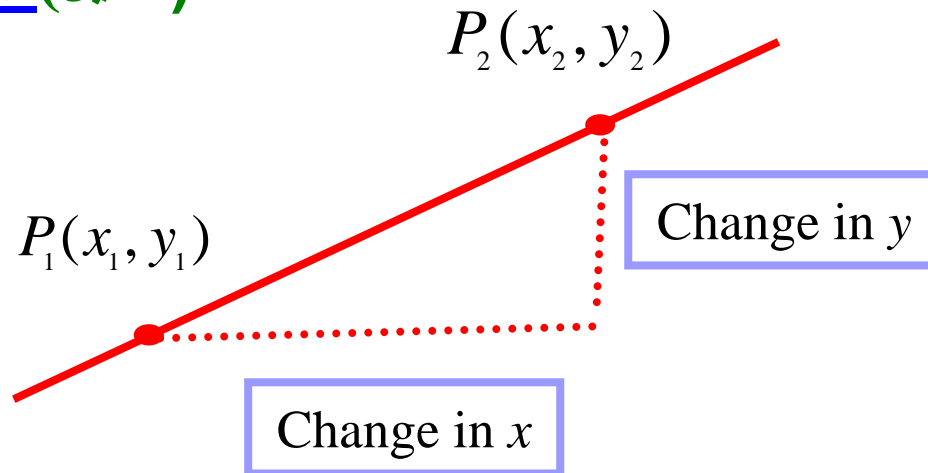
2.3 LINEAR FUNCTIONS



Def:

A function of the form $f(x) = mx + b$, $m \neq 0$ where m and b are real numbers is called a linear function

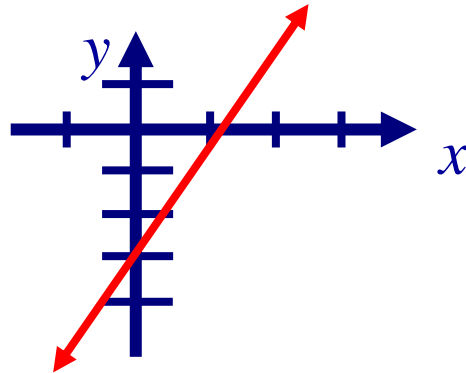
• Slopes of Lines (الميل)



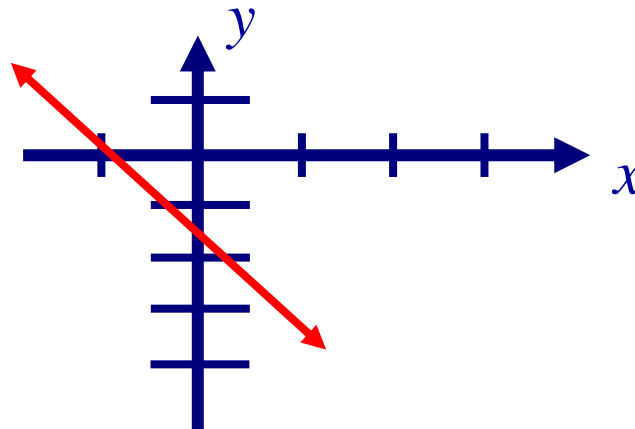
$$\text{The slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Cases:

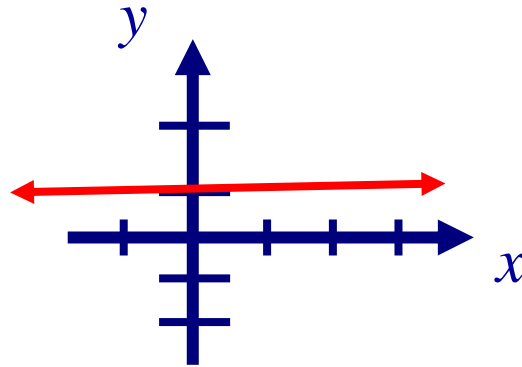
1) If m is positive, then the line is increasing



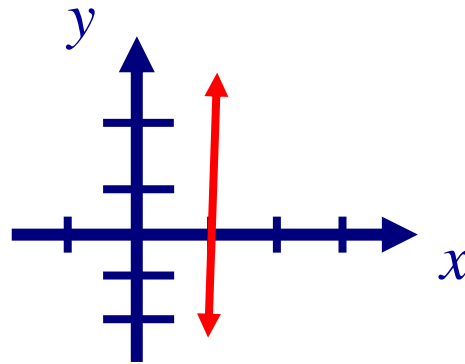
2) If m is negative, then the line is decreasing



3) If $m = 0$, then the line is **horizontal** (أفقي)



