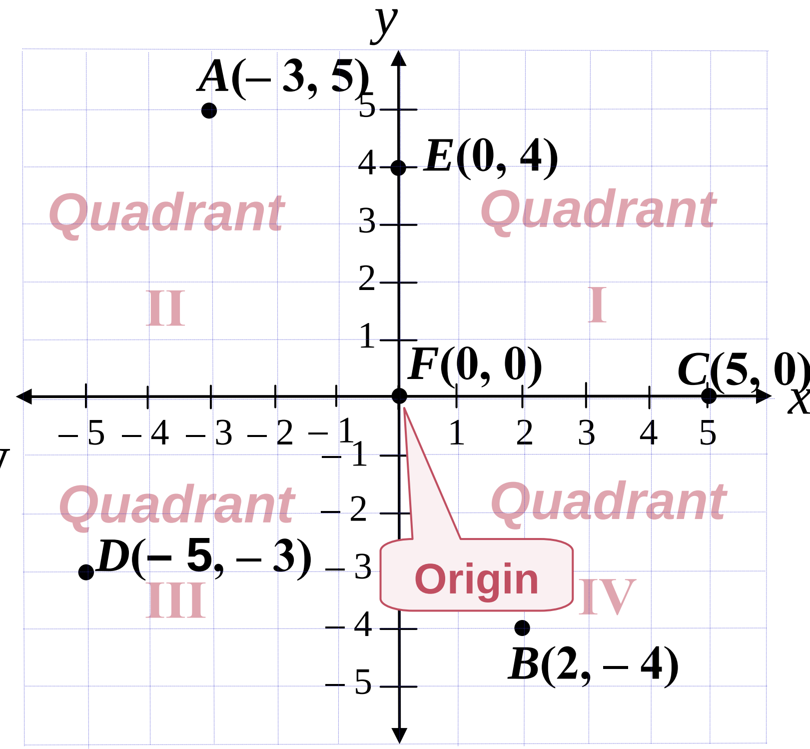
2.1 A TWO-DIMENSIONAL COORDINATE SYSTEM AND GRAPHS

Plotting Points in the Cartesian Coordinate System:الاحداثيات الكارتيزية

Plot: We move from the origin and plot points in the following way:



A(– 3, 5): 3 units left, 5 units up

B(2, – 4): 2 units right, 4 units down

C(5, 0): 5 units right, 0 units vertically

D(– 5,– 3): 5 units left, 3 units down

E(0, 4): 0 units horizontally, 4 units up

F(0, 0): 0 units horizontally, 0 units vertically

where we find the corresponding <u>ordered pairs</u> of real numbers (*x*,*y*).

The ordered Pairs (a,b) and (c,d) are equal

If and only if

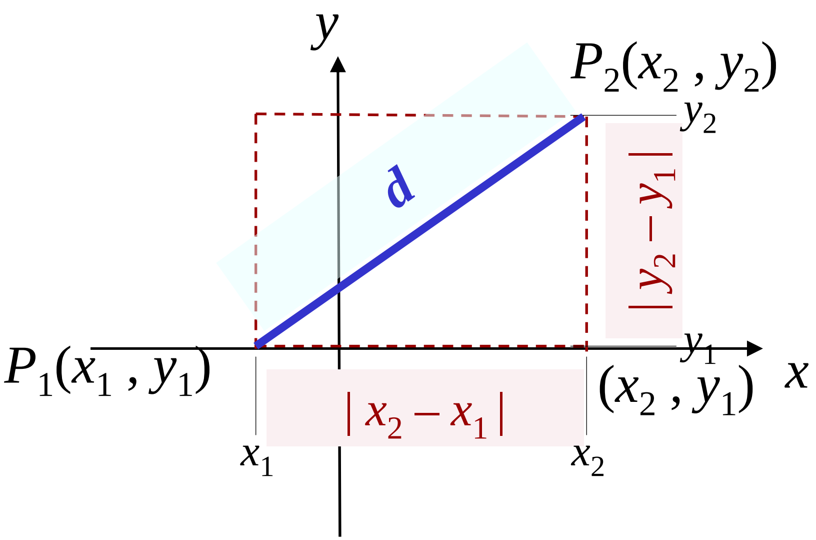
$a=c$ and $b=d$

For instance, $if\ \ (x,4)=(-2,y),\ then\ \ x=-2\ \ and\ \ y=4$

Distance Between Two Points

Using the Pythagorean Theorem, we can find the distance between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the Cartesian coordinate system.