4) If m is undefined, then the line is **vertical** (عمودي)





Ex: Find the slope of the lines in the following cases:

- 1) Passing through the points $(3, 4)$ and $(5, 10)$
- 2) Passing through the points $(-3, 4)$ and $(1, -2)$
- 3) Passing through the points $(-2, 4)$ and $(1, 4)$
- 4) Passing through the points $(2, 4)$ and $(2, 10)$

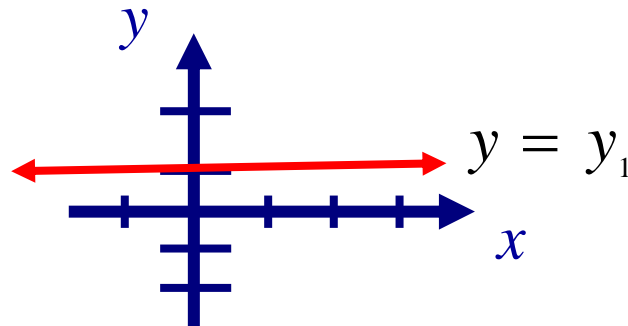
The Equation of a Line

Case1: Point-slope form

The equation of a line with slope m and passing through the point (x_1, y_1) is given by: $y - y_1 = m(x - x_1)$

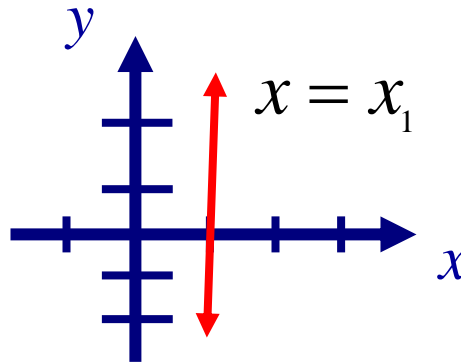
Case2:

The equation of horizontal line (slope =0) is give by $y = y_1$



Case3:

The equation of horizontal line (slope is undefined) is given by $x = x_1$



Case4: Slope-Intercept Form

The equation of a line with slope m and y -intercept $(0, b)$ is given by $y = mx + b$

Important Note:

If $y = mx + b$, then the slope is the coefficient of x

Ex2: Find the equation of the straight line in the following cases. Write the answer in the form $y=mx+b$

1) has slope $\frac{2}{3}$ and passes through the point $(-3, 4)$

2) passes through the points $(5, -6)$ and $(2, -8)$

3) passes through the points $(-1, 2)$ and $(3, 2)$

4) has undefined slope and passes through the point $(-2, 3)$

Parallel (متوازي) and Perpendicular (متعامد) Lines

Consider the two lines

$$l_1 : m_1 x + b \quad \text{AND} \quad l_2 : m_2 x + b$$

1) l_1 and l_2 are **parallel** if and only if $m_1 = m_2$

2) l_1 and l_2 are **perpendicular** if and only if $m_1 = -\frac{1}{m_2}$
[OR $m_1 \cdot m_2 = -1$]

Ex3: Determine whether the two lines

$$2x + 3y = 1 \quad \text{AND} \quad 3x - 2y = 4$$

are parallel or perpendicular.

Ex4: Find the equation of a line that passes through the point $(1, 3)$ and parallel to the line $3x + 4y = -24$

Ex5: Find the equation of a line that passes through the point $(-3, 4)$ and perpendicular to the line $2x - y = 7$

Ex6: If the two lines $3x-5y+1=0$ and $ky-2x+4=0$ are perpendicular, then find k .

Ex7: Find the value of x in the domain of $f(x)=4x-3$ for which $f(x)=-2$

Ex8: If $(2,3)$, $(4,2)$ and $(k,0)$ are three point on the same straight line , then find k .

Ex9: Find a point $P(x,y)$ of the graph of the equation $y = x^2$ Such that the slope of the line through the point $(3, 9)$ and P is $15/2$.

Point of Intersection (نقطة التقاطع)

To find the point of intersection between two lines solve the two equations simultaneously.

Ex8: Find the point of intersection between the two lines $2y - 3x = 12$ and $y + 2x + 1 = 0$.

Solution

$$y + 2x + 1 = 0 \Rightarrow y = -2x - 1 \quad \text{Substitute in the other equation}$$

$$\Rightarrow 2(-2x - 1) - 3x = 12$$

$$-4x - 2 - 3x = 12$$

$$-7x = 14$$

$$x = -2$$

$$\therefore y = -2(-2) + 1 = 3$$

Hence, the point of intersection is $(-2, 3)$

The previous question can be also solved graphically