Thus we can find the distance d by the Pythagorean Theorem.



$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

The Distance Formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$

Ex1. Find the distance between (– 1, – 3) and (2, 3).

Solution: Let $(x_1, y_1) = (-1, -3)$ and $(x_2, y_2) = (2, 3)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{[2-(-1)]^2 + [3-(-3)]^2}$$

$$d = \sqrt{(2+1)^2 + (3+3)^2}$$

$$d = \sqrt{3^2 + 6^2}$$

$$d = \sqrt{9+36}$$

$$d = \sqrt{45}$$

$$d = 3\sqrt{5} \approx 6.71$$

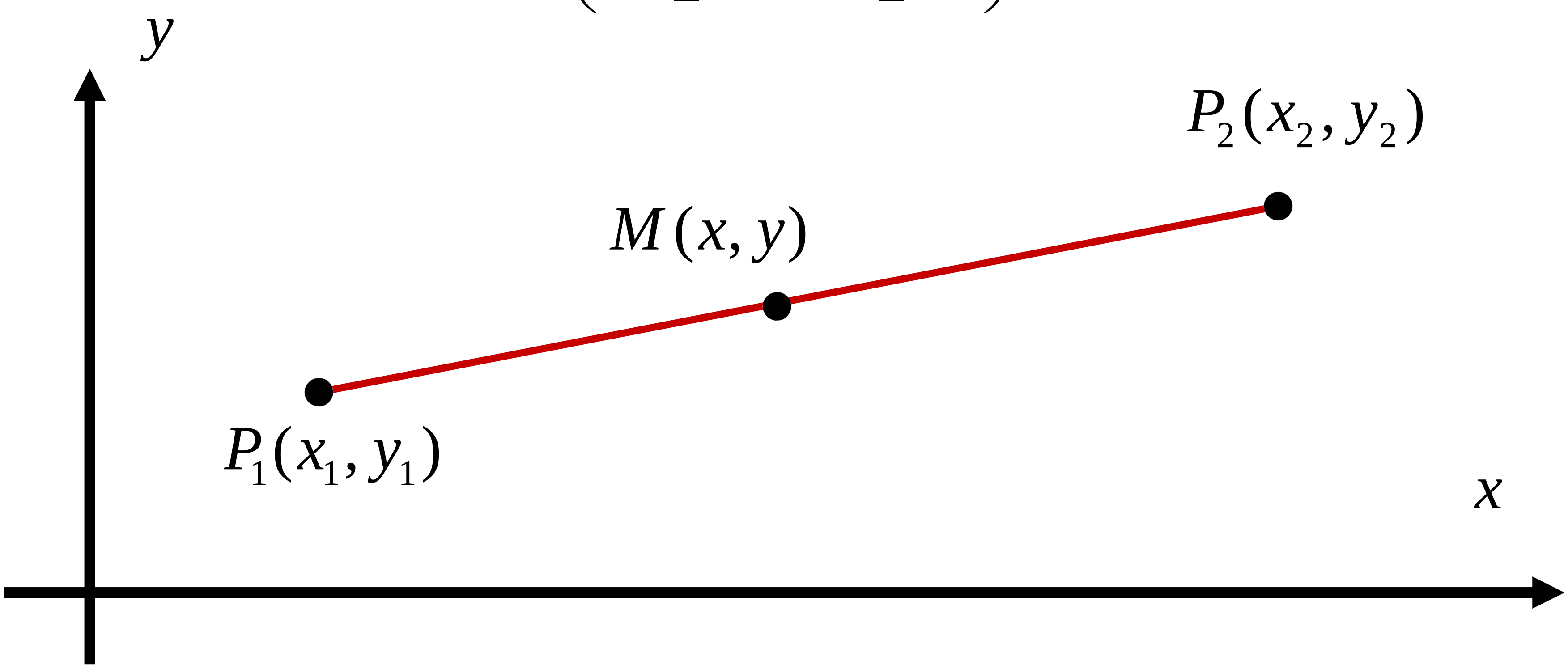
Ex. Find the distance between the points:

$(x, 4x)$ and $(-2x, 3x)$, where $x < 0$

The Midpoint Formula

The Midpoint M of the line segment from $P_1(x_1, y_1)$ to $P_2(x_2, y_2)$

Is given by: $$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$



Ex2. Find the midpoint of the line segment with endpoints (1, – 6) and (– 8, – 4).

Solution:

Let $(x_1, y_1) = (1, -6)$ and $(x_2, y_2) = (-8, -4)$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{1 + (-8)}{2}, \frac{-6 + (-4)}{2}\right)$$

$$= \left(\frac{-7}{2}, \frac{-10}{2}\right)$$

$$= \left(\frac{-7}{2}, -5\right)$$

Ex. Find the midpoint with segment with endpoints (6, – 3) and (6, 11).

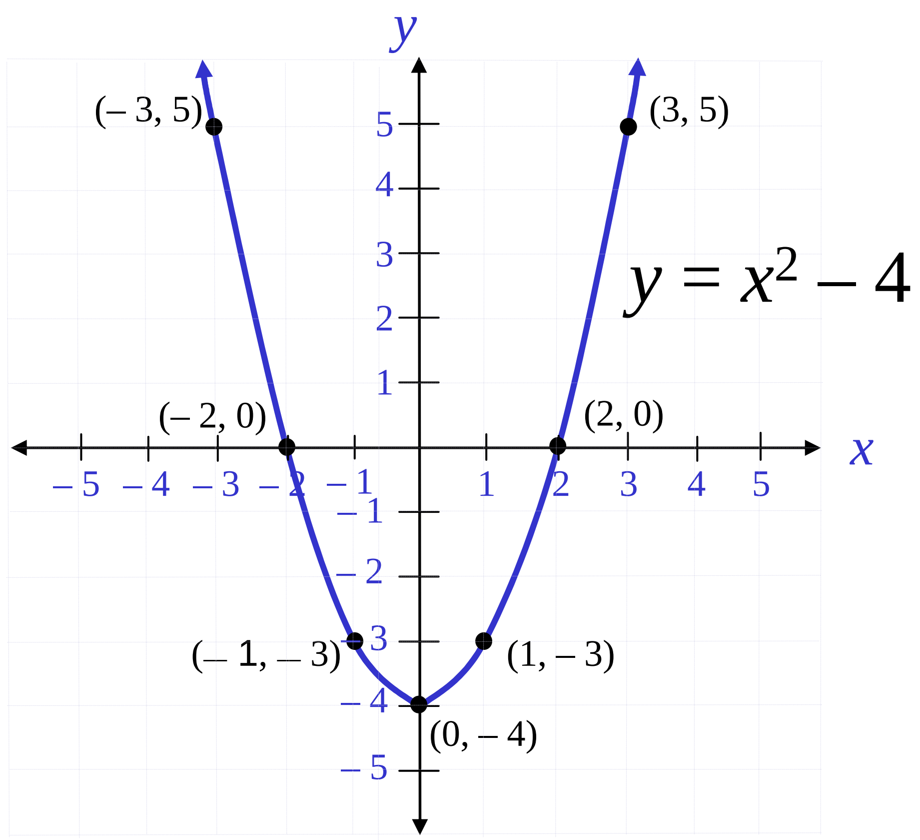
Graphing an Equation

The graph of an equation in two variables *x* and *y* is the set of all points whose coordinates satisfy the equation.

Ex3. Graph $y = x^2 - 4$

Solution: Select integers for *x*:

x	y	$= x^2 - 4$
– 3	5	$(-3)^2 - 4 = 5$
– 2	0	$(-2)^2 - 4 = 0$
– 1	–3	$(-1)^2 - 4 = -3$
0	–4	$(0)^2 - 4 = -4$
1	–3	$(1)^2 - 4 = -3$
2	0	$(2)^2 - 4 = 0$
3	5	$(3)^2 - 4 = 5$



Ex. Graph the equation $2y+2=|x-1|$

(Hint: locate the breaking point)

Intercepts

An ***x-intercept*** of a graph is an *x*-coordinate of the point where the graph intersects the *x*-axis.

- If $(x_1,0)$ satisfies an equation, then the point $(x_1,0)$ is called an ***x-intercept*** of the equation.

The ***y-intercept*** of a graph is a y-coordinate of the point where the graph intersects the y-axis.

- If $(0,y_1)$ satisfies an equation, then the point $(0,y_1)$ is called an ***y-intercept*** of the equation.

Procedure for finding the *x*- and *y*- intercepts of the graph of an equation *algebraically*:

- To find the *x*-intercepts of the graph of an equation, substitute 0 for *y* in the equation and solve for *x*.
- To find the y-intercepts of the graph of an equation, substitute 0 for *x* in the equation and solve for y.

Ex.4 Find the x- and y-intercepts of the graph of $y = x^2 + 4x - 5$.

Solution:

- To find the x-intercepts, let $y = 0$ and solve for x.

$0 = x^2 + 4x - 5$		Substitute 0 for y.
$0 = (x - 1)(x + 5)$		Factor.
$x - 1 = 0$	$x + 5 = 0$	Set each factor equal to 0.
$x = 1$	$x = -5$	Solve for x.

So, the x-intercepts are (1, 0) and (–5, 0).

- To find the y-intercept, let $x = 0$ and solve for y.

$$y = 0^2 + 4(0) - 5 = -5$$

So, the y-intercept is (0, –5).

Ex.5 Find the x- and y-intercepts of the graph of $x = |y| - 2$

Solution:

- To find the x-intercepts, let $y = 0$ and solve for x.

$$x = 0 - 2 = -2$$

So, the x-intercept is (-2, 0)

- To find the y-intercept, let $x = 0$ and solve for y.

$$0 = |y| - 2$$

$$|y| = 2$$

$$y = \pm 2$$

So, the y-intercepts are (0, 2) and (0, -2).

Ex. Find the *x*- and *y*-intercepts of the graph of

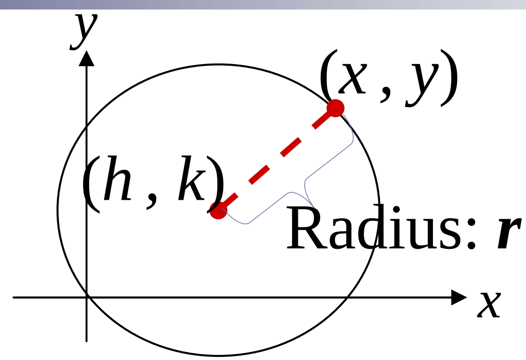
1. $|x-4y|=8$
2. $x=y^3-2$

Ex. Find the other endpoint of the line segment thatr has endpoint (5,1) and midpoint (9,3).

CIRCLES, THEIR EQUATIONS, AND THEIR GRAPHS

Definition of a Circle الدائرة

A ***circle*** is a set of all points in a plane that are a fixed distance from a fixed point called the ***center***. The fixed distance from the circle's center to any point on the circle is called the ***radius***.



The distance between (h, k) and (x, y)

$$\sqrt{(x-h)^2+(y-k)^2}=r$$

Squaring both sides of this equation yields:

Standard Form of the Equation of a Circle $(x-h)^2+(y-k)^2=r^2$.

Ex.6 Write the standard form of the equation of the circle with center (0, 0) and radius 2.

Solution: Let center $(h, k) = (0, 0)$ and let radius $\boldsymbol{r} = 2$.

$(x - h)^2 + (y - k)^2 = r^2$ Standard form of a circle's equation.

$(x - 0)^2 + (y - 0)^2 = 2^2$ Substitute the given values.

$x^2 + y^2 = 4$ Simplify.

Ex.7 Write the standard form of the equation of the circle with center (-2, 3) and radius 4.

Solution: $(x - h)^2 + (y - k)^2 = r^2$ Standard form of a circle's equation.

$[x - (-2)]^2 + (y - 3)^2 = 4^2$ Substitute the given values.

$(x + 2)^2 + (y - 3)^2 = 16$ Simplify.

Ex.7 Find the center and radius of the circle whose equation is

$$(x - 2)^2 + (y + 4)^2 = 9$$

Solution: $(x - 2)^2 + [y - (-4)]^2 = 3^2$

Center $(h, k) = (2, -4)$ and Radius $\boldsymbol{r} = 3$

General Form of the Equation of a Circle

The ***general form of the equation of a circle*** is

$$\boldsymbol{x^2 + y^2 + Ax + By + C = 0.}$$

$$(x - h)^2 + (y - k)^2 = r^2$$

by squaring and combining like terms

$$\boldsymbol{x^2 + y^2 + Ax + By + C = 0.}$$

by completing the square on *x* and on *y*.

$$(x - h)^2 + (y - k)^2 = r^2$$

Ex.8 Find the center and radius of the circle whose equation is

Solution: $x^2 + y^2 + 4x - 6y - 23 = 0$

$x^2 + 4x + y^2 - 6y = 23$ Complete the squares on x and on y

$$[x^2 + 4x + (2)^2] + [y^2 - 6y + (-3)^2] = 23 + 4 + 9$$

$$(x + 2)^2 + (y - 3)^2 = 36$$

$$[x - (-2)]^2 + (y - 3)^2 = 6^2$$

Center $(h, k) = (-2, 3)$ and Radius $\boldsymbol{r} = 6$

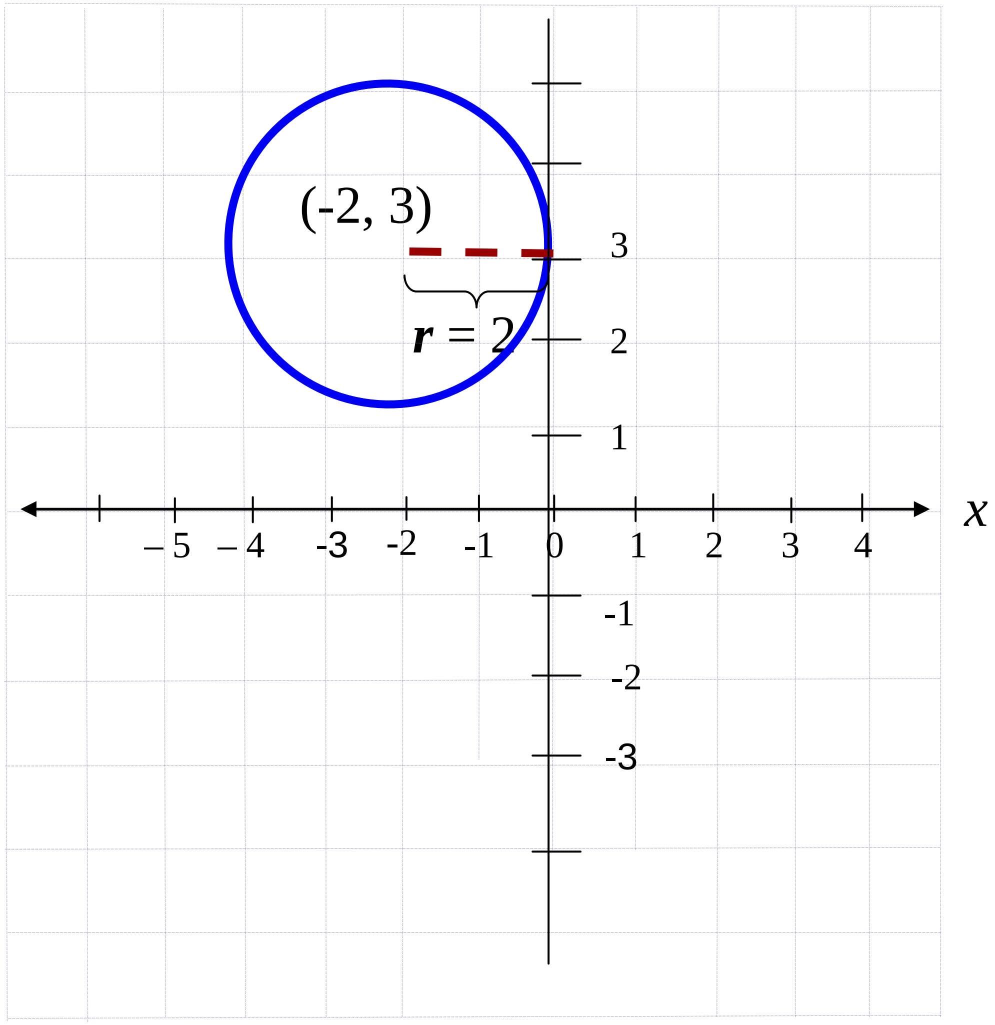
Ex.9 Find an equation of a circle that has its center at (-2, 3), and is tangent to the y-axis. Write your answer in standard from.

Solution:

$$(x-h)^2 + (y-k)^2 = r^2$$

$$[x-(-2)]^2 + (y-3)^2 = 2^2$$

$$(x+2)^2 + (y-3)^2 = 4$$



Ex. Find an equation of a circle that has center (4,1) and radius $r = 2$

Ex. Find the center and radius of the circle whose equation is

$$(x - 3)^2 + (y - 9)^2 = 144$$

Ex. Find the center and radius of the circle whose equation is

$$9x^2 + 9y^2 - 6y - 17 = 0$$

Ex. Find the equation of the circle that has a diameter with end points (2,3) and (-4,11).

Ex. Find the equation of the circle that is tangent to both axis, has its center in the third quadrant, and has a diameter of $\sqrt{5}$